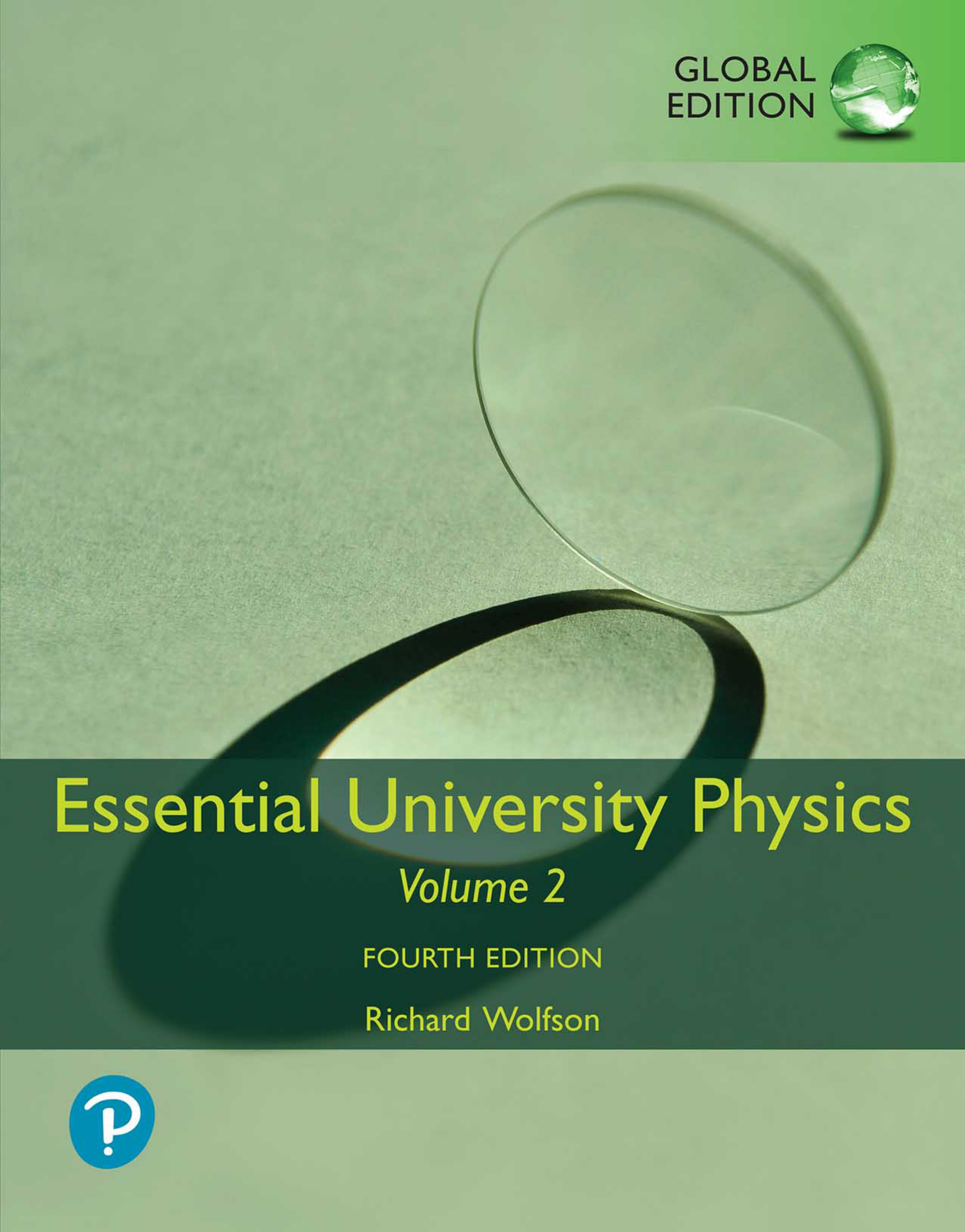

GLOBAL EDITION

Essential University Physics

Volume 2

FOURTH EDITION

Richard Wolfson

VOLUME TWO

Chapters 20–39

Essential University Physics

FOURTH EDITION

GLOBAL EDITION

Richard Wolfson

Middlebury College

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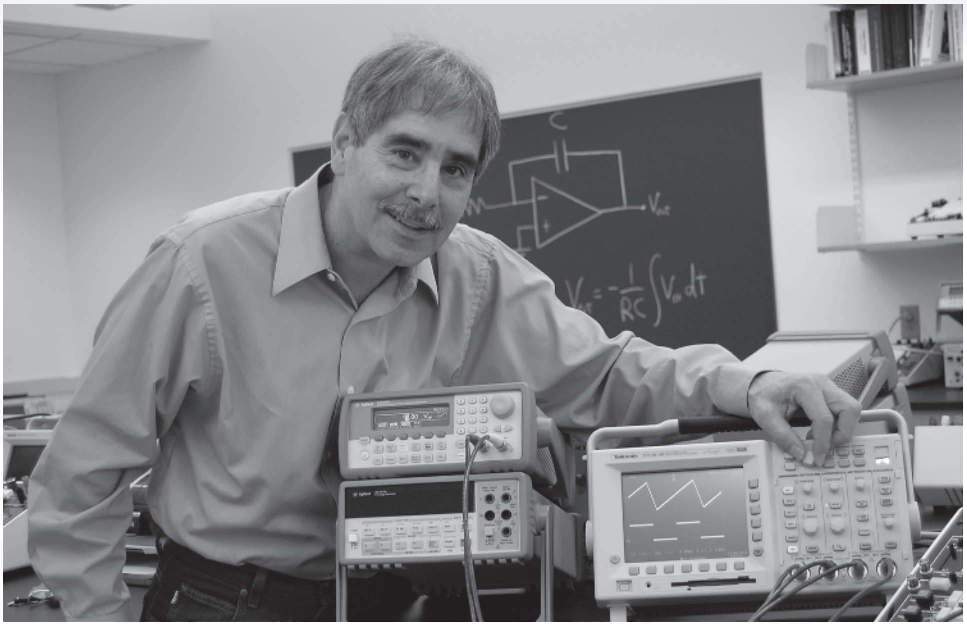
Brief Contents

Detailed Contents

About the Author



Richard Wolfson

Richard Wolfson is the Benjamin F. Wissler Professor of Physics at Middlebury College, where he has taught since 1976. He did undergraduate work at MIT and Swarthmore College, and he holds an M.S. from the University of Michigan and a Ph.D. from Dartmouth. His ongoing research on the Sun's corona and climate change has taken him to sabbaticals at the National Center for Atmospheric Research in Boulder, Colorado; St. Andrews University in Scotland; and Stanford University.

Rich is a committed and passionate teacher. This is reflected in his many publications for students and the general public, including the video series *Einstein's Relativity and the Quantum Revolution: Modern Physics for Nonscientists* (The Teaching Company, 1999), *Physics in Your Life* (The Teaching Company, 2004), *Physics and Our Universe: How It All Works* (The Teaching Company, 2011), and *Understanding Modern Electronics* (The Teaching Company, 2014); books *Nuclear Choices: A Citizen's Guide to Nuclear Technology* (MIT Press, 1993), *Simply Einstein: Relativity Demystified* (W. W. Norton, 2003), and *Energy, Environment, and Climate* (W. W. Norton, third edition, 2018); and articles for *Scientific American* and the *World Book Encyclopedia.*

Outside of his research and teaching, Rich enjoys hiking, canoeing, gardening, cooking, and watercolor painting.

Preface to the Instructor

Introductory physics texts have grown ever larger, more massive, more encyclopedic, more colorful, and more expensive. *Essential University Physics* bucks that trend—without compromising coverage, pedagogy, or quality. The text benefits from the author's four decades of teaching introductory physics, seeing firsthand the difficulties and misconceptions that students face as well as the GOT IT? moments when big ideas become clear. It also builds on the author's honing multiple editions of a previous calculus-based textbook and on feedback from hundreds of instructors and students.

Goals of This Book

Physics is the fundamental science, at once fascinating, challenging, and subtle—and yet simple in a way that reflects the few basic principles that govern the physical universe. My goal is to bring this sense of physics alive for students in a range of academic disciplines who need a solid calculus-based physics course—whether they're engineers, physics majors, premeds, biologists, chemists, geologists, mathematicians, computer scientists, or other majors. My own courses are populated by just such a variety of students, and among my greatest joys as a teacher is having students who took a course only because it was required say afterward that they really enjoyed their exposure to the ideas of physics. More specifically, my goals include:

- Helping students build the analytical and quantitative skills and confidence needed to apply physics in problem solving for science and engineering.
- Addressing key misconceptions and helping students build a stronger conceptual understanding.
- Helping students see the relevance and excitement of the physics they're studying with contemporary applications in science, technology, and everyday life.
- Helping students develop an appreciation of the physical universe at its most fundamental level.
- Engaging students with an informal, conversational writing style that balances precision with approachability.

New to the Fourth Edition

The emphasis in this fourth-edition revision has been on pedagogical features, including substantial updates to the end-of-chapter problem sets, learning outcomes, annotated equations, and new, contemporary applications. In addition, I've responded—as I have in previous editions—to the many suggestions made by my colleagues, by instructors around the world, and by reviewers engaged to help make this the most student-friendly and pedagogically useful edition of *Essential University Physics*. And, as always, I've been on the lookout for new developments in physics and technology to incorporate into the text.

- Chapter opening pages have been redesigned to include explicit lists of **learning outcomes** associated with each chapter. Learning outcomes appear at the appropriate section headings and are also keyed with specific problems.
- End-of-chapter problem sets each have over 15% **new problems**. Many of the new problems are of intermediate difficulty, featuring multiple steps and requiring a clear understanding of problem-solving strategies. I've also increased the number of estimation problems and of problems involving symbolic rather than numerical answers. Still other new problems feature contemporary real-world situations.

