Reflections on Ethnomathematics

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Looking back historically we may see that ethnomathematics has been defined at different levels. Defined as the cultural anthropology of mathematics and mathematical education, it is a relatively new field of interest. Because the view of mathematics as "culture-free", as a "universal, basically aprioristic form of knowledge" has been dominant, ethnomathematics emerged later than other ethnosciences. Among mathematicians, ethnographers, psychologists, and educationalists, R. Wilder [1950], L. White [1947], E. Fettweis, G. Luquet [1929] and O. Raum [1938] may be registered as isolated forerunners of ethnomathematics. [1] Their reflections did not find much of an echo. A reductionist tendency tended to dominate mathematics education, relying on culture-free cognition models. [2]

Ethnomathematics as the mathematics practiced among identifiable cultural groups

At the end of the 1970s and the beginning of the 1980s, there developed a growing awareness among mathematicians of the societal and cultural aspects of mathematics and mathematical education. It is in that period that U. D'Ambrosio proposed his *ethnomathematical program* as a methodology to track and analyse the processes of generation, transmission, diffusion, and institutionalization of (mathematical) knowledge in diverse cultural systems. [3] In contrast to academic mathematics, i.e. the mathematics which is taught and learned in the schools, D'Ambrosio calls ethnomathematics "the mathematics which is practiced among identifiable cultural groups, such as national-tribal societies, labor groups, children of a certain age bracket, professional classes, and so on" [1985b, p. 45]. [4] "The mechanism of schooling replaces these practices by other equivalent practices which have acquired the status of mathematics, which have been expropriated in their original forms and returned in a codified version" [1985b, p. 47]. Before and outside school almost all children in the world become "matherate", i.e. they develop the "capacity to use numbers, quantities, the capability of qualifying and quantifying and some patterns of inference" [1985a, p. 43]. In school the "learned" matheracy eliminates the so-called spontaneous matheracy: the former spontaneous abilities are downgraded, repressed, and forgotten while the learned ones are not assimilated, either as a consequence of a learning blockage, or of an early drop-out, or even as a consequence of failure or for other reasons. The question which arises, then, is what to do: "should we ... give up school mathematics and remain with ethnomathematics? Clearly not ... " [1985a, p. 70]. In D'Ambrosio's view one should compatibilize cultural forms, i.e. "... the mathematics in schools shall be such that it facilitates knowledge, understanding, incorporation, and compatibilization of known and current popular practices into the curriculum. In other words, recognition and incorporation of ethnomathematics into the curriculum" [1985a, p. 71].

Let us now briefly review other concepts that have been proposed and that are related to D'Ambrosio's ethnomathematics.

Emergence of concepts related to ethnomathematics

During the 1970s and 1980s, there emerged among the teachers and didacticians of mathematics in developing countries, and later also in other countries, a growing resistance against the racist and (neo)colonial prejudices related to mathematics, against the eurocentrism in mathematics and its history. [5] It was stressed that beyond the imported school mathematics there existed and continues to exist *other mathematics*.

In this context, various concepts have been proposed that contrast with the *academic* mathematics/school mathematics (i.e. the school mathematics of the transplanted, imported curriculum):

- *indigenous* mathematics [e.g. Gay & Cole, 1967 [6]; Lancy, 1976 [7]]: Criticizing the mathematics education of Kpelle children (Liberia) in Western-oriented schools—they "are taught things that have no point or meaning within their culture" [1967, p. 7]— Gay and Cole propose a creative education that uses the indigenous mathematics as a starting point;
- *sociomathematics* of Africa [Zaslavsky, 1973 [8]]: "The applications of mathematics in the lives of African people, and, conversely, the influence that African institutions had upon the evolution of their mathematics" [p. 7];
- *informal* mathematics [Posner, 1978, 1982 [9]]: mathematics that is transmitted and that one learns outside the formal system of education;
- *mathematics in the (African) socio-cultural environment* [S. Touré, S. Doumbia (Côte d'Ivoire), 1980]: integration of the mathematics of African games and craftwork that belongs to the socio-cultural environment of children into the mathematics curriculum;
- *spontaneous* mathematics [D'Ambrosio, 1982 [10]]: each human being and each cultural group develops certain mathematical methods spontaneously;
- *oral* mathematics [Carraher et al., 1982 [11]; Kane, 1987 [12]]: in all human societies there exists mathematical knowledge that is transmitted orally from one generation to the next;
- *oppressed* mathematics [Gerdes, 1982 [13]]: in class societies (e.g., in the countries of the Third World during the colonial occupation) there exist mathematical elements in the daily life of the populations that are not recognized as mathematics by the dominant ideology;

