Noncontact temperature measurement. II. Least squares based techniques

Mansoor A. Khan,[a] Charly Allemand, and Thomas W. Eagar
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

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A technique for the noncontact measurement of temperatures is described. The technique is based on the measurement of the emitted intensity at multiple wavelengths and the simultaneous calculation of emissivity and temperature through the use of least squares curve fitting techniques. The technique is shown to make no assumptions regarding the emissivity of the target except that it be possible to model it with an analytic function. A theory is developed to predict the errors in the estimation of temperatures based on both linear and nonlinear least-squares techniques. It is shown that the maximum error in the predicted temperature is related to the noise in the measured intensities in a quantifiable manner. It is shown using computer simulations that the theory and algorithms developed here can predict both the temperatures and the uncertainty associated with each temperature prediction with a very high accuracy. An instrument was developed to test this theory. This instrument, referred to as the MITTMA, was used to measure absolute temperatures of various sources from 900 °C to 2300 °C with an average accuracy of approximately 0.5%.

I. INTRODUCTION

Temperature is one of the most basic and important parameters in the analysis and control of both scientific and commercial processes and phenomena. As we try to achieve a better understanding of phenomena of scientific and industrial interest and as we try to better control and predict these phenomena, accurate measurement of temperature becomes more critical. Generally, with current technology the most accurate method of making these measurements is to use a contact technique such as thermocouples. Unfortunately, there are many phenomena where it is not desirable to use an intrusive or contact technique to make temperature measurements. This may be because the target is not easily accessible, because it is in an environment hostile to the measuring instrument or because it could be contaminated by the measuring instrument. One alternative to contact temperature measurement is the use of pyrometric techniques to achieve noncontact (and nonintrusive) temperature measurements.

Pyrometric noncontact temperature measurement has been known for at least sixty years. Such measurements makes use of the Planck radiance formula:

$$N(\lambda)=\epsilon(\lambda)C_1\lambda^{-5}[\exp(C_2/\lambda T)-1]^{-1},$$

where N is the spectral radiance emission, $\epsilon(\lambda)$ is the emissivity of the material at wavelength λ, T is the temperature, and C_1 and C_2 are Planck radiation constants. Strictly speaking, many pyrometry techniques use the Wien approximation to the Planck radiance formula which is

$$N(\lambda)=\epsilon(\lambda)C_1\lambda^{-5}\exp(-C_2/\lambda T).$$

By measuring the spectral radiance emission, N, at a wavelength λ, and by supplying the appropriate values for ϵ, C_1 and C_2, an estimate of the temperature of the source can be calculated.

Historically there have been two distinct techniques for such calculations. The older technique is sometimes known as ratio pyrometry. This technique involves measurement of radiance emission at a number of different wavelengths in an attempt to eliminate the emissivity term by making ratios of the measured radiances. The other, newer, method of pyrometry is known as multiwavelength pyrometry. In this method, again, measurements of the spectral radiance emissions of the object are taken at several wavelengths. The processing of this data is then performed by a variety of techniques, the most accurate of which have proven to be least-squares-based multiwavelength techniques. These involve fitting the radiance emission data to an assumed radiance functional form. Ratio techniques have not, in general, provided adequately accurate temperature estimates for broad industrial usage.[1] The often large inaccuracies of the ratio techniques have been attributed to the fact that they require unrealistic assumptions to be made about the nature of the emissivity, ϵ, in the Planck formula. Ratio techniques assume that both (in the case of two color) or certain (in the cases of three or four color) of the emissivities at the measured wavelengths be equal. Multiwavelength, particularly least-squares-based, techniques have been somewhat more successful, largely because they more reasonably assume a wavelength-dependent emissivity function, rather than that all or some of the emissivities are equal. In fact, with certain materials, these techniques have been reported to be accurate to within 1%. With other materials, however, results have been unsatisfactory. It has been assumed by previous investigators that the unsatisfactory results have been due to two sources: first, incorrect form or lack of function; and second, so-called "correlation effects" due to the inability of curve-fitting routines to distinguish in certain circum-

[a]Present address: Wyman Gordon Co., 244 Worcester St., N. Grafton, MA 01536.