- Among the most exciting of the new features—and one that gave me both great challenges and great professional satisfaction—are the **Example Variation (EV)** problems. These two sets of four related problems in each chapter, each set based on one of the chapter's worked examples, help the student make connections, enhance her understanding of physics, and build confidence in solving problems different from ones she's seen before. The first problem in each set is essentially the example problem but with different numbers. The second presents the same scenario as the example but asks a different question. The third and fourth problems repeat this pattern but with entirely different scenarios. Working these problems ensures first that the student understands the worked example and then gradually takes her out of her comfort zone to explore new physics, more challenging math, and more complex problem solving.
- Students should perceive a physics textbook as more than a list of equations to consult in solving assigned problems. *Essential University Physics* has always helped students avoid this unfortunate approach to physics. Earlier editions had a few instances where I felt an equation was so important that I developed a separate figure that was essentially an "anatomy" of the equation, with annotations pointing to and explaining the terms in the equation. The new edition extends this approach with **annotated key equations**, giving life to and understanding of all the most important and fundamental equations as statements about the physical universe rather than mere math into which numbers get plugged.
- A host of **new applications** connects physics concepts that students are learning with contemporary technological and biomedical innovations, as well as recent scientific discoveries. A sample of new applications includes the acceleration of striking rattlesnakes, gravitational wave detection and multimessenger astronomy, earthquake resonance effects, the *New Horizons* mission to Pluto, the audacious *Starshot* project, the graded-index lenses of squids' eyes, and environmental and energy issues.
- As with earlier revisions, I've incorporated new research results, new applications of physics principles, and findings from physics education research.
- Finally, this edition includes the 2019 revision of the SI—the international system of units—which represents the most significant change the SI has undergone in more than a century.

Pedagogical Innovations

This book is *concise*, but it's also *progressive* in its embrace of proven techniques from physics education research and *strategic* in its approach to learning physics. Chapter 1 introduces the IDEA framework for problem solving, and every one of the book's subsequent **worked examples** employs this framework. IDEA—an acronym for Identify, Develop, Evaluate, Assess—is not a "cookbook" method for students to apply mindlessly, but rather a tool for organizing students' thinking and discouraging equation hunting. It begins with an interpretation of the problem and an identification of the key physics concepts involved; develops a plan for reaching the solution; carries out the mathematical evaluation; and assesses the solution to see that it makes sense, to compare the example with others, and to mine additional insights into physics. In nearly all of the text's worked examples, the Develop phase includes making a drawing, and most of these use a hand-drawn style to encourage students to make their own drawings—a step that research suggests they often skip. IDEA provides a common approach to all physics problem solving, an approach that emphasizes the conceptual unity of physics and helps break the typical student view of physics as a hodgepodge of equations and unrelated ideas. In addition to IDEA-based worked examples, other pedagogical features include:

- **Problem-Solving Strategy boxes** that follow the IDEA framework to provide detailed guidance for specific classes of physics problems, such as Newton's second law, conservation of energy, thermal-energy balance, Gauss's law, or multiloop circuits.
- **Tactics boxes** that reinforce specific essential skills such as differentiation, setting up integrals, vector products, drawing free-body diagrams, simplifying series and parallel circuits, or ray tracing.

- **QR** codes at the end of each chapter link to resources on Mastering Physics, including video tutorials. These "Pause and predict" videos of key physics concepts ask students to submit a prediction before they see the outcome. The videos are also available in the Study Area of Mastering and in the Pearson eText.
- **GOT IT? boxes** that provide quick checks for students to test their conceptual understanding. Many of these use a multiple-choice or quantitative ranking format to probe student misconceptions and facilitate their use with classroom-response systems.
- **Tips** that provide helpful problem-solving hints or warn against common pitfalls and misconceptions.
- **Chapter openers** that include a graphical indication of where the chapter lies in sequence as well as lists of the learning outcomes and of skills and knowledge needed for the chapter. Each chapter also includes an opening photo, captioned with a question whose answer should be evident after the student has completed the chapter.
- **Applications**, self-contained presentations typically shorter than half a page, provide interesting and contemporary instances of physics in the real world, such as bicycle stability; flywheel energy storage; laser vision correction; ultracapacitors; noise-cancelling headphones; wind energy; magnetic resonance imaging; smartphone gyroscopes; combined-cycle power generation; circuit models of the cell membrane; CD, DVD, and Blu-ray technologies; radiocarbon dating; and many, many more.
- **For Thought and Discussion** questions at the end of each chapter designed for peer learning or for self-study to enhance students' conceptual understanding of physics.
- **Annotated figures** that adopt the research-based approach of including simple "instructor's voice" commentary to help students read and interpret pictorial and graphical information.
- **Annotated equations**, new to the fourth edition, that feature a similar format to the annotated figures.
- **End-of-chapter** problems that begin with simpler exercises keyed to individual chapter sections and ramp up to more challenging and often multistep problems that synthesize chapter material. Context-rich problems focusing on real-world situations are interspersed throughout each problem set.
- **Chapter summaries** that combine text, art, and equations to provide a synthesized overview of each chapter. Each summary is hierarchical, beginning with the chapter's "big ideas," then focusing on key concepts and equations, and ending with a list of "applications"—specific instances or applications of the physics presented in the chapter.

Organization

This contemporary book is *concise*, *strategic*, and *progressive*, but it's *traditional* in its organization. Following the introductory Chapter 1, the book is divided into six parts. Part One (Chapters 2–12) develops the basic concepts of mechanics, including Newton's laws and conservation principles as applied to single particles and multiparticle systems. Part Two (Chapters 13–15) extends mechanics to oscillations, waves, and fluids. Part Three (Chapters 16–19) covers thermodynamics. Part Four (Chapters 20–29) deals with electricity and magnetism. Part Five (Chapters 30–32) treats optics, first in the geometrical optics approximation and then including wave phenomena. Part Six (Chapters 33–39) introduces relativity and quantum physics. Each part begins with a brief description of its coverage, and ends with a conceptual summary and a challenge problem that synthesizes ideas from several chapters.

Essential University Physics is available in two paperback volumes, so students can purchase only what they need—making the low-cost aspect of this text even more attractive. Volume 1 includes Parts One, Two, and Three, mechanics through thermodynamics. Volume 2 contains Parts Four, Five, and Six, electricity and magnetism along with optics and modern physics.

Instructor Supplements

Note: For convenience, all of the following instructor supplements can be downloaded from the Instructor's Resource Area of Mastering™ Physics (www.masteringphysics.com).

Name of Supplement	Instructor or Student Supplement	Description
Mastering™ Physics (www.masteringphysics.com) (9781292350226)	Supplement for Students and Instructors	Mastering Physics from Pearson is the most advanced physics homework and tutorial system available. This online homework and tutoring system guides students through the toughest topics in physics with self-paced tutorials that provide individualized coaching. These assignable, in-depth tutorials are designed to coach students with hints and feedback specific to their individual errors. Instructors can also assign end-of-chapter problems from every chapter, including multiple-choice questions, section-specific exercises, and general problems. Quantitative problems can be assigned with numerical answers and randomized values (with significant figure feedback) or solutions. The Mastering gradebook records scores for all automatically graded assignments in one place, while diagnostic tools give instructors access to rich data to assess student understanding and misconceptions. http://www.masteringphysics.com.
Pearson eText	Supplement for Students and Instructors	The fourth edition of *Essential University Physics* features Pearson eText. The Pearson eText offers students the power to create notes, highlight text in different colors, create bookmarks, zoom, and view single or multiple pages. Professors also have the ability to annotate the text for their course and hide chapters not covered in their syllabi.
Instructor Solutions Manual–Download Only (9781292350196)	Supplement for Instructors	Prepared by John Beetar, the Instructor Solutions Manual contains solutions to all end-of-chapter exercises and problems, written in the Interpret/Develop/Evaluate/Assess (IDEA) problem-solving framework.
Instructor Resources Materials–Download Only (9781292350189)	Supplement for Instructors	Includes all the art, photos, and tables from the book in JPEG format for use in classroom projection or when creating study materials and tests. Also available are downloadable files of the Instructor Solutions Manual and "Clicker Questions," including GOT IT? Clickers, for use with classroom-response systems. Each chapter also has a set of PowerPoint® lecture outlines. These resources are downloadable from the 'Instructor's Resources' area within Mastering Physics. They are also downloadable from the catalog page for Wolfson's *Essential University Physics*, 4th edition, Global Edition at www.pearsonglobaleditions.com
TestGen Test Bank–Download Only (9781292350240)	Supplement for Instructors	The TestGen Test Bank contains more than 2000 multiple-choice, true-false, and conceptual questions. More than half of the questions can be assigned with randomized numerical values.

Acknowledgments

A project of this magnitude isn't the work of its author alone. First and foremost among those I thank for their contributions are the now several thousand students I've taught in calculus-based introductory physics courses at Middlebury College. Over the years your questions have taught me how to convey physics ideas in many different ways appropriate to your diverse learning styles. You've helped identify the "sticking points" that challenge introductory physics students, and you've showed me ways to help you avoid and "unlearn" the misconceptions that many students bring to introductory physics.

Thanks also to the numerous instructors and students from around the world who have contributed valuable suggestions for improvement of this text. I've heard you, and you'll find many of your ideas implemented in this fourth edition of *Essential University Physics*. And special thanks to my Middlebury physics colleagues who have taught from this text and who contribute valuable advice and insights on a regular basis: Jeff Dunham, Mike Durst, Angus Findlay, Eilat Glikman, Anne Goodsell, Noah Graham, Chris Herdmann, Paul Hess, Susan Watson, and especially Steve Ratcliff.

Experienced physics instructors thoroughly reviewed every chapter of this book, and reviewers' comments resulted in substantive changes—and sometimes in major rewrites—to the first drafts of the manuscript. We list these reviewers below. But first, special thanks are due to several individuals who made exceptional contributions to the quality and in some cases the very existence of this book. First is Professor Jay Pasachoff of Williams College, whose willingness more than three decades ago to take a chance on an inexperienced coauthor has made writing introductory physics a large part of my professional career. Dr. Adam Black, former physics editor at Pearson, had the vision to see promise in a new introductory text that would respond to the rising chorus of complaints about massive, encyclopedic, and expensive physics texts. Brad Patterson, developmental editor for the first edition, brought his graduate-level knowledge of physics to a role that made him a real collaborator. Brad is responsible for many of the book's innovative features, and it was a pleasure to work with him. John Murdzek continued Brad's excellent tradition of developmental editing on this fourth edition. We've gone to great lengths to make this book as error-free as possible, and much of the credit for that happy situation goes to John Beetar, who solved every new and revised end-of-chapter problem and updated the solutions manual, and to Edward Ginsberg, who blind-solved all the new problems and thus provided a third check on the answers.

