

Multiwavelength pyrometry: an improved method

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Abstract. It is demonstrated that the temperature of a thermal radiator can be determined without prior knowledge of the emissivity of the source by curve-fitting techniques using multiple spectral radiance measurements. This new passive measurement technique assumes only that a smooth function exists between spectral emissivity and wavelength. The spectral radiance values are fitted to a Planck radiation law relation to yield the temperature of the source. Error analysis shows that relative errors in the temperature measurements are generally an order of magnitude less than in the spectral radiance measurements and in the simultaneously calculated spectral emissivity values. Computer simulations are included that show the effects of varying different parameters, such as the number of data pairs, the wavelength range, the spectral emissivity behavior, the source temperature, and the measurement noise, on the accuracy of the temperature determination. Experimental confirmation of this technique is presented, showing temperature measurements within 1% of the actual temperatures on a platinum surface within a temperature range of 1273 to 1724 K.

Subject terms: optical pyrometry; multiwavelength pyrometry; temperature measurement; curve-fitting analysis; optical data processing; spectral emissivity.

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1. INTRODUCTION

One of the greater problems faced by spectrometric methods of temperature measurement is characterization of the emissivity of the source. The most widely used types of radiation pyrometry are based on an approximation of the Planck radiation law and on an assumption of constant spectral emissivity in the observed wavelength band. Ratio pyrometry techniques, such as the two-color method, are examples of this.¹⁻³ Other methods require that emissivity be determined separately from the temperature to obtain accurate thermal measurements.

One class of techniques, referred to as multiwavelength pyrometry, has been developed to calculate both the temperature and emissivity of a thermal radiator employing one set of radiation measurements from several wavelengths.³⁻⁶ Typically, these techniques involve assuming a functional dependence between spectral emissivity and wavelength and then using the radiation measurement data to determine the adjustable emissivity parameters and temperature by solving simultaneous equations. The main disadvantage with these methods is the sensitivity of the accuracy to radiation measurement errors.⁶

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An improvement to this approach is to use a much larger number of radiation measurements to determine the temperature and emissivity parameters using curve-fitting techniques. The sensitivity to measurement errors is greatly reduced by the statistics involved with curve fitting a large number of measurements. The only assumption necessary for this technique is that a smooth function exists between spectral emissivity and wavelength over the wavelength range of the radiation data.

This paper describes the improved multiwavelength technique. A second paper describing the particular application and measurements in greater detail is being prepared for publication.

2. THEORY

Provided that the source is a thermal radiator, the spectral radiance (N_λ , in units of $\text{W}/\text{cm}^2 \cdot \mu\text{m} \cdot \text{sr}$) emitted is a function of the wavelength (λ , in μm) of the radiation, the temperature (T , in K) of the source, and the spectral emissivity (ϵ_λ) of the emitting surface, according to the Planck radiation law:

$$N_\lambda = \frac{\epsilon_\lambda C_1}{\lambda^5 [\exp(C_2/\lambda T) - 1]} \quad (1)$$

where C_1 ($11,910 \text{ W} \cdot \mu\text{m}^4/\text{cm}^2 \cdot \text{sr}$) and C_2 ($14,387.9 \mu\text{m} \cdot \text{K}$) are the Planck radiation constants.^{1,2,7} Typical Planck radiation law curves are shown in Fig. 1. For this application, the term ϵ_λ is properly named the "spectral emittance," referring to the property of a particular surface, rather than "spectral emissivity," the intrinsic material property. However, to avoid the possibility of confusion with the radiant energy flux terms that also use "emittance," "emissivity" is used for this paper.⁷

Emissivity is a function of many variables⁸; however, all but wavelength are constant if the spectral radiance measurements are simultaneous and optically identical. Because it has been difficult to establish an analytical expression, spectral emissivity has been represented in many studies by a simple series of terms with adjustable parameters—a polynomial, for instance—valid in a restricted