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stances between changes in emissivity and changes in temperature.[2–5]

We have developed in the laboratory a new multiwavelength technique, which appears to be more accurate than thermocouples. Typical measurement errors can be significantly less than 1% of the absolute temperature. A unique feature of this method is the ability to estimate the maximum error in the temperature measurement, thus providing the operator with confirmation of the accuracy of this temperature sensor.

The system uses least-squares-based multiwavelength pyrometry techniques. Radiances from an object are detected by a spectrograph. These radiance values are input to a computer that curve-fits these signals to a theoretical emissivity function that has variable coefficients of the emissivity function and the temperature.

II. THEORY

All multiwavelength pyrometry techniques work in the same manner, i.e., some sort of functional form is chosen to model the emissivity function and is then combined with the Planck radiation law to give a model for the measured radiance. This model is then curve fit to the measured radiance data to get values for the unknown parameters, one of which is temperature.

The curve fit can be performed in one of two ways; the first method uses Lagrangian interpolation, i.e., n data points are fit with a function with n degrees of freedom. For example, if 10 data points are measured, a 10th order polynomial is used to fit the data. This method has the disadvantage that fitting more than 3 or 4 data points can lead to severe over fitting! In fact, Coates[6] showed that such a technique could lead to errors of around 1000% for as few as 10 measurements. Fortunately the second multiwavelength technique does not suffer from this severe limitation. This approach uses a least-squares technique to fit the data to the model. The least-squares technique requires that radiance measurements be made at many more wavelengths than there are degrees of freedom in the model. These extra data then have the affect of reducing the noise level and random fluctuations in the measurements.

The least-squares-based pyrometric techniques can themselves be divided into two classes, namely linear and nonlinear least squares. Linear least-squares techniques require that the fitting function be linear in the unknown parameters, i.e., a polynomial. Linear least squares has the advantage of being fast and the disadvantage of being limited to using exponential polynomials to model the emissivity of the target material. The non-linear least squares can use any type of function that is deemed appropriate by the user but it has the disadvantage of being slow. The following sections describe both of these least squares based multiwavelength techniques in detail.

A. Linear least-squares method

The linear least-squares method used linear algebra techniques to solve for the values of the unknown parameters. Consider the problem of fitting some data $(b_1,b_2...,b_n)$ to a polynomial of degree m. If $n > m$, we say that the problem is "overdetermined." If, for example, $n = 3$ and $m = 1$ then we will have to solve the following system of equations

$$a_0 + a_1y_1 = b_1,\ a_0 + a_1y_2 = b_2,\ a_0 + a_1y_3 = b_3.$$

Here, the unknown parameters are the a_i coefficients; because the equations are linear with respect to these parameters, we can derive the following equations. The overspecified system of equations is inconsistent and in general has no solution. One can get around this problem by defining an error function(E) and then minimizing it; more precisely, we will minimize the square root of the sum of the squares of the error:

$$\mathrm{E} = \mathrm{SQRT}[(a_0 + a_1y_1 - b_1)^2 + (a_0 + a_1y_3 - b_3)^2]$$

or in matrix form:

$$\mathrm{E} = \|\mathrm{AX} - \mathrm{B}\|_2,$$

where

$$\mathrm{A} = \begin{pmatrix} 1 & y_1 \\ 1 & y_2 \\ 1 & y_3 \end{pmatrix},\ \mathrm{X} = \begin{pmatrix} a_0 \\ a_1 \end{pmatrix},\ \mathrm{B} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix},$$

and so

$$E^2 = \|AX - B\|_2^2 = \sum_{i=j}^{n} \sum_{j=0}^{m-1} (a_jy_i - b_i)^2.$$

The minimum of E^2 will be found by solving the system of equations

$$\frac{\delta E^2}{\delta a_j} = 0$$

and in matrix form

$$A^TAX - A^TB = 0.$$

In general the method of least squares involves finding the minimum of $\|AX - B\|$. Thus,

