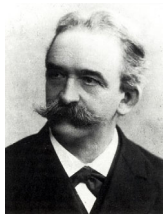
Chapter 9

Analysis of statically indeterminate structures by the **force method**

In 1874, Christian Otto Mohr formalized the idea of a statically indeterminate structure.

Mohr was an enthusiast for graphical tools and developed a method for visually representing stress.

In 1882, he famously developed the graphical method for analyzing stress known as Mohr's circle and used it to propose an early theory of strength based on shear stress.



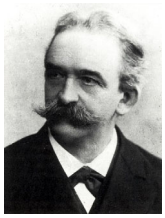
1

Chapter 9

Analysis of statically indeterminate structures by the **force method**

He also developed the Williot-Mohr diagram for truss displacements and the Maxwell-Mohr method for analyzing statically indeterminate structures.

He retired in 1900 yet continued his scientific work in Dresden until his death on October 2, 1918.



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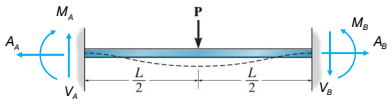
Analysis of statically indeterminate structures by the **force method**

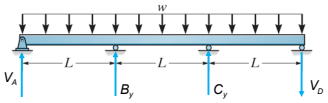
Objectives: To show how to apply the force or flexibility method to analyze statically indeterminate ***beams***.

6 Unknowns

3 Equations of equilibrium



4 Unknowns



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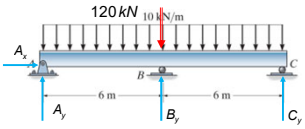

$\overset{+}{\rightarrow}\sum F_x = 0 = A_x$ (1)

$\circlearrowleft^+ \sum M_A = 0 = -120\,kN(6\,m) + B_y(6m) + C_y(12\,m)$ (2)

$\circlearrowleft^+ \sum M_B = 0 = -A_y(6m) + C_y(6\,m)$ (3)

$\circlearrowleft^+ \sum M_C = 0 = 120\,kN(6\,m) - B_y(6m) - A_y(12\,m)$ (4)

These equations are ***not*** independent.

4

Analysis of statically indeterminate structures by the **force method**

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(2) + (4) = (3)

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5

Analysis of statically indeterminate structures by the **force method**

- A structure of any type is classified as ***statically indeterminate*** when the ***number of unknowns*** exceeds the number of available ***equilibrium equations***.
- Many structures designed today are statically indeterminate.
- This is particularly true for ***reinforced concrete*** buildings since the columns and beams are poured as continuous members through the joints and over supports.
- In this section, we will discuss the merits of using indeterminate structures, and we will describe two fundamental ways in which they may be analyzed.

6

Analysis of statically indeterminate structures by the **force method**

Advantages and Disadvantages

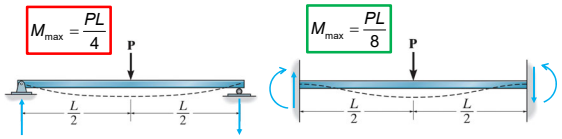
- Although the analysis of a ***statically indeterminate*** structure is more involved than that of a statically determinate one, there are usually several very important reasons for choosing this type of structure for design.
- Most importantly, for a given loading, the maximum stress and deflection of an ***indeterminate structure*** are generally ***smaller*** than those of its statically determinate counterpart.

7

Analysis of statically indeterminate structures by the **force method**

Advantages and Disadvantages

- Consider simply supported and fixed-supported beams, with the same point load **P** applied at the center.



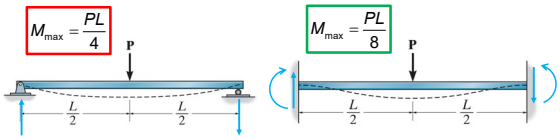
- The simply supported beam will be subjected to ***twice*** the moment as the statically indeterminate, fixed-supported beam.

8

Analysis of statically indeterminate structures by the **force method**

Advantages and Disadvantages

- Consider simply supported and fixed-supported beams, with the same point load **P** applied at the center.



- As a result, the fixed-supported beam has ***¼ the deflection*** and ***½ the stress*** at the center of the one that is pin support

9

Analysis of statically indeterminate structures by the **force method**

Advantages and Disadvantages

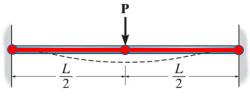
- Statically indeterminate structures tend to ***redistribute their load to their redundant supports*** in cases where faulty design or overloading occurs.
- In these cases, the structure maintains ***stability*** and prevents collapse.
- This is particularly important when ***sudden*** lateral loads, such as wind or earthquake, are imposed on the structure.