I also wish to thank Nancy Whilton, Jeanne Zalesky, and Tiffany Mok at Pearson Education, and Kim Fletcher at Integra, for their highly professional efforts in shepherding this book through its vigorous production schedule. Finally, as always, I thank my family, my colleagues, and my students for the patience they showed during the intensive process of writing and revising this book.

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Preface to the Student

Welcome to physics! Maybe you're taking introductory physics because you're majoring in a field of science or engineering that requires a semester or two of physics. Maybe you're premed, and you know that medical schools are interested in seeing calculus-based physics on your transcript. Perhaps you're really gung-ho and plan to major in physics. Or maybe you want to study physics further as a minor associated with related fields like math, computer science, or chemistry or to complement a discipline like economics, environmental studies, or even music. Perhaps you had a great high-school physics course, and you're eager to continue. Maybe high-school physics was an academic disaster for you, and you're approaching this course with trepidation. Or perhaps this is your first experience with physics. Whatever your reason for taking introductory physics, welcome!

And whatever your reason, my goals for you are similar: I'd like to help you develop an understanding and appreciation of the physical universe at a deep and fundamental level; I'd like you to become aware of the broad range of natural and technological phenomena that physics can explain; and I'd like to help you strengthen your analytic and quantitative problem-solving skills. Even if you're studying physics only because it's a requirement, I want to help you engage the subject and come away with an appreciation for this fundamental science and its wide applicability. One of my greatest joys as a physics teacher is having students tell me after the course that they had taken it only because it was required, but found they really enjoyed their exposure to the ideas of physics.

Physics is fundamental. To understand physics is to understand how the world works, both in everyday life and on scales of time and space so small and so large as to defy intuition. For that reason I hope you'll find physics fascinating. But you'll also find it challenging. Learning physics will challenge you with the need for precise thinking and language; with subtle interpretations of even commonplace phenomena; and with the need for skillful application of mathematics. But there's also a simplicity to physics, a simplicity that results because there are in physics only a very few really basic principles to learn. Those succinct principles encompass a universe of natural phenomena and technological applications.

I've been teaching introductory physics for decades, and this book distills everything my students have taught me about the many different ways to approach physics; about the subtle misconceptions students often bring to physics; about the ideas and types of problems that present the greatest challenges; and about ways to make physics engaging, exciting, and relevant to your life and interests.

I have some specific advice for you that grows out of my long experience teaching introductory physics. Keeping this advice in mind will make physics easier (but not necessarily easy!), more interesting, and, I hope, more fun:

- *Read* each chapter thoroughly and carefully before you attempt to work any problem assignments. I've written this text with an informal, conversational style to make it engaging. It's not a reference work to be left alone until you need some specific piece of information; rather, it's an unfolding "story" of physics—its big ideas and their applications in quantitative problem solving. You may think physics is hard because it's mathematical, but in my long experience I've found that failure to *read* thoroughly is the biggest single reason for difficulties in introductory physics.
- *Look for the big ideas.* Physics isn't a hodgepodge of different phenomena, laws, and equations to memorize. Rather, it's a few big ideas from which flow myriad applications, examples, and special cases. In particular, don't think of physics as a jumble of equations that you choose among when solving a problem. Rather, identify those few big ideas and the equations that represent them, and try to see how seemingly distinct examples and special cases relate to the big ideas.
- *When working problems, re-read* the appropriate sections of the text, paying particular attention to the worked examples. Follow the IDEA strategy described in Chapter 1 and used in every subsequent worked example. Don't skimp on the final Assess step. Always ask: Does this answer make sense? How can I understand my answer in relation to the big principles of physics? How was this problem like others I've worked, or like examples in the text?
- *Don't confuse physics with math.* Mathematics is a tool, not an end in itself. Equations in physics aren't abstract math, but statements about the physical world. Be sure you understand each equation for what it says about physics, not just as an equality between mathematical terms.
- *Work with others.* Getting together informally in a room with a blackboard is a great way to explore physics, to clarify your ideas and help others clarify theirs, and to learn from your peers. I urge you to discuss physics problems together with your classmates, to contemplate together the "For Thought and Discussion" questions at the end of each chapter, and to engage one another in lively dialog as you grow your understanding of physics, the fundamental science.

Video Tutor Demonstrations

Video tutor demonstrations can be accessed by scanning the QR code at the end of each chapter in the textbook using a smartphone. They are also available in the Study Area and Instructor's Resource Area on Mastering Physics and in the eText. Practice Exams and Dynamic Study Modules, which can be used to prepare for exams, are also available in Mastering Physics.

Chapter	Video Tutor Demonstration
2	Balls Take High and Low Tracks
3	Dropped and Thrown Balls
3	Ball Fired from Cart on Incline
3	Ball Fired Upward from Accelerating Cart
3	Range of a Gun at Two Firing Angles
3	Independent Horizontal and Vertical Motion
4	Cart with Fan and Sail
4	Ball Leaves Circular Track
4	Suspended Balls: Which String Breaks?
4	Weighing a Hovering Magnet
5	Tension in String between Hanging Weights
5	Rotational Motion—Loop the Loop
6	Work and Kinetic Energy
7	Chin Basher?
9	Balancing a Meter Stick
9	Water Rocket
9	Happy/Sad Pendulums
9	Conservation of Linear Momentum
10	Canned Food Race
11	Spinning Person Drops Weights
11	Off-Center Collision
11	Conservation of Vector Angular Momentum
11	Conservation of Angular Momentum
12	Walking the Plank
13	Vibrating Rods
13	Resonance of Everyday Items
14	Out-of-Phase Speakers
15	Pressure in Water and Alcohol
15	Water Level in Pascal's Vases
15	Weighing Weights in Water
15	Air Jet Blows between Bowling Balls
15	Bernoulli's Principle—Venturi Tubes
16	Heating Water and Aluminum
16	Water Balloon Held over Candle Flame
16	Candle Chimneys
20	Charged Rod and Aluminum Can
21	Electroscope in Conducting Shell
22	Charged Conductor with Teardrop Shape
23	Discharge Speed for Series and Parallel Capacitors
24	Resistance in Copper and Nichrome
25	Bulbs Connected in Series and in Parallel
26	Magnet and Electron Beam
26	Current-Carrying Wire in Magnetic Field
27	Eddy Currents in Different Metals
29	Parallel-Wire Polarizer for Microwaves
29	Point of Equal Brightness between Two Light Sources
31	Partially Covering a Lens
36	Illuminating Sodium Vapor with Sodium and Mercury Lamps

PART FOUR

Electromagnetism

Electricity constitutes a significant portion of humankind's energy, as evidenced by this composite satellite image of Earth at night. Nearly all that electrical energy is produced by generators, devices that exploit an intimate relation between electricity and magnetism.

OVERVIEW

Electromagnetism is one of the fundamental forces, and it governs the behavior of matter from the atomic scale to the macroscopic world. Electromagnetic technology, from computer microchips to cell phones and on to large electric motors and generators, is essential to modern society. Even our bodies rely heavily on electromagnetism: Electric signals pace our heartbeat, electrochemical processes transmit nerve impulses, and the electric structure of cell membranes mediates the flow of materials into and out of the cell.

Four fundamental laws describe electricity and magnetism. Two deal separately with the two phenomena, while the others reveal profound connections that make electricity and magnetism aspects of a single phenomenon that we call electromagnetism. In this part you'll come to understand those fundamental laws, learn how electromagnetism determines the structure and behavior of nearly all matter, and explore the electromagnetic technologies that play so important a role in your life. Finally, you'll see how the laws of electromagnetism lead to electromagnetic waves and thus help us understand the nature of light.

Electric Charge, Force, and Field

Learning Outcomes

After finishing this chapter you should be able to:

LO 20.1 Describe electric charge as a fundamental property of matter.

LO 20.2 Use Coulomb's law to calculate the forces between charges.

LO 20.3 Use the superposition principle to calculate forces involving multiple charges.

LO 20.4 Describe the concept of electric field.

LO 20.5 Determine the fields of electric charge distributions using superposition.

LO 20.6 Describe the electric dipole and the field it produces.

LO 20.7 Determine the fields of continuous charge distributions by integration.

LO 20.8 Determine the motion of charged particles in electric fields.

LO 20.9 Determine forces and torques on electric dipoles in electric fields.

Skills & Knowledge You'll Need

- The concept of force and Newton's second law (Sections 4.2 and 4.3)
- The gravitational field (Section 8.5)
- Integration techniques for physics (Tactics 9.1)
- The concept of torque, expressed as a cross product (Section 11.2)

What holds your body together? What keeps a skyscraper standing? What holds your car on the road as you round a turn? What governs the electronic circuitry in your computer or smartphone, or provides the tension in your climbing rope? What enables a plant to make sugar from sunlight and simple chemicals? What underlies the awesome beauty of lightning? The answer, in all cases, is the **electric force**. With the exception of gravity, all the forces we've encountered in mechanics—including tension forces, normal forces, compression forces, and friction—are based on electric interactions; so are the forces responsible for all of chemistry and biology. The electric force, in turn, involves a fundamental property of matter—namely, electric charge.

What's the fundamental criterion for initiating a lightning strike?

20.1 Electric Charge

LO 20.1 *Describe electric charge as a fundamental property of matter.*

Electric charge is an intrinsic property of the electrons and protons that, along with uncharged neutrons, make up ordinary matter. What is electric charge? At the most fundamental level we don't know. We don't know what mass "really" is either, but we're familiar with it because we've spent our lives pushing objects around. Similarly, our knowledge of electric charge results from observing the behavior of charged objects.

Charge comes in two varieties, which Benjamin Franklin designated *positive* and *negative*. Those names are useful because the total charge on an object—the object's **net charge**—is the algebraic sum of its constituent charges. Like charges repel, and opposites attract, a fact that constitutes a qualitative description of the electric force.

Quantities of Charge

All electrons carry the same charge, and all protons carry the same charge. The proton's charge has *exactly* the same magnitude as the electron's, but with opposite sign. Given that electrons and protons differ substantially in other properties—like mass—this electric relation is remarkable. Exercise 11 shows how dramatically different our world would be if there were even a slight difference between the magnitudes of the electron and proton charges.

The magnitude of the electron or proton charge is the **elementary charge** e. Electric charge is **quantized**; that is, it comes only in discrete amounts. In a famous experiment in 1909, the American physicist R. A. Millikan measured the charge on small oil drops and found it was always a multiple of a basic value we now know as the elementary charge.