$$\rho = \|AX - B\|$$

and at the minimum

$$\rho_{LS} = \|AX_{LS} - B\|.$$

From trigonometry, we can define

$$\sin\theta = \frac{\rho_{LS}}{\|B\|}.$$

$$\cos\theta = \frac{AX - B}{\|B\|},$$

then it can be shown that in the worst case

$$\frac{\|\delta X\|}{\|X\|} \leqslant u_B(X) \cdot \frac{\|\delta B\|}{\|B\|}, \tag{1}$$

$$\frac{\|\delta X\|}{\|X\|} u_A(X) \cdot \frac{\|\delta A\|}{\|A\|}, \tag{2}$$

where $u_s(t)$ is called the "condition number" of t with respect to s, and

$$u_B(X)=\frac{u(A)}{\cos\theta}, \tag{3}$$

$$u_A(X)=u^2(A)\tan\theta, \tag{4}$$

$$u(A)=\|A\|\cdot\|A^+\|=\sigma_1/\sigma_n,$$

where A^+ is the pseudoinverse of A, σ_1 is the largest singular value of A, and, σ_n is the smallest singular value of A.

Equations (1) and (2) describe the effects of perturbations in the data B and the coefficients matrix A on the solution X. In the simplest terms the condition numbers can be thought of as amplification factors for the translation of a perturbation in B or A on to a perturbation in X_{LS}. Whereas θ is a measure of how closely the equations AX can match the data B.

For the case being discussed here, perturbations in B would arise from noise in the measured data. Perturbations in A would be caused primarily by roundoff error during the LS calculations and would not be very significant unless A was terribly conditioned (i.e., had a very large condition number).

So far we have made no mention of the number of data points, n, with which we are dealing. In general, the actual condition number for perturbations in the data is lower by an approximate multiple of $n^{-1/2}$, i.e.,

$$u_B(X)\sim\frac{1}{n^{1/2}}\frac{u(A)}{\cos\theta}.$$

It should be kept in mind that this is only an order of magnitude approximation.

The measured data (f_i) for multiwavelength pyrometry (assuming perfect measurements) is the product of two functions:

$$f_i=\epsilon(\lambda_i)C_1\lambda_i^{-5}[\exp(C_2/\lambda_i T)-1]^{-1},$$

where $\lambda_i=i$th wavelength and $\epsilon(\lambda)=$ emissivity.

In general we do not know much about $\epsilon(\lambda)$, but we can make some simplifying assumptions: (i) $\epsilon(\lambda)$, $\epsilon'(\lambda)$, $\epsilon''(\lambda)$ are continuous over some sufficiently small (λ_1,λ_2), (ii) $\epsilon(\lambda)$ is single valued over some sufficiently small $(\lambda_1\,\lambda_2)$. Given these assumptions we can approximate the emissivity with any suitable analytic function such as a polynomial in λ or an exponential raised to a polynomial in λ. This function f is to be modeled by a fitting function g. At this point, however, f is nonlinear in at least one of the unknowns, the temperature T. So we rearrange it, after making the Wien approximation, to get

$$\ln(C_1^{-1}f_i\lambda_i^5)=\ln\,\epsilon-C_2\lambda_i^{-1}T^{-1}.$$

All the quantities on the left side of this equation are known and on the right side the unknowns are ϵ and temperature. If we define

$$g=\ln\left[\exp\left(\sum_{i=0}^{m-2}\alpha_i\lambda^i\right)\right]-\alpha_{m-1}\lambda^{-1},\quad \alpha_{m-1}=C_2/T,$$

the emissivity is then modeled by

$$\epsilon=\exp\left(\sum_{i=0}^{m-2}\alpha_i\lambda^i\right)$$

and the unknown parameters are the $m-1$ coefficients $(\alpha_0,\alpha_1,\ldots,\alpha_{m-2})$ of the $m-2$ degree polynomial and the temperature T. We now have g as a linear function of the unknown parameters.