- *non-standard* mathematics [Carraher et al., 1982; Gerdes, 1982, 1985 [14]; Harris, 1987 [15]]: beyond the dominant standard forms of academic and school mathematics there develops and developed in the whole world and in each culture mathematical forms that are distinct from the established patterns;
- *hidden* or *frozen* mathematics [Gerdes, 1982, 1985]: although, probably, most of the mathematical knowledge of the formerly colonized peoples has been lost, one may try to reconstruct or unfreeze the mathematical thinking that is hidden or frozen in old techniques, like, e.g., that of basketmaking;
- *folk* mathematics [Mellin-Olsen, 1986 [16]]: the mathematics (although often not recognized as such) that develops in the working activity of the peoples may serve as a starting point in the teaching of mathematics;
- mathematics *codified in know-hows* [Ferreira, 1987 [17]].

These proposals are provisional. They belong to a tendency that emerged in the context of the Third World and that later on found an echo in other countries. The various aspects illuminated by the aforementioned provisional concepts have been gradually united under the more general common denominator of D'Ambrosio's ethnomathematics. This process has been accelerated by the creation of the *International Study Group on Ethnomathematics* (ISGEm) in 1985.

Ethnomathematics as the field of research that tries to study mathematics (or mathematical ideas) in its (their) relationship to the whole of cultural and social life

In the previous section ethnomathematics was the mathematics of a certain (sub)culture. In this sense so-called academic mathematics is also a concrete example of ethnomathematics. When all ethnomathematics is mathematics, why call it ethnomathematics—and not simply the mathematics of this or that (sub)culture? By the decision to call it so we have defined ethnomathematics at another level, as a research field. The emergence of this research field reflects the consciousness of the existence of many mathematics, particular in certain ways to a variety of (sub)cultures.

As a research field, ethnomathematics may be defined as the cultural anthropology of mathematics and mathematical education, or in D'Ambrosio's formulation in 1977: "*Ethnoscience* [is] the study of scientific and, by extension, technological phenomena in direct relation to their social, economic and cultural backgrounds" [1987, p.74]; D'Ambrosio sometimes uses the expression *anthropological mathematics* [e.g. 1985b]. In this sense ethnomathematics comes near to the *sociology of mathematics* (1942) of D. Struik. [18] It includes "the study of mathematical ideas of nonliterate peoples", which was the Aschers' definition of ethnomathematics in 1985. [19]

The ethnomathematical movement [20]

The scholars who are engaged in ethnomathematical research are normally socially actively engaged. In this sense I should like to speak of an *ethnomathematical movement* that might be characterized by the following features, among others:

- *Ethnomathematicians* use broad conceptions of mathematics, including, in particular, counting, locating, measuring, designing, playing, explaining; [21]
- *Ethnomathematicians* emphasize and analyse the influences of socio-cultural factors on the teaching, learning, and development of mathematics;
- *Ethnomathematicians* draw attention to the fact that mathematics (its techniques and truths) is a *cultural product*. They stress that every people—every culture and every subculture—develops its own particular mathematics. Mathematics is considered to be a *universal, pan-human activity*. As a cultural product mathematics has a history. Under certain economic, social, and cultural conditions, it emerged and developed in certain directions; under other conditions, it emerged and developed in other directions. In other words, the development of mathematics is *not unilinear*. [22]
- *Ethnomathematicians* emphasize that the school mathematics of the transplanted, imported curriculum is *apparently* alien to the cultural traditions of Africa, Asia, and South America. Apparently this mathematics comes from outside the Third World. *In reality*, however, a substantial part of the contents of this school mathematics is of African and Asian origin; [23]
- *Ethnomathematicians* try to contribute to knowledge of the mathematical realizations of the formerly colonized peoples. They look for cultural elements that survived colonialism and that reveal mathematical and other scientific thinking. They try to *reconstruct these* mathematical thoughts;
- *Ethnomathematicians* in Third World countries look for mathematical traditions that survived colonization and for mathematical activities in people's daily life and analyse ways to incorporate them into the curriculum;
- In the educational context *ethnomathematicians* generally favor a critical mathematics education that enables the students to reflect about the reality they live in and empowers them to develop and use mathematics in an emancipatory way. The influence of the well-known Third World pedagogue Paulo Freire is visible. [24]

Ethnomathematical research in Mozambique

Ethnomathematical research started in Mozambique in the late 1970s. As most mathematical traditions that survived colonization and most mathematical activities in the daily life of the Mozambican people are not explicitly mathematical, i.e. the mathematics is hidden, the first aim of this research was to uncover the hidden mathematics. The first results of this uncovering are included in the book *On the awakening of geometrical thinking* (1985) [25] and slightly extended in *Ethnogeometry: cultural-anthropological contributions to the genesis and didactics of geometry* (1992). [26] These studies forced us to reflect more profoundly on the role of human labour (and) activity on the historical

(and possibly educational) development of mathematical reflection and reasoning.