10

Analysis of statically indeterminate structures by the **force method**

Advantages and Disadvantages

- Consider again a fixed-end beam



- As **P** is increased, the beam's material at the walls and the center of the beam begins to ***yield*** and forms localized "plastic hinges."
- This causes the beam to deflect as if it were hinged or pin-connected at these points.

11

Analysis of statically indeterminate structures by the **force method**

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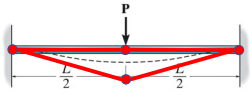

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12

Analysis of statically indeterminate structures by the **force method**

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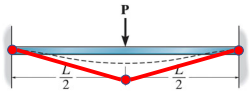


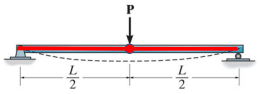
- Although the deflection becomes large, the fixed supports will develop ***horizontal force and moment reactions*** that will hold the beam and thus prevent it from totally collapsing.

13

Analysis of statically indeterminate structures by the **force method**

Advantages and Disadvantages

- Consider again a simply supported beam



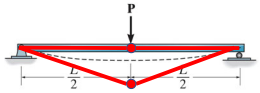
- As **P** is increased, a "plastic hinge" forms only at the center of the beam
- Due to the large vertical deflection, the supports will not develop the horizontal force and moment reactions that are necessary to prevent total collapse.

14

Analysis of statically indeterminate structures by the **force method**

Advantages and Disadvantages

- Consider again a simply supported beam



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15

Analysis of statically indeterminate structures by the **force method**

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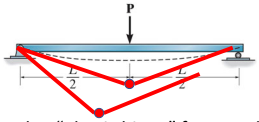

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16

Analysis of statically indeterminate structures by the **force method**

Advantages and Disadvantages

- Although a statically indeterminate structure can support a loading with ***thinner members*** and with ***increased stability***, there are cases when these advantages may instead become disadvantages.
- The cost savings in material must be compared with the added cost necessary to ***fabricate the structure and construct its supports and joints***.

17

Analysis of statically indeterminate structures by the **force method**

Advantages and Disadvantages

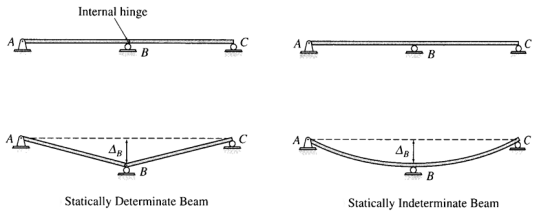
- Although a statically indeterminate structure can support a loading with ***thinner members*** and with ***increased stability***, there are cases when these advantages may instead become disadvantages.
- Also, at redundant support reactions, one must be very careful to prevent ***relative displacement*** of the supports, or changes in length caused by ***fabrication errors or temperature***, since these effects will introduce internal stress in the structure.

18

Analysis of statically indeterminate structures by the **force method**

Advantages and Disadvantages

- For example, consider the stress induced into the beams due to support ***settlements***

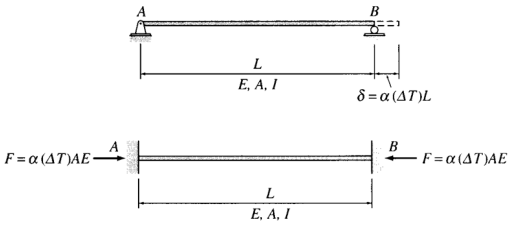


19

Analysis of statically indeterminate structures by the **force method**

Advantages and Disadvantages

- For example, consider the stress induces into the beams due to ***temperature***



20

Analysis of statically indeterminate structures by the **force method**

Advantages and Disadvantages

- It is worth mentioning a case where engineers have designed a structure that has the advantages of being statically ***indeterminate***. Yet, the analysis of it is reduced to one that is statically ***determinate***.
- It is a ***trussed cantilever bridge***, such as the one shown here.

21

Analysis of statically indeterminate structures by the **force method**

Advantages and Disadvantages

- It consists of two cantilevered sections *AB* and *CD*, and a center suspended span *BC* that was floated out and lifted in place.
- This span was pinned at *B* and suspended from *C* by a primary vertical member *CE*.