Translating this into the terms used to describe the linear LS technique we get

$$B=\begin{pmatrix}N_1\\N_2\\\cdot\\\cdot\\\cdot\\N_n\end{pmatrix},\quad N_i=\ln(C_1^{-1}f_i\lambda_i^5),\quad X=\begin{pmatrix}\alpha_0\\\alpha_1\\\cdot\\\cdot\\\cdot\\\alpha_{m-1}\end{pmatrix},$$

where

$$A=\begin{pmatrix}1&\lambda_1&\lambda_1^2&\cdot&\cdot&\cdot&\cdot&\lambda^{m-2}&\lambda_1^{-1}\\1&\lambda_2&\lambda_2^2&\cdot&\cdot&\cdot&\cdot&\lambda_2^{m-2}&\lambda_2^{-1}\\\cdot&&&&&&&&\\\cdot&&&&&&&&\\\cdot&&&&&&&&\\1&\lambda_n&\lambda_n^2&\cdot&\cdot&\cdot&\cdot&\lambda_n^{m-2}&\lambda_n^{-1}\end{pmatrix}.$$

Using this method, the values of the unknown parameters X can be evaluated. As shown, the value for the temperature is inversely proportional to the value of α_{m-1}:

$$T=C_2/\alpha_{m-1}.$$

The error is determined from the condition number and the closeness of fit as determined by θ. The larger the condition number, the more inaccurate the estimate of the temperature will be for a given noise level.

B. Nonlinear least-squares method

If the emissivity is not a linear exponential function of the wavelength, nonlinear least-squares method are used. The technique is as follows.

Consider the case where one is trying to fit the data f with the function N. From the above

$$X^2(\alpha_j)=1/2\sum_{i=1}^{n}[N_i(\alpha_j,\lambda_i)-f_i]^2 d\lambda_i,$$
$$(j=0,1,\ldots,m-1).$$

One now wishes to evaluate the parameters α_j at the minimum of X^2. If N is nonlinear in the α_j, then the system of equations resulting from differentiating X^2 with respect to the α_j will be nonlinear; and linear techniques such as Gaussian elimination, cannot be used to solve this system of equations. One would then have to look for some sort of iterative technique to locate the minimum in X^2.

The solution technique used here is described by Bevington.[7] The algorithm used is termed the gradient expansion algorithm and was first presented by Marquardt. This algorithm searches the surface defined by X^2 over the m

dimensions for a minimum. When X^2 is large this method follows the gradient of X^2. As X^2 gets smaller, the algorithm linearizes X^2 by a Taylor expansion. Finally when X^2 is very small the algorithm does a linear fit to the data. How easy this task will be depends upon the shape of the surface defined by X^2.

To study the behavior of X^2, we expand it in a Taylor series around the minimum following Bard.[8] We start by defining the following

$$\beta=\begin{pmatrix}\alpha_0\\ \alpha_1\\ \cdot\\ \cdot\\ \cdot\\ \alpha_{m-1}\end{pmatrix},$$

where β' is the true value of beta,

$$J=\frac{\partial X^2}{\partial \alpha_i},\quad H=\frac{\partial^2 X^2}{\partial \alpha_i \partial \alpha_j},$$

where J' is the gradient at $\beta=\beta'$, and $H'=$ Hessian at $\beta=\beta'$:

$$\delta\beta=\beta-\beta'.$$

Assuming X^2 is quadratic in a sufficiently small neighborhood around $\beta=\beta'$, then, expanding X^2 in a Taylor series, we can write

$$X^2(\beta)\sim X^2(\beta')+J'^T\delta\beta+1/2\delta\beta^T H'\delta\beta.$$

If β' is an unconstrained minimum, then $J(\beta')=0$ and $X^2(\beta')=0$ by definition. The above equation then reduces to

$$X^2(\beta)\geqslant 1/2\delta\beta^T H'\delta\beta.$$

This inequality describes an m dimensional ellipsoid for each value ν of $X^2(\beta)$. The ellipsoids are concentric and centered around $\delta\beta=0$. The ellipsoid of interest covers a volume of uncertainty in the parameter space, i.e., we cannot determine any of the parameters with an error less than L multiplied by the smallest axis of the ellipsoid. The quantity L is a constant on the order of $n^{1/2}$. We can predict L only in order of magnitude terms.