Elementary particle theories show that the fundamental charge is actually $\frac{1}{3}e$. Such "fractional charges" reside on quarks, the building blocks of protons, neutrons, and many other particles. Quarks always join to produce particles with integer multiples of the full elementary charge, and it seems impossible to isolate individual quarks.

The SI unit of charge is the **coulomb** (C), named for the French physicist Charles Augustin de Coulomb (1736–1806). From the late 19th century to the early 21st century, the coulomb was defined in terms of electric current and time—a definition that was difficult to implement in practice. The 2019 revision of the SI gave the coulomb a much simpler definition. Now, the elementary charge is defined to be exactly $1.602176634 \times 10^{-19}$ C. The coulomb is therefore the number of elementary charges equal to the inverse of this number. For our purposes, that's about 6.24×10^{18} elementary charges.

Charge Conservation

Electric charge is a conserved quantity, meaning that the net charge in a closed region remains constant. Charged particles may be created or annihilated, but always in pairs of equal and opposite charge. The net charge always remains the same.

GOT IT?

20.1 The proton is a composite particle composed of three quarks, all of which are either *up quarks* (u; charge $+\frac{2}{3}e$) or *down quarks* (d; charge $-\frac{1}{3}e$). (More on quarks in Chapter 39.) Which of these quark combinations is the proton? (a) *udd*; (b) *uuu*; (c) *uud*; (d) *ddd*

20.2 Coulomb's Law

LO 20.2 *Use Coulomb's law to calculate the forces between charges.*

LO 20.3 *Use the superposition principle to calculate forces involving multiple charges.*

Rub a balloon; it gets charged and sticks to your clothing. Charge another balloon, and the two repel (Fig. 20.1). Socks cling to your clothes as they come from the dryer, and bits of Styrofoam cling annoyingly to your hands. Walk across a carpet, and you'll feel a shock when you touch a doorknob. All these are common examples where you're directly aware of electric charge.

Electricity would be unimportant if the only significant electric interactions were these obvious ones. In fact, the electric force dominates all interactions of everyday matter, from the motion of a car to the movement of a muscle. It's just that matter on a large scale

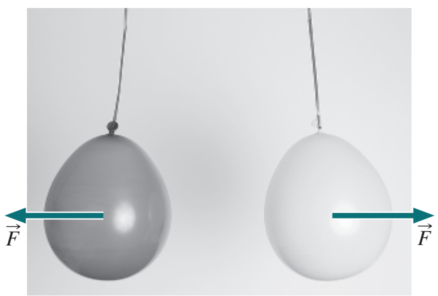


FIGURE 20.1 Two balloons carrying similar electric charges repel each other.

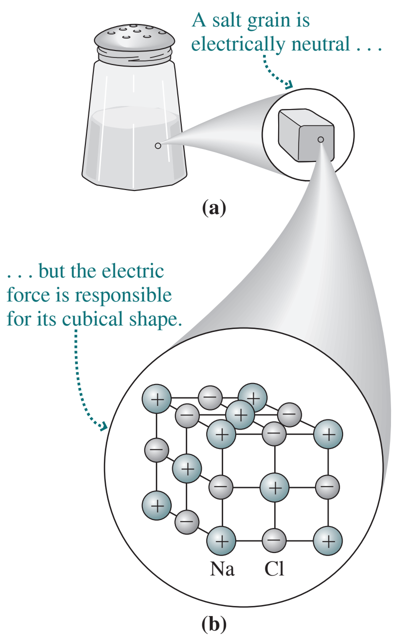


FIGURE 20.2 (a) A single salt grain is electrically neutral, so the electric force isn't obvious. (b) Actually, the electric force determines the structure of salt.

is almost perfectly neutral, meaning it carries zero net charge. Therefore, electric effects aren't obvious. But at the molecular level, the electric nature of matter is immediately evident (Fig. 20.2).

Attraction and repulsion of electric charges imply a force. Joseph Priestley and Charles Augustin de Coulomb investigated this force in the late 1700s. They found that the force between two charges acts along the line joining them, with the magnitude proportional to the product of the charges and inversely proportional to the square of the distance between them. **Coulomb's law** summarizes these results:

$\vec{F}_{12}$ is the force charge q_1 exerts on charge q_2.

k is approximately $9.0 \times 10^9\,\mathrm{N{\cdot}m^2/C^2}$.

q_1 and q_2 are two charges.

$$\vec{F}_{12} = \frac{kq_1q_2}{r^2}\hat{r} \quad \text{(Coulomb's law)} \tag{20.1}$$

r is the distance between the two charges.

$\hat{r}$ is a unit vector that points from q_1 toward q_2 regardless of the signs of the charges.

where $\vec{F}_{12}$ is the force charge q_1 exerts on q_2 and r is the distance between the charges. In SI the proportionality constant k has the approximate value $9.0 \times 10^9\,\mathrm{N{\cdot}m^2/C^2}$. Force is a vector, and $\hat{r}$ is a unit vector that helps determine its direction. Figure 20.3 shows that $\hat{r}$ lies on a line passing through the two charges and points in the direction *from* q_1 *toward* q_2. Reverse the roles of q_1 and q_2, and you'll see that $\vec{F}_{21}$ has the same magnitude as $\vec{F}_{12}$ but the opposite direction; thus Coulomb's law obeys Newton's third law. Figure 20.3 also shows that the force is in the same direction as the unit vector when the charges have the same sign, but opposite the unit vector when the charges have different signs. Thus Coulomb's law accounts for the fact that like charges repel and opposites attract.

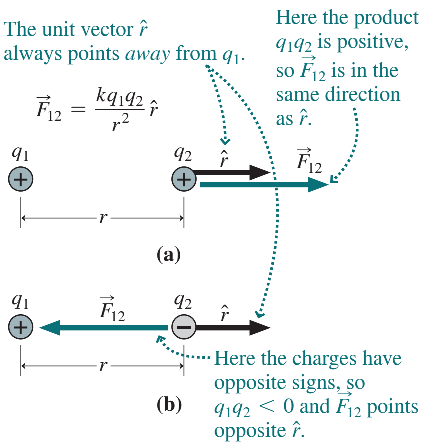


FIGURE 20.3 Quantities in Coulomb's law for calculating the force $\vec{F}_{12}$ that q_1 exerts on q_2.

PROBLEM-SOLVING STRATEGY 20.1 Coulomb's Law

The key to using Coulomb's law is to remember that force is a vector, and to realize that Coulomb's law in the form of Equation 20.1 gives both the magnitude and direction of the electric force. Dealing carefully with vector directions is especially important in situations with more than two charges.

INTERPRET First, make sure you're dealing with the electric force alone. Identify the charge or charges on which you want to calculate the force. Next, identify the charge or charges producing the force. These comprise the **source charge**.

DEVELOP Begin with a drawing that shows the charges, as in Fig. 20.4. If you're given charge coordinates, place the charges on the coordinate system; if not, choose a suitable coordinate system. For each source charge, determine the unit vector(s) in Equation 20.1. If the charges lie along or parallel to a coordinate axis, then the unit vector will be one of the unit vectors $\hat{\imath}$, $\hat{\jmath}$, or $\hat{k}$, perhaps with a minus sign. In Fig. 20.4, the force on q_3 due to q_1 is such a case. When the two charges don't lie on a coordinate axis, like q_1 and q_2 in Fig. 20.4, you can find the unit vector by noting that the displacement vector $\vec{r}_{12}$ points in the desired direction, from the source charge to the charge experiencing the force. Dividing $\vec{r}_{12}$ by its own magnitude then gives the unit vector in the direction of $\vec{r}_{12}$; that is, $\hat{r} = \vec{r}_{12}/r_{12}$.

EVALUATE For each source charge, determine the electric force using Equation 20.1,

$$\vec{F}_{12} = (kq_1q_2/r^2)\hat{r}$$

with $\hat{r}$ the unit vector you've just found.

ASSESS As always, assess your answer to see that it makes sense. Is the direction of the force you found consistent with the signs and placements of the charges giving rise to the force?

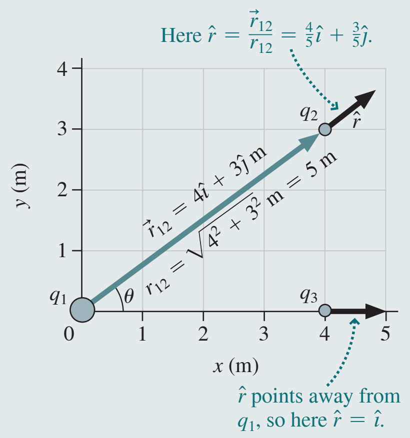


FIGURE 20.4 Finding unit vectors.

GOT IT? **20.2** Charge q_1 is located at $x = 1$ m, $y = 0$. What should you use for the unit vector $\hat{r}$ in Coulomb's law if you're calculating the force that q_1 exerts on a charge q_2 located (1) at the origin and (2) at the point $x = 0$, $y = 1$ m? Explain why you can answer without knowing the sign of either charge.

EXAMPLE 20.1 Finding the Force: Two Charges

A 1.0-μC charge is at $x = 1.0$ cm, and a -1.5-μC charge is at $x = 3.0$ cm. What force does the positive charge exert on the negative one? How would the force change if the distance between the charges tripled?

INTERPRET Following our strategy, we identify the -1.5-μC charge as the one on which we want to find the force and the 1-μC charge as the source charge.

DEVELOP We're given the coordinates $x_1 = 1.0$ cm and $x_2 = 3.0$ cm. Our drawing, Fig. 20.5, shows both charges at their positions on the x-axis. With the source charge q_1 to the left, the unit vector in the direction from q_1 toward q_2 is $\hat{\imath}$.

EVALUATE Now we use Coulomb's law to evaluate the force:

$$\vec{F}_{12} = \frac{kq_1q_2}{r^2}\hat{r}$$

$$= \frac{(9.0 \times 10^9\ \mathrm{N{\cdot}m^2/C^2})(1.0 \times 10^{-6}\ \mathrm{C})(-1.5 \times 10^{-6}\ \mathrm{C})}{(0.020\ \mathrm{m})^2}\hat{\imath}$$

$$= -34\hat{\imath}\ \mathrm{N}$$

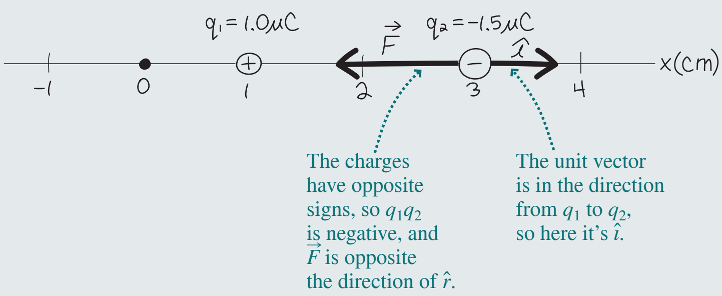


FIGURE 20.5 Sketch for Example 20.1.