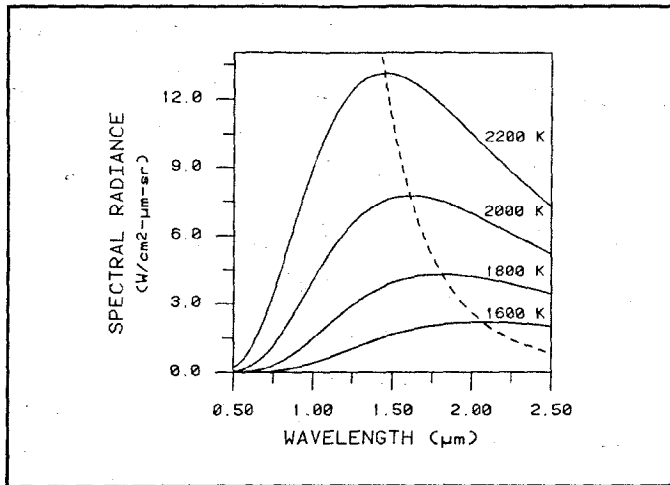


Fig. 1. Planck radiation law curves with a constant spectral emissivity ($\epsilon_\lambda = 1$) for four temperatures. The dotted line denotes the locus of curve maxima for all temperatures and this spectral emissivity.

wavelength range.^{3,6} Substituting the assumed spectral emissivity function into Eq. (1) yields the spectral radiance relation as a function of only wavelength at a constant temperature. The temperature and coefficients of the emissivity relation are obtained simultaneously by curve fitting the spectral radiance measurements for a large number of wavelengths to the spectral radiance function. This eliminates the need to determine emissivity separately from the temperature.

The insensitivity of the temperature determination to errors in the spectral radiance measurements can be shown analytically by differentiating Eq. (1) and substituting the relations

$$Z = \frac{C_2}{\lambda T}, \quad (2)$$

$$Y = \frac{Z}{1 - \exp(-Z)} \quad (3)$$

into the result to obtain

$$\frac{dN_\lambda}{N_\lambda} = \frac{d\epsilon_\lambda}{\epsilon_\lambda} + Y \frac{dT}{T}. \quad (4)$$

For the range of wavelengths and temperatures examined in this study, Y is approximately 10. Thus, a relative error in the spectral radiance data will cause at most an equal relative error in spectral emissivity or about an order of magnitude less relative error in temperature. The precision in the emissivity and temperature determinations is further increased due to the statistical nature of the curve-fitting analysis.

As can be deduced from Eq. (4), correlation effects between the adjustable parameters, even in the absence of experimental error in the spectral radiance ($dN_\lambda/N_\lambda = 0$), can cause errors in the temperature determination:

$$\frac{d\epsilon_\lambda}{\epsilon_\lambda} = -Y \frac{dT}{T}. \quad (5)$$

Thus, the relative error in spectral emissivity will be about an order of magnitude greater than that in temperature.

3. APPLICATION

The eventual goal of this work is to produce an instrument and

technique to measure surface temperatures of molten gas-tungsten arc weld pools. Most previous studies of weld pool temperatures have employed thermocouples.^{9,10} These techniques have been far from optimal since the thermocouples impede fluid motion in the pool and may alter the thermal profile.¹ To avoid contact with the weld pool, some investigators have used total radiation pyrometry to map isotherms on and around the weld pool¹¹⁻¹³; however, the difficulty in determining the emissivity has prevented them from accurately associating these isotherms with absolute temperatures.

Recent advances in multichannel photoelectric detectors allow the simultaneous spectral radiance measurements at multiple wavelengths. Coupled with the rapid data processing capabilities of computers, these detectors make practical the improved pyrometric technique described above. This technique has the potential for eliminating the problems encountered with thermocouples or emissivity in other studies, as well as making accurate thermal measurements.