Our interest, however, is more in the magnitude of the largest possible error, i.e., we wish to locate the points on the ellipsoid farthest from the center. Thus we wish to find the vector $\delta\beta$ which maximizes

$$\delta\beta^T H'\delta\beta=2\nu.$$

It can be shown, using the method of Lagrange multipliers, that the desired vector is an unnormalized eigenvector of H' with an eigenvalue of η. So,

$$\delta\beta^T\delta\beta=2\nu/\eta.$$

Each one of these eigenvectors forms an axis of the ellipsoid, and so the worst error will lie in the direction corresponding to the eigenvector with the smallest eigenvalue:

$$\|\delta\beta\|=[2\nu/\eta_{\text{smallest}}]^{1/2}.$$

In general,

$$\|\delta\beta\|\leqslant[2\nu/\eta_{\text{smallest}}]^{1/2}$$

and in particular,

$$\|\delta\beta\| \text{ is of the order of } \left[\frac{2\nu}{n\eta_{\text{smallest}}}\right]^{1/2}.$$

If the data f_i are measured with an uncertainty Δ_i, then we can tolerate a change of

$$\frac{1}{2}\sum_{i=1}^{n}\Delta_i^2$$

in X^2 without having any reason to prefer one value of $\delta\beta$ over another, as long as they all lie in the region defined by

$$\nu+\frac{1}{2}\sum_{i=1}^{n}\Delta_i^2.$$

This means that the m dimensional ellipsoid described earlier is now defined by

$$\nu+\frac{1}{2}\sum_{i=1}^{n}\Delta_i^2$$

rather than just by ν. So the error in the parameter estimation is now given by

$$\|\delta\beta\|=\left(\frac{2\nu+\sum_{i=1}^{n}\Delta_i^2}{\eta_{\text{smallest}}}\right)^{1/2}.$$

In general

$$\|\delta\beta\|\leqslant\left(\frac{2\nu+\sum_{i=1}^{n}\Delta_i^2}{\eta_{\text{smallest}}}\right)^{1/2}$$

and in particular $\|\delta\beta\|$ is of the order of

$$\left(\frac{2\nu+\sum_{i=1}^{n}\Delta_i^2}{n\eta_{\text{smallest}}}\right)^{1/2}.$$

Eigenvalue calculations are difficult and sometimes unstable, so we wish to avoid them if possible. Fortunately, for the cases we deal with here, H' is a Hermitian matrix. Hermitian matrices have the property that the singular values are equal to the absolute values of the eigenvalues. Additionally the singular vectors are equal to the eigenvectors if the matrix is positive semidefinite, otherwise, the singular and eigenvectors agree to within a factor of $(-1)^{1/2}$. So, we can write the equation as:

$$\|\delta\beta\| \text{ is of the order of } \left(\frac{2\nu+\sum_{i=1}^{n}\Delta_i^2}{R\sigma_{\text{smallest}}}\right)^{1/2},$$

where the σ_{smallest} is the smallest singular value of H.

In plain English what these equations are saying is simply this: "for a given model of the emissivity we can predict the maximum possible and approximate errors in

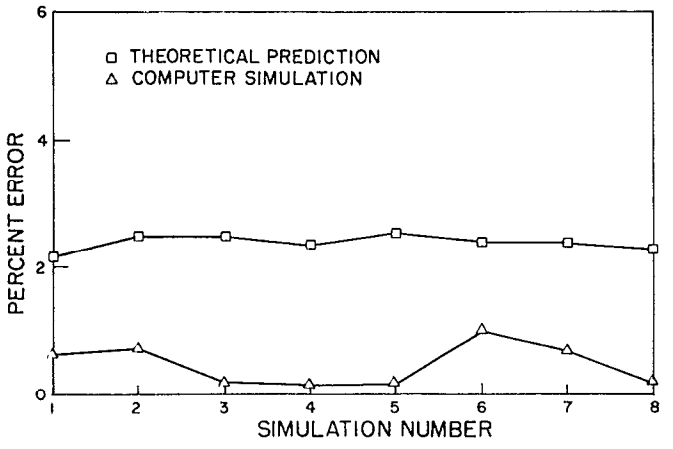

FIG. 1. Linear least-squares simulation results for platinum at 1600 K and 0.1% rms noise. Results from eight simulations all with the same rms noise but different noise vectors are reported.