This force is for a separation of 2 cm; if that distance tripled, the force would drop by a factor of $1/3^2$, to $-3.8\hat{\imath}$ N.

ASSESS Make sense? Although the unit vector $\hat{\imath}$ points in the $+x$-direction, the charges have opposite signs and that makes the force direction opposite the unit vector, as shown in Fig. 20.5. In simpler terms, we've got two opposite charges, so they attract. That means the force exerted on a charge at $x = 3$ cm by an opposite charge at $x = 1$ cm had better be in the $-x$-direction.

CONCEPTUAL EXAMPLE 20.1 **Gravity and the Electric Force**

The electric force between elementary particles is far stronger than the gravitational force, yet gravity is much more obvious in everyday life. Why?

EVALUATE Gravity and the electric force obey similar inverse-square laws, and the magnitude of the force is proportional to the product of the masses or charges. There's a big difference, though: There's only one kind of mass, and gravity is always attractive, so large concentrations of mass—like a planet—result in strong gravitational forces. But charge comes in two varieties, and opposites attract, so large accumulations of matter tend to be electrically neutral, in which case large-scale electrical interactions aren't obvious.

ASSESS Ironically, it's the very strength of the electric force that makes it less obvious in everyday life. Opposite charges bind strongly, making bulk matter electrically neutral and its electrical interactions subtle.

MAKING THE CONNECTION Compare the magnitudes of the electric and gravitational forces between an electron and a proton.

EVALUATE Equation 8.1 gives the gravitational force: $F_g = Gm_e m_p/r^2$. Equation 20.1 gives the electric force: $|F_E| = ke^2/r^2$, where we wrote e^2 because the electron and proton charges have the same magnitude. We aren't given the distance, but that doesn't matter because both forces have the same inverse-square dependence. The ratio of the force magnitudes is huge: $F_E/F_g = ke^2/Gm_e m_p = 2.3 \times 10^{39}$!

Point Charges and the Superposition Principle

Coulomb's law is strictly true only for **point charges**—charged objects of negligible size. Electrons and protons can usually be treated as point charges; so, approximately, can any two charged objects if their separation is large compared with their size. But often we're interested in the electric effects of **charge distributions**—arrangements of charge spread over space. Charge distributions are present in molecules, memory cells in your computer, your heart, and thunderclouds. We need to combine the effects of two or more charges to find the electric effects of such charge distributions.

Figure 20.6 shows two charges q_1 and q_2 that constitute a simple charge distribution. We want to know the net force these exert on a third charge q_3. To find that net force, you might calculate the forces $\vec{F}_{13}$ and $\vec{F}_{23}$ from Equation 20.1, and then vectorially add them. And you'd be right: The force that q_1 exerts on q_3 is unaffected by the presence of q_2, and vice versa, so you can apply Coulomb's law separately to the pairs q_1q_3 and q_2q_3 and then combine the results. That may seem obvious, but nature needn't have been so simple.

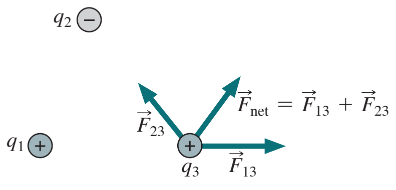


FIGURE 20.6 The superposition principle lets us add vectorially the forces from two or more charges.

The fact that electric forces add vectorially is called the **superposition principle**. Our confidence in this principle is ultimately based on experiments showing that electric and indeed electromagnetic phenomena behave according to the principle. With superposition we can solve relatively complicated problems by breaking them into simpler parts. If the superposition principle didn't hold, the mathematical description of electromagnetism would be far more complicated.

Although the force that one point charge exerts on another decreases with the inverse square of the distance between them, the same is not necessarily true of the force resulting from a charge distribution. The next example provides a case in point.

EXAMPLE 20.2 **Finding the Force: Raindrops**
Worked Example with Variation Problems

Charged raindrops are ultimately responsible for lightning, producing substantial electric charge within specific regions of a thundercloud. Suppose two drops with equal charge q are on the x-axis at $x = \pm a$. Find the electric force on a third drop with charge Q at an arbitrary point on the y-axis.

INTERPRET Coulomb's law and the superposition principle apply, and we identify Q as the charge for which we want the force. The two charges q are the source charges.

DEVELOP Figure 20.7 is our drawing, showing the charges, the individual force vectors, and their sum. The drawing shows that the distance r in Coulomb's law is the hypotenuse $\sqrt{a^2 + y^2}$. It's clear from symmetry that the net force is in the y-direction, so we need to find only the y-components of the unit vectors. The y-components are clearly the same for each, and the drawing shows that they're given by $\hat{r}_y = y/\sqrt{a^2 + y^2}$.

EVALUATE From Coulomb's law, the y-component of the force from each q is $F_y = (kqQ/r^2)\hat{r}_y$, and the net force on Q becomes

$$\vec{F} = 2\left(\frac{kqQ}{a^2 + y^2}\right)\left(\frac{y}{\sqrt{a^2 + y^2}}\right)\hat{j} = \frac{2kqQy}{(a^2 + y^2)^{3/2}}\hat{j}$$

The factor of 2 comes from the two charges q, which contribute equally to the net force.

ASSESS Make sense? Evaluating $\vec{F}$ at $y = 0$ gives zero force. Here, midway between the two charges, Q experiences equal but opposite forces and the net force is zero. At large distances $y \gg a$, on the other hand, we can neglect a^2 compared with y^2, and the force becomes $\vec{F} = k(2q)Q\hat{j}/y^2$. This is just what we would expect from a single charge $2q$ a distance y from Q—showing that the system of two charges acts like a single charge $2q$ at distances that are large compared with the charge separation. In between our two extremes the behavior of force with distance is more complicated; in fact, its magnitude initially increases as Q moves away from the origin and then begins to decrease.

In drawing Fig. 20.7, we tacitly assumed that q and Q have the same signs. But our analysis holds even if they don't; then the product qQ is negative, and the forces actually point opposite the directions shown in Fig. 20.7.

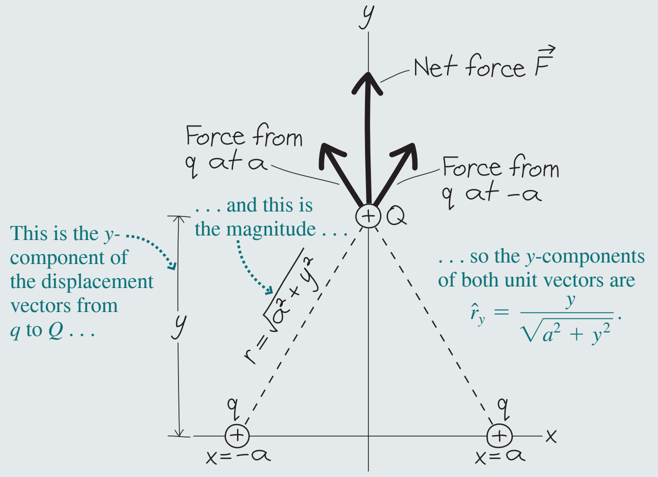


FIGURE 20.7 The force on Q is the vector sum of the forces from the individual charges.

20.3 The Electric Field

LO 20.4 *Describe the concept of electric field.*

In Chapter 8 we defined the gravitational field at a point as the gravitational force per unit mass that an object at that point would experience. In that context, we can think of $\vec{g}$ as the *force per unit mass* that any object would experience due to Earth's gravity. So we can picture the gravitational field as a set of vectors giving the magnitude and direction of the gravitational force per unit mass at each point, as shown in Fig. 20.8*a* below.

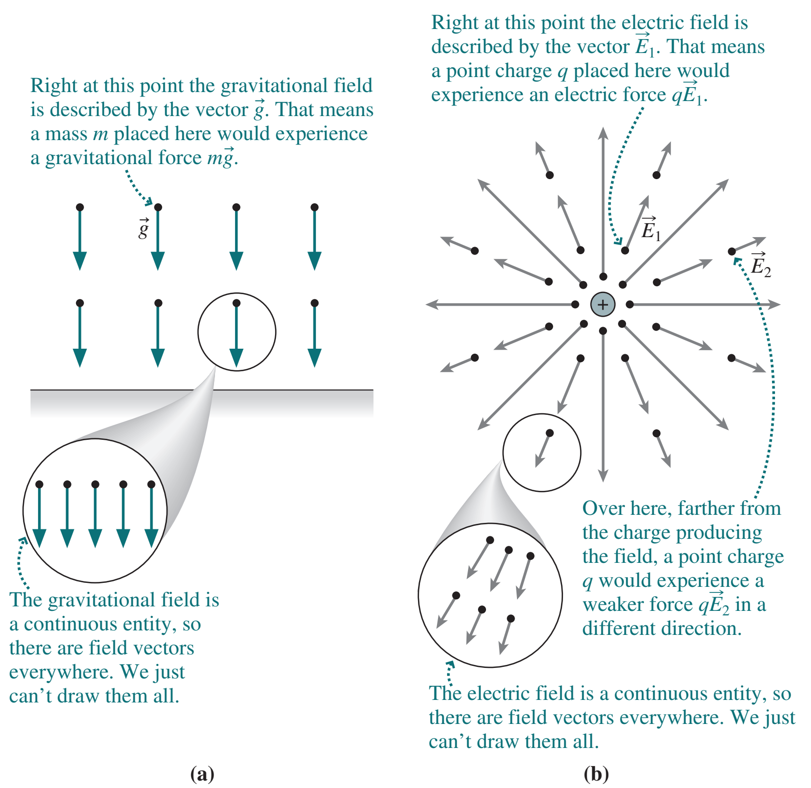
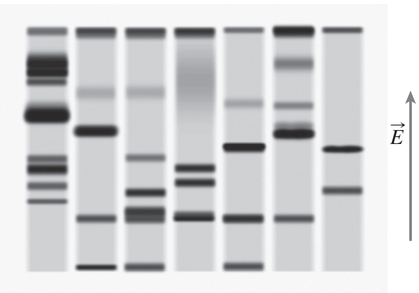


FIGURE 20.8 (a) Gravitational and (b) electric fields, here represented as sets of vectors.