An instrument was built to experimentally demonstrate the technique and to serve as a prototype for eventually measuring weld pool surface temperatures.¹⁴ The instrument is named the Multichannel Infrared-Red Temperature Micro-Analyzer (MIRTMA). The MIRTMA is divided functionally into three parts: the infrared microscope, the detector system, and the data processor. The infrared microscope is comprised of the spectrograph, which disperses the radiation onto the photodetector surface, and the focusing optics, which image a 100 μm effective source area onto the spectrograph. The detector system measures the photon irradiance and provides experiment synchronization capabilities, while the data processor is used to calibrate the detector response to spectral radiance and analyze the calibrated data to determine the source temperature. The detector selected for this study is a Silicon Intensified Target (SIT) vidicon detector purchased from EG&G Princeton Applied Research Corporation.^{14,15} The spectral radiance from approximately 200 channels (of the available 500) of wavelength in the range 0.60 to 0.80 μm was measured since only these channels registered significant signals.

It should be noted that this method assumes that only emitted radiation is being measured from an opaque source. Thus, care must be taken to restrict reflected radiation from the source area. Since the source area is normally at a high temperature relative to its surroundings, reflected radiation can be neglected. However, in the case of a weld pool, the pool surface will probably have a high reflectance, and the hot electrode will have to be masked or removed to avoid the reflected radiation. The welding arc also will have to be extinguished to avoid measuring radiation not emitted by the weld pool.

A direct substitution method is used to calibrate the detector response to spectral radiation, so the reflectance and transmittance of the intervening optics are unimportant if they remain constant. However, the vapors and dust in the optical path between the source and detector will absorb variable amounts of light between measurements, especially in the weld environment. Assuming that the dust is too cool to emit significant radiation compared to the source, this variable absorption will affect only the emissivity terms and not the measured temperature.

Once the spectral radiance data have been obtained, the data processor uses a curve-fitting routine to calculate the temperature. For the purposes of curve fitting, the spectral emissivity is assumed to follow each of three relations in turn:

$$\epsilon_\lambda = \epsilon_{01} + \epsilon_{11}\lambda, \quad (6)$$

$$\epsilon_\lambda = \epsilon_{02} + \epsilon_{12}\lambda + \epsilon_{22}\lambda^2, \quad (7)$$

$$\epsilon_\lambda = \exp(\epsilon_{03} + \epsilon_{13}\lambda), \quad (8)$$

where ϵ_{ij} are constant coefficients for the particular experimental conditions. Equation (8) is employed by Svet for his multiwavelength technique.³

The curve-fitting routine is two-step process. Initially, a value is

TABLE I. Schedules of Parameters for the Computer Simulations (Using Eqs. (1) and (6))

Schedule	T (K)	ϵ_{01}	ϵ_{11}	Data pairs	λ range (μm)
1	1800	1.00	0.00	200	0.60 to 0.80
2	1273	1.00	0.00	200	0.60 to 0.80
3	1800	1.60	-1.00	200	0.60 to 0.80
4	1800	1.00	0.00	100	0.60 to 0.80
5	1800	1.00	0.00	200	0.50 to 1.50

assumed for the temperature in order to determine the emissivity coefficients of the linear spectral emissivity function [Eq. (6)] by a least squares method.¹⁶ By repeating this process, the routine searches for a temperature that will yield a minimum ϵ_{11} term. The temperature that is found is then used to calculate a mean emissivity value:

$$\epsilon_{\lambda} = \epsilon_{00} \quad (9)$$

This provides approximations of temperature and emissivity that are used as the starting point for the second step of the process.

Next, the routine uses a least squares fit with a gradient expansion search algorithm to find the emissivity coefficients and the temperature using each spectral emissivity function [Eq. (6) through (8)] in turn. The method involves expanding the fitting function in a Taylor's expansion with the adjustable parameters of the spectral emissivity function and temperature, then uses a least squares method to find the optimum values of these parameters. These optimum values are chosen by minimizing the reduced χ^2 value for the fit so that the relative decrease in the χ^2 value for small changes in the parameters is reduced below a threshold (0.01%). This algorithm is explained in greater detail in Ref. 17.