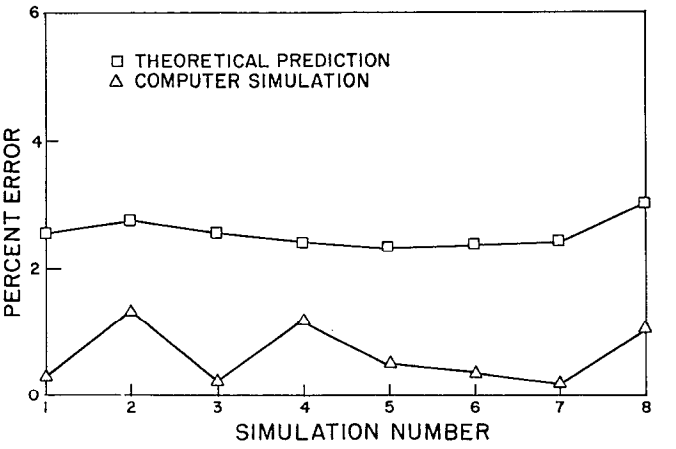

FIG. 2. Linear least-squares simulation results for molybdenum at 1600 K and 0.5% rms noise. Results from eight simulations all with the same rms noise but different noise vectors are reported.

the predicted temperature for each set of measurements." More precisely, we have shown that both for the linear and nonlinear methods the error in the predicted temperature is smaller than some multiple of the perturbation of the radiance data due to the measuring instrument, i.e., noise.

The experimental results presented later will show that we can measure temperatures with an accuracy better than that of thermocouples, and if these techniques fail to predict the correct temperature we have a very good estimate of the uncertainty in the predicted temperature.

An additional advantage of these techniques is that they can estimate the emissivity of the target simultaneously with the temperature. This aspect can prove to be very important when a fast method of generating reflectivity versus wavelength or emissivity versus wavelength data is required. It is presently both difficult and time consuming to generate such data.

III. COMPUTER SIMULATIONS

A computer model was developed to test the ability of the aforementioned techniques to predict temperatures, emissivities and the associated errors. The following flow chart describes the simulation procedure:

1. Pick a temperature and a wavelength range.
2. Pick published emissivity data.
3. Use the Planck law and the emissivity data to generate radiances.
4. Add pseudorandom noise.
5. Pick a fitting function for the emissivity.[9]
6. Calculate the temperature *and* the emissivity using both linear and nonlinear least-square techniques.
7. Repeat for different temperatures and emissivities.

Figures 1 and 2 present the results of the linear least-squares simulations for platinum (0.1% rms noise) and molybdenum (0.5% rms noise) at 1600 K. Each figure depicts the results of eight simulations, each of which used a different noise vector. It should be pointed out that even though each simulation used different noise vectors, the rms value of these vectors was about the same for each set of eight simulations. These figures show that the theory is indeed capable of predicting the error in the parameter estimation. Similarly Figs. 3 and 4 present the results of six nonlinear least squares simulations for each of two different noise levels for platinum (0.1% rms) and molybdenum (0.5% rms) at 1600 K. Here again we see that the error is very well predicted by the theory.

A large number of these simulations were performed, using both techniques, for different materials, temperatures, and noise levels.

IV. EXPERIMENTAL PROGRAM

The experimental program was developed to test the validity of the theory and to develop an instrument to implement the algorithms developed here. Figure 5 depicts a schematic of the instrument developed here and named the MITTMA (M.I.T. Temperature Micro Analyzer). The MITTMA is based upon a photoradiometer and a personal computer. References 10 and 11 describe the instrument in greater detail.