APPLICATION Electrophoresis

Electrophoresis is a widely used application of electric fields for separating molecules by size and molecular weight. It's especially useful in biochemistry and molecular biology for distinguishing larger molecules like proteins and DNA fragments. In the commonly used *gel electrophoresis*, molecules carrying electric charge move through a semisolid but permeable gel under the influence of a uniform electric field; the greater the charge, the greater the electric force. The gel exerts a retarding force that increases with increasing molecular size, with the result that each molecular species moves at a velocity that depends on its size and charge. After a given time, the electric field is switched off. The locations of the molecules then serve as indicators of their size, with the molecules that traveled farthest being the smallest. The photo shows a typical gel electrophoresis result. Here DNA fragments were introduced into the seven channels at the top of the gel and then moved downward; their final locations indicate molecular size. The smaller molecules—those with fewer nucleotide base pairs—end up farther down on the gel. The electric field is shown by the arrow; it needs to point upward because DNA fragments carry a negative charge.



We can do the same thing with the electric force, defining the **electric field** as the force per unit charge:

> The electric field at any point is the force per unit charge that a charge would experience at that point. Mathematically,

$\vec{E}$ is the electric field at any point.

You can determine $\vec{E}$ by measuring the electric force $\vec{F}$ …

$$\vec{E} = \frac{\vec{F}}{q} \quad \text{(electric field)} \tag{20.2a}$$

… on a small test charge q.

The electric field exists at every point in space. When we represent the field by vectors, we can't draw one everywhere, but that doesn't mean there isn't a field at all points. Furthermore, we draw vectors as extended arrows, but each vector represents the field at only one point—namely, the tail end of the vector. Figure 20.8*b* illustrates this for the electric field of a point charge.

The field concept leads to a shift in our thinking about forces. Instead of the action-at-a-distance idea that Earth reaches across empty space to pull on the Moon, the field concept says that Earth creates a gravitational field and the Moon responds to the field at its location. Similarly, a charge creates an electric field throughout the space surrounding it. A second charge then responds to the field at its immediate location. Although the field reveals itself only through its effect on a charge, the field nevertheless exists at all points, whether or not charges are present. Right now you probably find the field concept a bit abstract, but as you advance in your study of electromagnetism you'll come to appreciate that fields are an essential feature of our universe, every bit as real as matter itself.

We can use Equation 20.2a as a prescription for measuring electric fields. Place a point charge at some location, measure the electric force it experiences, and divide by the charge to get the field. In practice, we need to be careful because the field generally arises from some distribution of source charges. If the charge we're using to probe the field—the **test charge**—is large, the field it creates may disturb the source charges, altering their configuration and thus the field they create. For that reason, it's important to use a very small test charge.

If we know the electric field $\vec{E}$ at a point, we can rearrange Equation 20.2a to find the force on any point charge q placed at that point:

$\vec{F}$ is the electric force … … on a charge q …

$$\vec{F} = q\vec{E} \quad \text{(electric force and field)} \tag{20.2b}$$

… at a point where the electric field is $\vec{E}$.

If the charge q is positive, then this force is in the same direction as the field, but if q is negative, then the force is opposite to the field direction.

Equations 20.2 show that the units of electric field are newtons per coulomb. Fields of hundreds to thousands of N/C are commonplace, while fields of 3 MN/C will tear electrons from air molecules. Sometimes we're interested in the magnitude of the field but not its direction. Then we can use Equations 20.2a and 20.2b without the vector signs. We'll often use the term *field strength* to be synonymous with the field's magnitude.

EXAMPLE 20.3 Force and Field: Inside a Lightning Storm

A charged raindrop carrying 10 μC experiences an electric force of 0.30 N in the $+x$-direction. What's the electric field at its location? What would the force be on a −5.0-μC drop at the same point?

INTERPRET In this problem we need to distinguish between electric force and electric field. The electric field exists with or without the charged raindrop present, and the electric force arises when the charged raindrop is in the electric field.

DEVELOP Knowing the electric force and the charge on the raindrop, we can use Equation 20.2a, $\vec{E} = \vec{F}/q$, to get the electric field. Once we know the field, we can use Equation 20.2b,

$\vec{F} = q\vec{E}$, to calculate the force that would act if a different charge were at the same point.

EVALUATE Equation 20.2a gives the electric field:

$$\vec{E} = \frac{\vec{F}}{q} = \frac{0.30\hat{\imath}\ \text{N}}{10\ \mu\text{C}} = 30\hat{\imath}\ \text{kN/C}$$

Acting on a -5.0-μC charge, this field would result in a force

$$\vec{F} = q\vec{E} = (-5.0\ \mu\text{C})(30\hat{\imath}\ \text{kN/C}) = -0.15\hat{\imath}\ \text{N}$$

ASSESS Make sense? The force on the second charge is opposite the direction of the field because now we've got a negative charge in the same field.

TIP **THE FIELD IS INDEPENDENT OF THE TEST CHARGE** Does the electric field in this example point in the $-x$-direction when the charge is negative? No. The field is independent of the particular charge experiencing that field. Here the electric field points in the $+x$-direction *no matter what charge* you put in the field. For a positive charge, the force $q\vec{E}$ points in the *same* direction as the field; for a negative charge, $q < 0$, the force is *opposite* the field.

The Field of a Point Charge

Once we know the field of a charge distribution, we can calculate its effect on other charges. The simplest charge distribution is a single point charge. Coulomb's law gives the force on a test charge q_{test} located a distance r from a point charge q: $\vec{F} = (kqq_{\text{test}}/r^2)\hat{r}$, where $\hat{r}$ is a unit vector pointing *away* from q. The electric field arising from q is the force per unit charge, or

The electric field $\vec{E}$ is the force per unit charge.

For a point charge, $\vec{E}$ depends on the charge q …

$$\vec{E} = \overbrace{\frac{\vec{F}}{q_{\text{test}}}} = \frac{kq}{r^2}\hat{r} \quad \text{(field of a point charge)} \tag{20.3}$$

… and on the distance r from the charge to the point where the field is being evaluated.

The unit vector $\hat{r}$ always points away from q, regardless of q's sign.

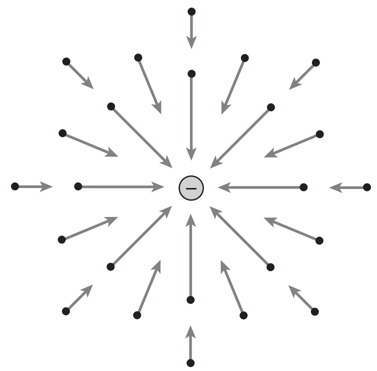


FIGURE 20.9 Field vectors for a negative point charge.

Since it's so closely related to Coulomb's law for the electric force, we also refer to Equation 20.3 as Coulomb's law. The equation contains no reference to the test charge q_{test} because the field of q exists independently of any other charge. Since $\hat{r}$ always points *away* from q, the direction of $\vec{E}$ is radially outward if q is positive and radially inward if q is negative. Figure 20.9 shows some field vectors for a negative point charge, analogous to those of the positive point charge in Fig. 20.8*b*.

GOT IT? **20.3** A positive point charge is located at the origin of an x–y coordinate system, and an electron is placed at a location where the electric field due to the point charge is given by $\vec{E} = E_0(\hat{\imath} + \hat{\jmath})$, where E_0 is positive. Is the direction of the force on the electron (a) toward the origin, (b) away from the origin, (c) parallel to the x-axis, or (d) impossible to determine without knowing the coordinates of the electron's position?

20.4 Fields of Charge Distributions

LO 20.5 *Determine the fields of electric charge distributions using superposition.*

LO 20.6 *Describe the electric dipole and the field it produces.*

LO 20.7 *Determine the fields of continuous charge distributions by integration.*

Since the electric force obeys the superposition principle, so does the electric field. That means the field of a charge distribution is the vector sum of the fields of the individual point charges making up the distribution:

The electric field $\vec{E}$ of a distribution of point charges… … is the sum of the fields of the individual point charges.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots = \sum_i \vec{E}_i = \sum_i \frac{kq_i}{r_i^2}\hat{r}_i \qquad (20.4)$$

Here the $\vec{E}_i$'s are the fields of the point charges q_i located at distances r_i from the point where we're evaluating the field—called, appropriately, the **field point**. The $\hat{r}_i$'s are unit vectors pointing *from* each point charge *toward* the field point. In principle, Equation 20.4 gives the electric field of *any* charge distribution. In practice, the process of summing the individual field vectors is often complicated unless the charge distribution contains relatively few charges arranged in a symmetric way.

Finding electric fields using Equation 20.4 involves the same strategy we introduced for finding the electric force; the only difference is that there's no charge to experience the force. The first step then involves identifying the field point. We still need to find the appropriate unit vectors and form the vector sum in Equation 20.4. Example 20.4 shows how this is done.

Sometimes we're interested in finding not the electric field but a point or points where the field is zero. Conceptual Example 20.2 explores such a case.

EXAMPLE 20.4 Finding the Field: Two Protons

Two protons are 3.6 nm apart. Find the electric field at a point between them, 1.2 nm from one of the protons. Then find the force on an electron at this point.

INTERPRET We follow our electric-force strategy, except that instead of identifying the charge experiencing the force, we identify the field point as being 1.2 nm from one proton. The source charges are the two protons; they produce the field we're interested in.

DEVELOP Let's have the protons define the x-axis, as drawn in Fig. 20.10. Then the unit vector $\hat{r}_1$ from the left-hand proton toward the field point (which we've marked P) is $+\hat{\imath}$, while $\hat{r}_2$ from the right-hand proton toward P is $-\hat{\imath}$.

EVALUATE We now evaluate the field at P using Equation 20.4:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{ke}{r_1^2}\hat{\imath} + \frac{ke}{r_2^2}(-\hat{\imath}) = ke\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right)\hat{\imath}$$

We wrote e for q here because the protons' charge is the elementary charge.