1. COMPUTER SIMULATIONS

To demonstrate the improved multiwavelength technique, a series of computer simulations was devised. Theoretical spectral radiance data were generated using the Planck radiation law [Eq. (1)] for a given schedule of temperature, emissivity function [Eq. (6)], wavelength range, and number of data pairs (Table I). A temperature of 1800 K was chosen for most schedules since this is the approximate melting temperature of steel. To simulate experimental noise, the generated Planck law values were multiplied by a random number series of uniform distribution around unity. This is not necessarily the form of noise expected in practice, but for simplicity the noise was assumed proportional to the spectral radiance. The rms value of the difference between the random number series and unity was used to describe the three simulated experimental noise levels: 0.00%, 2.85%, and 5.70%.

These generated data sets were used to simulate measured data for the curve-fitting routines. For each noise level, a mean and a standard deviation for the adjustable parameters were calculated by repeating the procedure with 20 different sets of data for each schedule. The mean and the standard deviation of the temperature parameter, which give an indication of the errors and scatter in the temperature measurements, are shown relative to the actual temperature in Figs. 2 through 5. The results for two noise levels of schedule 3 are given in Table II.

As can be seen, the introduction of as much as 5.70% rms noise causes less than 0.5% relative mean temperature error, with less than a 5% relative standard deviation, for most schedules and spectral emissivity assumptions. Table II shows that the errors and scatter tend to be much larger for the emissivity parameters than for the temperature determination, which is typical of all of the simulations. This behavior is as expected from the analysis of Eqs. (4) and (5). Also consistent with this analysis is the reduction in most errors and scatter for the lower temperature simulation (schedule 2) due to the increase in Y value [Eq. (3)] compared to the higher temperature (schedule 1). Reducing the number of data pairs from 200 (schedule

1) to 100 (schedule 4) generally caused an increase in the mean error and standard deviation, as is expected by curve-fitting statistics.

In the wavelength regions and temperatures examined, the Planck radiation law appears roughly exponential in nature, as can be seen in Fig. 1. Thus, especially with the exponential fitting function [Eq. (8)], there is a possible ambiguity between the effects of a change in the temperature or emissivity parameters that cannot be resolved by the curve-fitting routines and results in a correlation between these parameters. As is expected from this, the mean errors and standard deviations are usually greatest for the exponential emissivity function. Extending the wavelength range to 0.5 to 1.5 μm (schedule 5) greatly reduces these values, especially for the exponential function, since the Planck radiation law curve does not approximate a linear exponential function in the extended range.

The generated spectral emissivity values for schedule 3 drop linearly from 1.0 to 0.8 as the wavelength increases from 0.6 to 0.8 μm . The constant and exponential emissivity fitting functions are expected to have difficulty fitting this behavior and thus cause errors in the temperature measurement. However, the two- and three-term polynomial emissivity fitting functions would be expected to make more accurate temperature measurements. In fact, aside from the two-term polynomial emissivity with no noise, the largest mean temperature measurement errors occur with schedule 3. Correlation effects between the fitting parameters are again the cause. A clear example of this is seen in Table II, comparing the measured polynomial emissivity parameters for data with no noise. In the absence of correlation between the parameters, the results for the two- and three-term polynomial functions should be identical [with $\epsilon_{22} = 0$ in Eq. (7)].

The effect of a temperature gradient within the source area was also simulated. For simplicity, the source is assumed to be divided in equal parts symmetrically to an average temperature so that half of the source is at a lower temperature and half of the source is at a higher temperature. This is a condition that is more severe than would actually exist. Three gradients are compared using the parameters of schedule 1 of Table I: 0.0% (1800 K), 1.0% (1791 and 1809 K), and 10% (1710 and 1890 K). The results of these simulations are shown in Table III. Even with this extreme approximation for the gradient, the measurement errors relative to the average temperature (2.4 to 3.8% for the 10% gradient) are less than the gradient itself. In addition, it can be seen that even with thermal gradients on the order of 1% in the measured area, the fitting routine will find the correct average temperature.