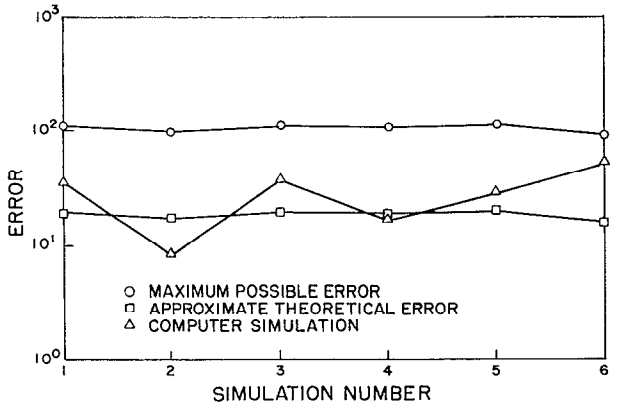

FIG. 3. Nonlinear least-squares simulation results for platinum at 1600 K and 0.1% rms noise. Results from six simulations all with the same rms noise but different noise vectors are reported.

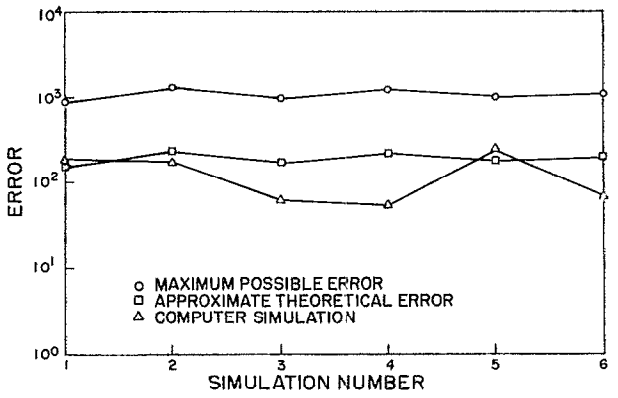

FIG. 4. Nonlinear least-squares simulation results for molybdenum at 1600 K and 0.5% rms noise. Results from six simulations all with the same rms noise but different noise vectors are reported.

The sensitivity of the silicon detectors provides a lower limit on the measurable temperature of around 900 °C in some cases. Because the theory developed here has no temperature limitations a detector such as InSb or PbS could be used to measure temperature down to around 500 °C. The previous experimental program described herein is limited to the range of temperatures that can be accessed by a silicon detector based instrument.

Table I lists the results of measurements on a number of different sources in the temperature range 913 to 2625 K.

In addition to using the MITTMA, the temperature of these sources was independently measured with a thermocouple. Table I lists the thermocouple and the MITTMA temperatures and the percent difference between them. As can be seen here the MITTMA measurements agree very well with the thermocouple results except at the two lowest temperatures where the signal-to-noise ratio was extremely low.

In order to simulate the effect of emissivity changes, i.e., a shiny surface being oxidized, a black body radiation source viewed through filters was used. The transmissivities of these filters are shown in Fig. 6. Filter B can be thought of as a shiny metal surface that becomes oxidized and changes to filter A.

As far as the MITTMA was concerned, these transmissions can be thought of as emissivities of a real body at the temperature of the black body. Again, as Table I

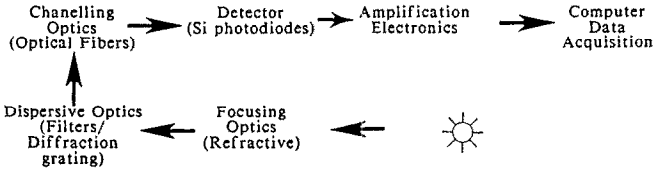

FIG. 5. Schematic of the MITTMA.

TABLE I. The results of measurements on a number of different sources in the temperature range 913 to 2625 K.

Source	Thermocouple Temperature	MITTMA Temperature	Difference (%)
Tungsten Strip	2625	2617	0.3
Black body with filter A	1234	1241	0.6
Black body with filter B	1233	1251	1.5
Pt strip	1255	1251	0.32
Pt strip	1121	1155	3.0
Pt strip	913	970	6.24

shows, this massive change in the emissivity did not affect the accuracy of the predicted temperature very substantially.