Unit vectors point *from* the source charges *toward* the field point P.

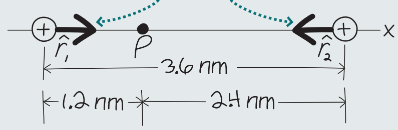


FIGURE 20.10 Finding the electric field at P.

Using $e = 1.6 \times 10^{-19}$ C, $r_1 = 1.2$ nm, and $r_2 = 2.4$ nm gives $\vec{E} = 750\hat{\imath}$ MN/C. An electron at P will therefore experience a force $\vec{F} = q\vec{E} = -e\vec{E} = -0.12\hat{\imath}$ nN.

ASSESS Make sense? The field points in the positive x-direction, reflecting the fact that P is closer to the left-hand proton with its stronger field at P. The force on the electron, on the other hand, is in the $-x$-direction; that's because the electron is negative (we used $q = -e$ for its charge), so the force it experiences is opposite the field. That field of almost 1 GN/C sounds huge—but that's not unusual at the microscopic scale, where we're close to individual elementary particles.

CONCEPTUAL EXAMPLE 20.2 Zero Field, Zero Force

A positive charge $+2Q$ is located at the origin, and a negative charge $-Q$ is at $x = a$. In which region of the x-axis is there a point where the force on a test charge—and therefore the electric field—is zero?

INTERPRET We're asked to locate qualitatively a point where the field is zero. Our sketch of the situation, Fig. 20.11, shows that the two charges divide the x-axis into three regions: (1) to the left of $2Q$ ($x < 0$), (2) between the charges ($0 < x < a$), and (3) to the right of $-Q$ ($x > a$). We need to determine which region could include a point where the electric force on a test charge is zero.

EVALUATE Consider what would happen to a positive test charge placed in each of these three regions. Anywhere in region (1), the test charge is closer to the charge with greater magnitude ($2Q$). That charge dominates throughout region (1), where our test charge would experience a repulsive force (to the left). The electric field, then, can't be zero in region (1). Between the two charges, the repulsive force from $2Q$ on a positive test charge points to the right; so does the attractive force from $-Q$. The field, therefore, can't be zero in region (2). That leaves region (3). Could the field be zero here? Put a positive test charge very close to $-Q$, and it experiences an attractive force toward the left. But far away, the distance between $2Q$ and $-Q$ becomes negligible. The fields of both charges drop off as the inverse square of the distance, so at large distances the field of the stronger charge will dominate. Therefore there *is* a point somewhere to the right of $-Q$ where the force on a test charge, and therefore the electric field, will be zero.

ASSESS This answer is consistent with our insight from Example 20.2 that when we get far from a charge distribution, it begins to resemble a point charge with the net charge of the distribution. Here that net charge is $2Q - Q = +Q$, so at large distances we should indeed have a field pointing away from the charge distribution—and that's to the right in region (3). Although we considered a positive test charge, you'll reach the same conclusion with a negative test charge.

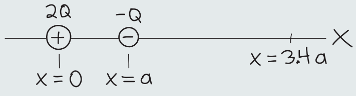


FIGURE 20.11 Where is the electric field zero? We've marked the answer, at $x = 3.4a$.

MAKING THE CONNECTION Find an expression for the position where the electric field in this example is zero.

EVALUATE In Fig. 20.11, we've taken the origin at $2Q$, so at any position x in region (3) we're a distance x from $2Q$ and a distance $x-a$ from $-Q$. Since we're to the right of both charges, the unit vector in Equation 20.3 for the point-charge field—a vector that always points away from the point charge—becomes $+\hat{\imath}$ for both charges. Applying Equation 20.3, $\vec{E} = (kq/r^2)\hat{r}$, for the fields of the two charges and summing gives

$$\vec{E} = \frac{k(2Q)}{x^2}\hat{\imath} + \frac{k(-Q)}{(x-a)^2}\hat{\imath}$$

If we set this expression to zero, we can cancel k, Q, and $\hat{\imath}$; inverting both sides of the remaining equation gives $x^2/2 = (x-a)^2$. Finally, taking the square root and solving for x gives the answer: $x = a\sqrt{2}/(\sqrt{2} - 1) \simeq 3.4a$. As a check, note that this point does indeed lie to the right of $x = a$. We've marked this point in Fig. 20.11.

The Electric Dipole

One of the most important charge distributions is the **electric dipole**, consisting of two point charges of equal magnitude but opposite sign. Many molecules are essentially dipoles, so understanding the dipole helps explain molecular behavior (Fig. 20.12). During contraction the heart muscle becomes essentially a dipole, and physicians performing electrocardiography are measuring, among other things, the strength and orientation of that dipole. Antennas used in wireless communications—including radio, TV, wifi, and cellphones—are often based on the dipole configuration.

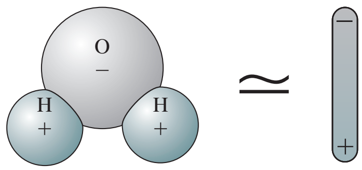


FIGURE 20.12 A water molecule behaves like an electric dipole. Its net charge is zero, but regions of positive and negative charge are separated.

EXAMPLE 20.5 The Electric Dipole: Modeling a Molecule

A molecule may be modeled approximately as a positive charge q at $x = a$ and a negative charge $-q$ at $x = -a$. Evaluate the electric field on the y-axis, and find an approximate expression valid at large distances ($|y| \gg a$).

INTERPRET Here's another example where we'll use our strategy in applying Equation 20.4 to calculate the field of a charge distribution. We identify the field point as being anywhere on the y-axis and the source charges as being $\pm q$.

DEVELOP Figure 20.13 is our drawing. The individual unit vectors point from the two charges toward the field point, but the *negative* charge contributes a field *opposite* its unit vector; we've indicated the individual fields in Fig. 20.13. Here symmetry makes the y-components cancel, giving a net field in the $-x$-direction. So we need only the x-components of the unit vectors, which Fig. 20.13 shows are $\hat{r}_{x-} = a/r$ for the negative charge at $-a$ and $\hat{r}_{x+} = -a/r$ for the positive charge at a.

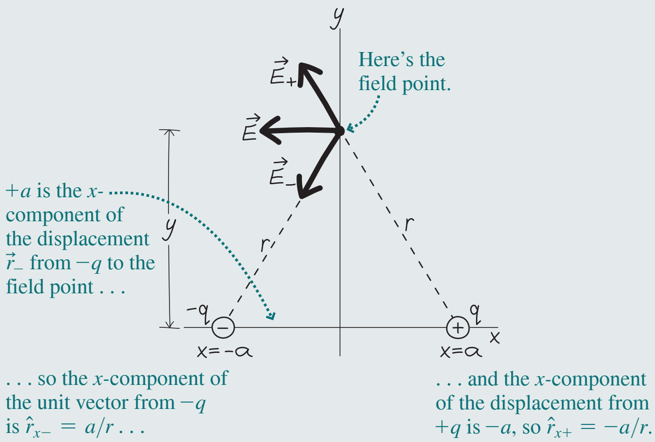


FIGURE 20.13 Finding the field of an electric dipole.

EVALUATE We then evaluate the field using Equation 20.4:

$$\vec{E} = \frac{k(-q)}{r^2}\left(\frac{a}{r}\right)\hat{\imath} + \frac{kq}{r^2}\left(-\frac{a}{r}\right)\hat{\imath} = -\frac{2kqa}{(a^2+y^2)^{3/2}}\hat{\imath}$$

where in the last step we used $r = \sqrt{a^2 + y^2}$. For $|y| \gg a$ we can neglect a^2 compared with y^2, giving

$$\vec{E} \simeq -\frac{2kqa}{|y|^3}\hat{\imath} \qquad (|y| \gg a)$$

(*continued*)

ASSESS Make sense? The dipole has no net charge, so at large distances its field can't have the inverse-square drop-off of a point-charge field. Instead the dipole field falls faster, here as $1/|y|^3$. Note that we were careful to put absolute value signs on y^3; that way, our result applies for both positive and negative values of y.

TIP **APPROXIMATIONS** Making approximations requires care. Here we're basically asking for the field when y is so large that a is negligible *compared with y*. So we neglect a^2 compared with y^2 when the two are summed, but we *don't* neglect a when it appears in the numerator, where it isn't being directly compared with y.

Example 20.5 shows that the dipole field at large distances decreases as the inverse *cube* of distance. Physically, that's because the dipole has zero *net* charge. Its field arises entirely from the slight separation of two opposite charges. Because of this separation, the dipole field isn't exactly zero, but it's weaker and more localized than the field of a point charge. Many complicated charge distributions exhibit the essential characteristic of a dipole—that is, they're neutral but consist of separated regions of positive and negative charge—and at large distances, such distributions all have essentially the same field configuration.

At large distances the dipole's physical characteristics q and a enter the equation for the electric field only through the product qa. We could double q and halve a, and the dipole's electric field would remain unchanged. At large distances, therefore, a dipole's electric properties are characterized completely by its **electric dipole moment** p, defined as the product of the charge q and the separation d between the two charges making up the dipole:

$$p = qd \quad \text{(dipole moment)} \tag{20.5}$$

In Example 20.5 the charge separation was $d = 2a$, so there the dipole moment was $p = 2aq$. In terms of the dipole moment, the field in Example 20.5 can then be written

$$\vec{E} = -\frac{kp}{|y|^3}\hat{\imath} \quad \left(\begin{array}{c}\text{dipole field for } |y| \gg a,\\ \text{on perpendicular bisector}\end{array}\right) \tag{20.6a}$$

You can show in Problem 54 that the field on the dipole axis is given by

$$\vec{E} = \frac{2kp}{|x|^3}\hat{\imath} \quad \left(\begin{array}{c}\text{dipole field}\\ \text{for } |x| \gg a, \text{ on axis}\end{array}\right) \tag{20.6b}$$

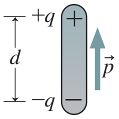


FIGURE 20.14 The dipole moment vector $\vec{p}$ has magnitude $p = qd$ and points from the negative toward the positive charge.

Because the dipole isn't spherically symmetric, its field depends not only on distance but also on orientation; for instance, Equations 20.6 show that the field along the dipole axis at a given distance is twice as strong as along the bisector. So it's important to know the orientation of a dipole in space, and therefore we generalize our definition of the dipole moment to make it a vector of magnitude $p = qd$ in the direction from the negative toward the positive charge (Fig. 20.14).