5. EXPERIMENTAL DEMONSTRATION

To demonstrate the technique experimentally, the temperature of a resistively heated platinum strip was measured using the MIRTMA system. The spectral emissivity behavior of this source was unknown. The temperature of the source area on the strip was monitored with a thermocouple for comparison with the MIRTMA results. Measurements were made at four temperatures within 1270 to 1725 K, and the results are shown in Fig. 6. The results are presented here only to indicate the temperature measurement accuracy that has been obtained in practice. More detailed descriptions of these measurements and the MIRTMA are being prepared for publication and are also included in Ref. 14.

The MIRTMA temperature measurements were found to be generally within 5% of the thermocouple measurements for all functions, and within 0.5% for the three-term polynomial emissivity fitting function. The inability of the constant function to adequately fit the actual spectral emissivity behavior (which is variable with wavelength) accounts for the errors with that function, and correlation effects are the cause of the large errors with the exponential function. By further optimizing the technique and choosing the proper emissivity fitting function, greater accuracy is expected.

6. SUMMARY

This paper describes an improved method of multiwavelength

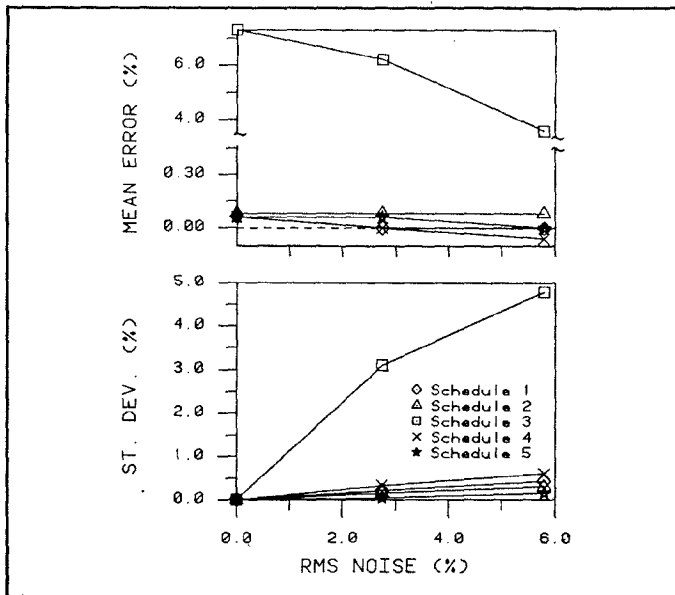


Fig. 2. Computer simulation results for the constant emissivity assumption [Eq. (9)]. The mean temperature measurement error relative to the actual temperature is above, with the relative standard deviation of the measurements below (note the change of scale).