The behavior of the MITTMA over time was studied by taking a number of readings of the sources at a constant temperature and comparing the results. Figure 7 shows the results of this experiment for the resistance treated Pt strip. This figure plots the percent error (compared to the thermocouple) for nine sets of experiments. The figure also presents the approximate error as predicted by the theory developed here. As can be seen here these two results match extremely well! The average theoretical error is about 0.7% as compared to the average actual error of about 0.8%. Because these techniques can predict both the temperature and the emissivity, each one of the temperatures shown here has an emissivity versus wavelength curve associated with it. Figure 8 depicts four of these curves for the Pt measurements of Fig. 7.

Specifically the emissivity curves corresponding to temperature errors of − 16, − 8, − 1, and + 4K are presented here. If these measurements are correct, then one would expect the true emissivity of Pt to fall somewhere between the − 1 and + 4 curves (the bottom two in Fig. 8). When this data is compared to the published data on Pt we find that the published emissivities for Pt with similar surface conditions and temperatures does indeed fall be-

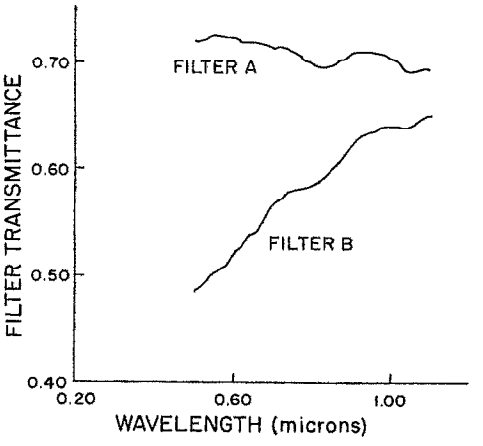

FIG. 6. Transmissivities of the filters used to simulate the effects of oxidation of a surface.

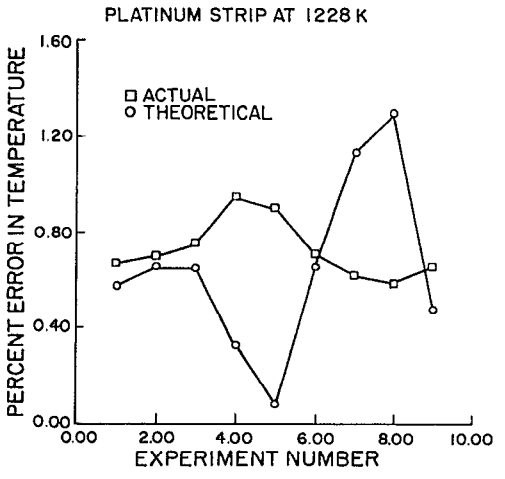

FIG. 7. Comparison of the experimental and theoretically predicted errors in the estimation of temperatures for the platinum strip source.

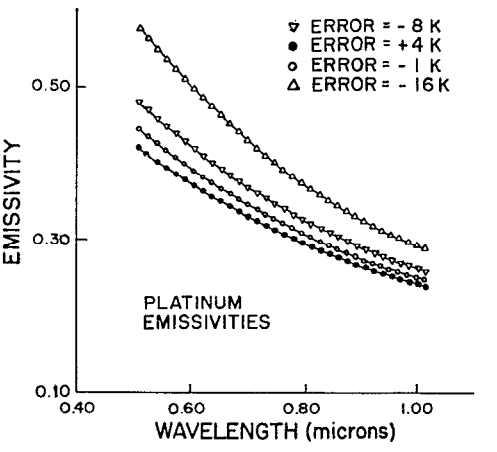

FIG. 8. Calculated spectral emissivities for platinum. The four curves correspond to four separate measurements and therefore to four distinct temperature errors. The true emissivity of platinum would lie between the curves corresponding to the − 1 and + 4 K errors.

tween these two curves and close to the one corresponding to an error of − 1K.

V. DISCUSSION

The theory developed here has been shown to be capable of calculating true temperatures of any material without requiring any *a priori* information about the source. The theory is also capable of predicting the approximate and absolute maximum error in the predicted temperature as well as the emissivity as a function of wavelength. All this is done by making radiance measurements at a number of wavelengths.

A commercial instrument has been adapted to implement this theory and has been used to make measurements on a number of sources 900 to 2300 °C with an accuracy of around 0.5%.

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