In the papers "On culture, mathematics and curriculum development in Mozambique" (1986) [27] and "On culture, geometrical thinking, and mathematics education" (1988) [28], I summarized my experimentation with the incorporation of traditional African cultural elements into mathematics education. They illustrate the methodology of *cultural conscientialization* in the context of teacher education. In the papers "A widespread decorative motif and the Pythagorean Theorem" (1988) [29] and "How many proofs of the Pythagorean Proposition do there exist?" [30], and in a more elaborated form in the book *African Pythagoras: a study in culture and mathematics education* (1992) [31], I tried to show how diverse African ornaments and artifacts may be used to create a rich context for the discovery and the demonstration of the so-called Pythagorean Theorem and of related ideas and propositions. A series of ethnomathematical papers are included in the books *Ethnomathematics: culture, mathematics, education* [32] and *Ethnomathematics and education in Africa* [33] In my book *SONA geometry: reflections on a drawing tradition among peoples in Africa south of the Equator* (1993) [34], I tried to reconstruct the mathematical components of the Tchokwe drawing tradition (Angola) and to explore their educational, artistic, and scientific potential. In an earlier article "On possible uses of traditional Angolan sand drawings in the mathematics classroom" (1988) [35] some possibilities for the educational incorporation of this tradition had already been analysed. In the paper "Find the missing figures" (1988) [36], and in the book *Lusona: geometrical recreations of Africa* (1991) [37], mathematical amusements inspired by the geometry of the sanddrawing tradition are presented. For children (age 10-15) the booklet "Living mathematics: drawings of Africa" (1990) [38] has been elaborated. Experimentation with the use of "sona" in teacher education is described in "Exploring Angolan sand drawings (sona): stimulating cultural awareness in mathematics teachers " [39] An overview of this research is given in "On mathematical elements in the Tchokwe "sona" tradition" (1990). [40]

In recent years more lecturers, and in particular young lecturers, who returned home after having studied abroad became interested in and started ethnomathematical research. Examples of papers they have presented at conferences are "The origin of the concepts of even and odd in Makhwa culture (Northern Mozambique)" (A. Ismael); "Mental Arithmetic and the Tsonga language (Southern Mozambique)", "A children's circle of interest in ethnomathematics" and "Children's mathematical activities stimulated by an analysis of African cultural elements" (M. Cherinda); "Popular counting practices in Mozambique" (D. Soares); "Mental addition and subtraction in Mozambique" (J. Draisma). Also a series of students at the Higher Pedagogical Institute in Mozambique (Maputo and Beira) became interested in ethnomathematical research. Two students have already completed master's theses in the field of ethnomathematics: "Symmetries of ornaments on baskets of the khuama type" (E. Uaila) and "Symmetries of ornaments on metallic window gratings in the city of Maputo" (A. Mapapá).

Perspectives

I consider ethnomathematics primarily as a (very new and quickly expanding) research field. It is rather early to draw (general) conclusions. In the concrete case of Mozambique and other African countries, ethnomathematical research will certainly reveal more historically, mathematically, educationally, and philosophically interesting practices, that will stimulate, in their turn, further research and pedagogical experimentation. Ethnomathematical research will oblige everyone to reconsider the history of mathematics [41]; to reconsider cognitive models of learning mathematics; to reconsider the goals, contents, and means of mathematical education; to reconsider the cultural role of mathematics; to reconsider what mathematics is all about.

References

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[23] Compare e.g. Bishop: "One of the greatest ironies ... is that several different cultures and societies contributed to the development of [the so-called] Western Mathematics—the Egyptians, the Chinese, the Indians, the Moslems, the Greeks, as well as the Western Europeans. Yet when Western cultural imperialism imposed its version of Mathematics on the colonized societies, it was scarcely recognisable as anything to which these societies might have contributed ..." Cultural imperialism and Western mathematics: the hidden persuader, in: Mangan, J.A. (ed.), *Education, Europe and Imperialism* Manchester, 1989; A. Bishop, Western Mathematics: the secret weapon of cultural imperialism, *Race and Class,* Vol. 32, No. 2. (1990)
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