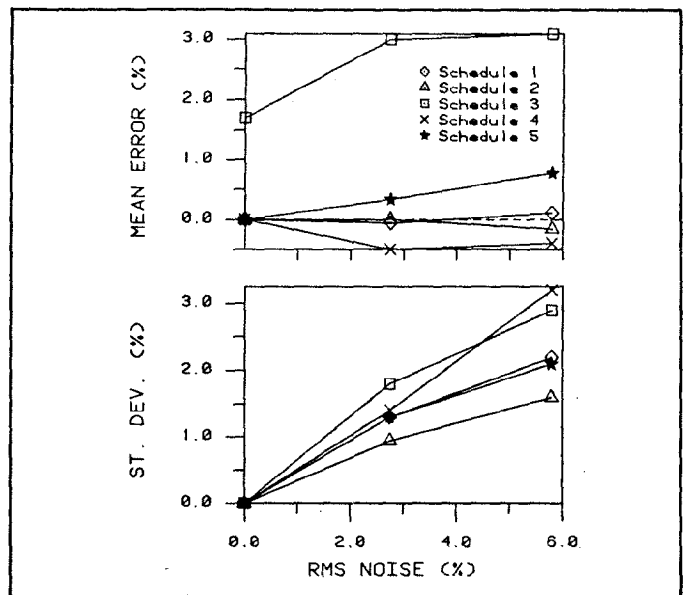


Fig. 4. Computer simulation results for the three-term polynomial emissivity assumption [Eq. (7)]. The mean temperature measurement error relative to the actual temperature is above, with the relative standard deviation of the measurements below.

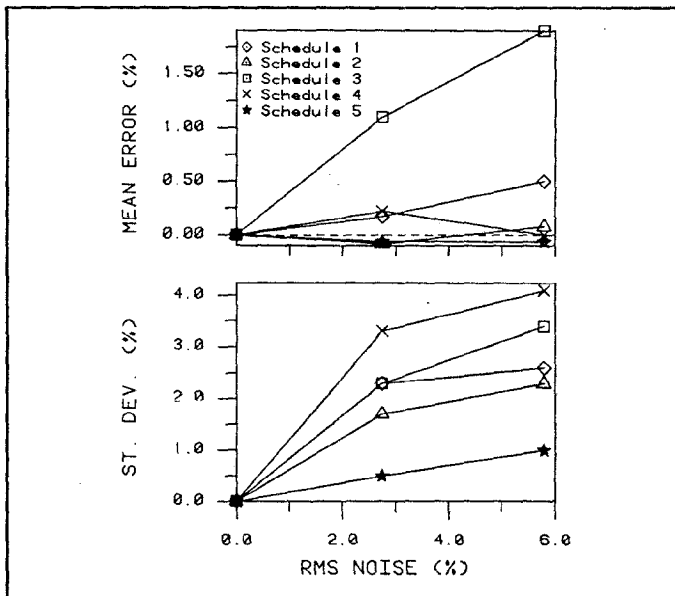


Fig. 3. Computer simulation results for the two-term polynomial emissivity assumption [Eq. (6)]. The mean temperature measurement error relative to the actual temperature is above, with the relative standard deviation of the measurements below.

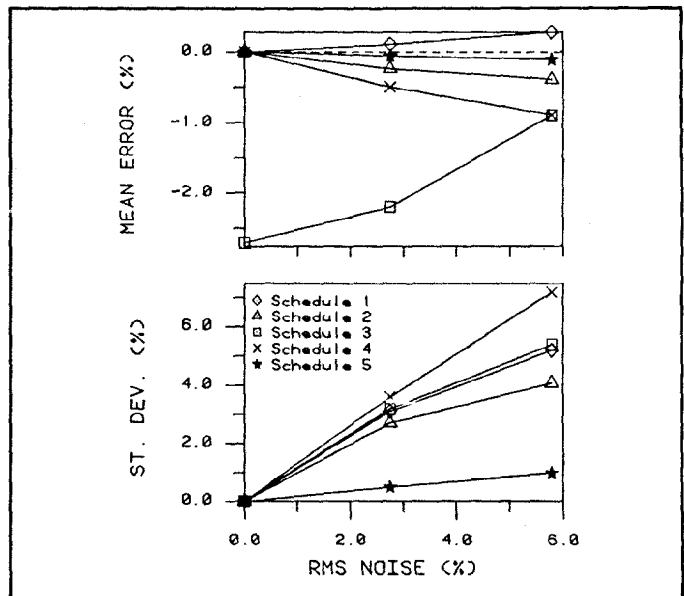


Fig. 5. Computer simulation results for the two-term exponential emissivity assumption [Eq. (8)]. The mean temperature measurement error relative to the actual temperature is above, with the relative standard deviation of the measurements below.