22

Analysis of statically indeterminate structures by the **force method**

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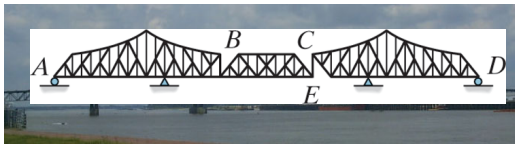
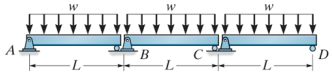


23

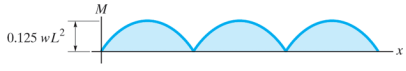
Analysis of statically indeterminate structures by the **force method**

Advantages and Disadvantages

- If the bridge were constructed as three simple beams resting on supports at *A*, *B*, *C*, and *D* it would look like this:



- Then the uniformly distributed loading on each beam would produce a maximum moment of **0.125 *wL*²** in the beam.

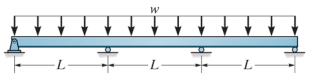


24

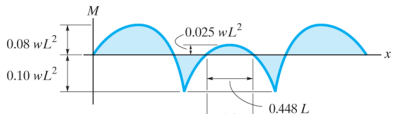
Analysis of statically indeterminate structures by the **force method**

Advantages and Disadvantages

- If instead a *continuous beam* were used



- Then a statically indeterminate analysis would produce a moment diagram that has a maximum moment in the beam of **0.10 *wL*²**.

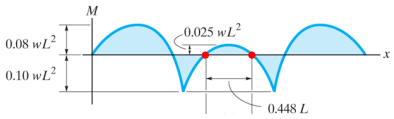


25

Analysis of statically indeterminate structures by the **force method**

Advantages and Disadvantages

- Although this is a 20% reduction in the maximum moment, unfortunately any ***slight settlement of one of the bridge piers*** would introduce larger reactions at the supports, and larger moments in the beam.
- To circumvent this problem, pins can be used on the span at the points *E* and *F*, where the moment is zero.

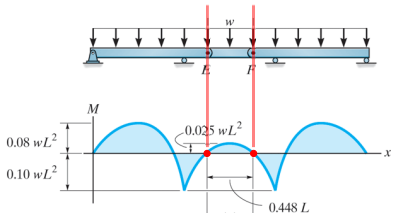


26

Analysis of statically indeterminate structures by the **force method**

Advantages and Disadvantages

- The beam then becomes ***statically determinate***.
- In this case any settlement of a support would not affect the reactions.



27

Analysis of statically indeterminate structures by the **force method**

Methods of Analysis

- When analyzing any indeterminate structure, it is necessary to satisfy ***equilibrium***, ***compatibility***, and ***force-displacement*** requirements.
- ***Equilibrium*** is satisfied when the reactive forces hold the structure at rest.
- ***Compatibility*** is satisfied when the various segments of the structure fit together without intentional breaks or overlaps.
- ***Force-Displacement*** requirements depend upon the way the material responds (in our case, linear-elastic).

28

Analysis of statically indeterminate structures by the **force method**

Flexibility or Force Method

- The ***flexibility or force method*** is a procedure for analyzing linear elastic indeterminate structures.
- This method was one of the first methods available for the analysis of statically indeterminate structures.
- Since compatibility forms the basis for this method, it has also been called the ***method of consistent deformations.***

29

Analysis of statically indeterminate structures by the **force method**

Flexibility or Force Method

- The method consists of writing equations that satisfy the ***compatibility*** requirements for the structure to determine ***redundant forces***.
- Once these redundant forces are determined, the remaining reactive forces on the structure are determined by satisfying ***equilibrium requirements***.

30

Analysis of statically indeterminate structures by the **force method**

Flexibility or Force Method

- The ***flexibility or force method*** was originally developed by James Clerk Maxwell in 1874 and later refined by Otto Mohr and Heinrich Müller-Breslau.


James Clerk Maxwell


Christian Otto Mohr


Heinrich Müller-Breslau

31

Analysis of statically indeterminate structures by the **force method**

Flexibility or Force Method

- All indeterminate analysis methods require that the solution satisfy ***equilibrium*** and ***compatibility*** requirements.
- By ***compatibility,*** we mean that the structure must fit together – no gaps can exist – and the deflected shape must be ***consistent with the constraints*** imposed by the supports.
- A key step in this method is the analysis of an indeterminate structure as a stable determinate structure with selected restraints (supports, for example) removed.

32

Analysis of statically indeterminate structures by the **force method**

Flexibility or Force Method

- While this method can be applied to any structure (beams, trusses, frames, etc.), the ***computational effort increases exponentially with the degree of indeterminacy***.
- Therefore, this method is most applicable to structures with a ***low degree of indeterminacy***.

33

Analysis of statically indeterminate structures by the **force method**

Any questions?

34