TABLE II. Mean Parameter Results of the Schedule 3 ($\epsilon_\lambda = 1.60 - 1.00\lambda$) Simulation (Standard Deviations in Parentheses)

Noise (%)	ϵ_λ Eq.	T (K)	ϵ_0	ϵ_1	ϵ_2
0.00	9	1931	0.41		
0.00	6	1800	1.60	-1.00	
0.00	7	1831	0.94	0.06	-0.48
0.00	8	1752	1.30	-1.56	
5.70	9	1864 (86)	0.72 (0.40)		
5.70	6	1834 (61)	1.37 (0.98)	-0.83 (0.93)	
5.70	7	1855 (53)	0.82 (0.39)	0.12 (0.39)	-0.47 (0.62)
5.70	8	1784 (98)	0.94 (1.33)	-1.31 (0.57)	

TABLE III. Temperature Results (in K) of the Thermal Gradient Simulations

Gradient of T (%)	ϵ_λ Equation			
	9	6	7	8
0.0	1801	1800	1800	1800
1.0	1801	1801	1801	1801
10.0	1844	1865	1868	1863

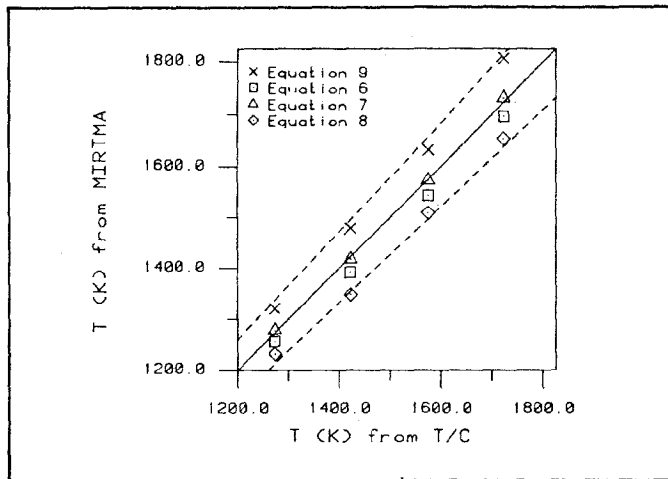


Fig. 6. Comparison of the MIRTMA measurements to the thermocouple (T/C) measurements of the platinum strip source (perfect agreement and 5% error bands are denoted by solid and dotted lines, respectively).

pyrometry that uses curve-fitting techniques. This method fits multiple spectral radiance measurements to the Planck radiation law relation to determine temperature and spectral emissivity simultaneously. It assumes only that a smooth function exists between spectral emissivity and wavelength over the range of the observed wavelength band. Relative errors in the temperature measurements are generally an order of magnitude less than random relative measurement errors of spectral radiance and than relative errors in the calculated spectral emissivity values.

Computer simulations show that the accuracy and precision of the technique increase as the wavelength range is extended toward the peak in spectral radiance and as the number of measured data pairs increases. Simulations also show that thermal gradients of approximately 1% of the average temperature within the source area cause negligible errors in the temperature measurement. The MIRTMA system has experimentally confirmed that this technique can be used to measure temperatures of a platinum strip within 1274 to 1724 K. These thermal measurements agreed with thermocouple measurements to within 5% for all assumed forms of the emissivity fitting function and within 0.5% for the three-term polynomial emis-

sivity function.

Errors may be caused in the temperature measurements if the assumed functional form of spectral emissivity with wavelength has too few terms to fit, or is otherwise inappropriate for, the actual behavior. This problem is exacerbated by correlation effects between the temperature and emissivity parameters during curve fitting with the current procedures. Further work is being conducted to improve the fitting procedure and minimize correlation effects between the adjustable parameters. Reducing correlation effects should increase the temperature measurement accuracy and shorten the computation time. Studies are also being conducted to determine the proper emissivity function and its limits of validity.

7. ACKNOWLEDGMENTS

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