GOT IT? **20.4** Far from a charge distribution, you measure an electric field strength of 800 N/C. What will the field strength be if you double your distance from the charge distribution, if the distribution consists of (1) a point charge or (2) a dipole?

Continuous Charge Distributions

Although any charge distribution ultimately consists of pointlike electrons and protons, it would be impossible to sum all the field vectors from the 10^{23} or so particles in a typical piece of matter. Instead, it's convenient to make the approximation that charge is spread continuously over the distribution. If the charge distribution extends throughout a volume, we describe it in terms of the **volume charge density** ρ, with units of C/m^3. For charge distributions spread over surfaces or lines, the corresponding quantities are the **surface charge density** σ (C/m^2) and the **line charge density** λ (C/m).

To calculate the field of a continuous charge distribution, we divide the charged region into very many small charge elements dq, each small enough that it's essentially a point charge. Each dq then produces an electric field $d\vec{E}$ given by Equation 20.3: $d\vec{E} = (k\,dq/r^2)\hat{r}$. We then form the vector sum of all the $d\vec{E}$'s (Fig. 20.15). In the limit of infinitely many infinitesimally small dq's and their corresponding $d\vec{E}$'s, that sum becomes an integral and we have

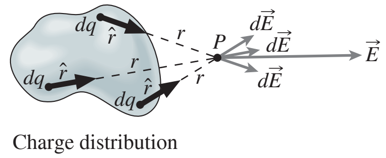


FIGURE 20.15 The electric field at P is the vector sum of the fields $d\vec{E}$ arising from the individual charge elements dq, each calculated using the appropriate distance r and unit vector $\hat{r}$.

The electric field $\vec{E}$ of a continuous distribution of charges …

dq is an infinitesimal charge element.

$\hat{r}$ is a unit vector that points away from dq, regardless of its sign.

$$\vec{E} = \int d\vec{E} = \int \frac{k\,dq}{r^2}\hat{r} \quad \left(\begin{array}{c}\text{field of a continuous}\\ \text{charge distribution}\end{array}\right) \qquad (20.7)$$

… is determined by integrating the fields $d\vec{E}$ of infinitesimal charge elements dq.

r is the distance from dq to the point where the field is being evaluated.

The limits of this integral include the entire charge distribution.

Calculating the field of a continuous charge distribution involves the same strategy we've already used: We identify the field point and the source charges—although now the source is a continuous charge distribution. Summing the individual field contributions now presents us with an integral, and that means writing the unit vectors $\hat{r}$ and distances r in terms of coordinates over which we can integrate. Setting up the integral involves the same strategy we outlined in Chapter 9 to find the center of mass of a continuous distribution of matter, and used again in Chapter 10 to find rotational inertias.

EXAMPLE 20.6 Evaluating the Field: A Charged Ring

A ring of radius a carries a charge Q distributed evenly over the ring. Find an expression for the electric field at any point on the axis of the ring.

INTERPRET We identify the field point as lying anywhere on the ring's axis, and the source charge as the entire ring.

DEVELOP Let's take the x-axis to coincide with the ring axis, with the center of the ring at $x = 0$ (Fig. 20.16). The figure shows that the y-components of the field contributions from pairs of charge elements on opposite sides of the ring cancel; therefore, the net field points in the $+x$-direction (for $x > 0$) and we need only the x-components of the unit vectors. Those are the same for all unit vectors—namely, $\hat{r}_x = x/r$.

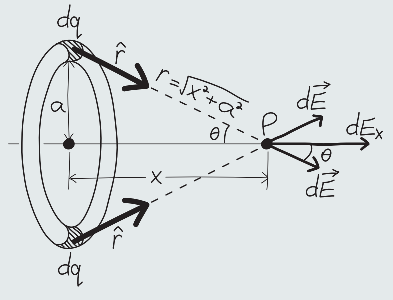


FIGURE 20.16 The electric field of a charged ring points along the ring axis, since field components perpendicular to the axis cancel in pairs.

EVALUATE We're now ready to set up the integral in Equation 20.7. Here each charge element contributes the same amount $dE_x = (k\,dq/r^2)\hat{r}_x = (k\,dq/r^2)(x/r)$ to the field. Figure 20.16 shows that $r = \sqrt{x^2 + a^2} = (x^2 + a^2)^{1/2}$, so the integral becomes

$$E = \int_{\text{ring}} dE_x = \int_{\text{ring}} \frac{kx\,dq}{(x^2 + a^2)^{3/2}} = \frac{kx}{(x^2 + a^2)^{3/2}} \int_{\text{ring}} dq$$

The last step follows because we have a fixed field point P, so its coordinate x is a constant for the integration. But the remaining integral is just the sum of all the charge elements on the ring—namely, the total charge Q. So our result becomes

$$E = \frac{kQx}{(x^2 + a^2)^{3/2}} \qquad \text{(on-axis field, charged ring)}$$

This is the magnitude; the direction is along the x-axis, away from the ring if Q is positive and toward it if Q is negative.

ASSESS Make sense? At $x = 0$ the field is zero. A charge placed at the ring center is pulled (or pushed) equally in all directions—no net force, so no electric field. But for $x \gg a$, we get $E = kQ/x^2$—just what we expect for a point charge Q. As always, a finite-sized charge distribution looks like a point charge at large distances. Problem 73 shows how you can use the result of this example to find the electric field on the axis of a charged disc, and Problem 75 shows that, once again, the field at large distances becomes that of a point charge.

EXAMPLE 20.7 **Line Charge: A Power Line's Field**
Worked Example with Variation Problems

A long, straight electric power line coincides with the x-axis and carries a uniform line charge density λ (unit: C/m). Find the electric field on the y-axis using the approximation that the wire is infinitely long.

INTERPRET We identify the field point as being a distance y from the wire, and the source charge as the whole wire.

DEVELOP Figure 20.17 is our drawing, showing a coordinate system with the field point P along the y-axis. We divide the wire into small charge elements dq and note that field contributions from two such elements dq on opposite sides of the y-axis contribute fields $d\vec{E}$ whose x-components cancel. Then we need only the y-component of each unit vector, and Figure 20.17 shows that's $\hat{r}_y = y/r$, where $r = \sqrt{x^2 + y^2}$.

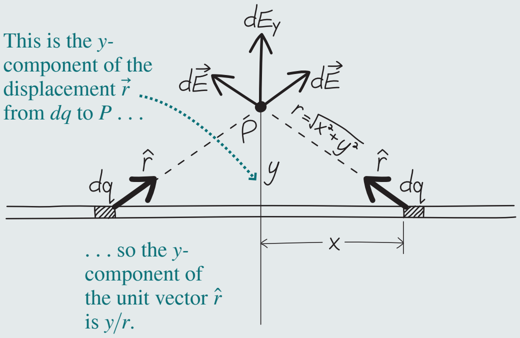


FIGURE 20.17 The field of a charged line is the vector sum of the fields $d\vec{E}$ from all the individual charge elements dq along the line.

EVALUATE We're now ready to set up the integral in Equation 20.7. As described in Chapter 9's integral strategy, we need to relate dq to a geometric variable so we can do the integral. Here our wire has charge density λ C/m, so if a charge element has length dx, then its charge is $dq = \lambda\, dx$. Putting all this together gives the y-component of the field from an arbitrary dq anywhere on the wire:

$$dE_y = \frac{k\,dq}{r^2}\hat{r}_y = \frac{k\lambda\,dx}{r^2}\frac{y}{r} = \frac{k\lambda y}{(x^2 + y^2)^{3/2}}dx$$

where we used $r = \sqrt{x^2 + y^2}$. Since the x-components cancel, we can sum—that is, integrate—the y-components to get the net field:

$$E = E_y = \int_{-\infty}^{+\infty} \frac{k\lambda y\,dx}{(x^2 + y^2)^{3/2}} = k\lambda y \int_{-\infty}^{+\infty} \frac{dx}{(x^2 + y^2)^{3/2}}$$

$$= k\lambda y\left[\frac{x}{y^2\sqrt{x^2 + y^2}}\right]_{-\infty}^{+\infty} = k\lambda y\left[\frac{1}{y^2} - \left(-\frac{1}{y^2}\right)\right] = \frac{2k\lambda}{y}$$

Here we used the integral table in Appendix A and applied the limits $x = \pm\infty$. Our result is the field's magnitude; the direction is away from the line for positive λ and toward the line for negative λ.

ASSESS Make sense? For an infinite line there's nothing to favor one direction along the line over another, so the only way the field can point is radially, away from or toward the line (Fig. 20.18). And because the line is infinite, it never resembles a point no matter how far away we are. As a result the field falls more slowly than the field of a point charge—in this case, as $1/y$. If we let r designate the radial distance from the line rather than the diagonal in Fig. 20.17, then the field decreases as $1/r$. An infinite line is impossible, but our result holds approximately for finite lines of charge as long as we're much closer to the line than its length, and not near an end. Far from a *finite* line, on the other hand, its field will resemble that of a point charge. You can explore the finite charged line in Problem 72.

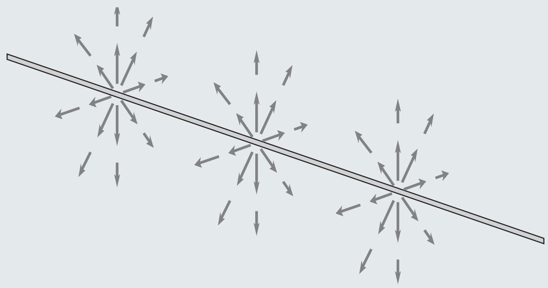


FIGURE 20.18 Field vectors for an infinite line of positive charge point radially outward, with magnitude decreasing inversely with distance.

20.5 Matter in Electric Fields

LO 20.8 *Determine the motion of charged particles in electric fields.*

LO 20.9 *Determine forces and torques on electric dipoles in electric fields.*

Electric fields give rise to forces on charged particles. Because matter consists of such particles, much of the behavior of matter is fundamentally determined by electric fields.

Point Charges in Electric Fields

The motion of a single charge in an electric field is governed by the definition of the electric field, $\vec{F} = q\vec{E}$, and Newton's law, $\vec{F} = m\vec{a}$. Combining these equations gives the acceleration of a particle with charge q and mass m in an electric field $\vec{E}$:

$$\vec{a} = \frac{q}{m}\vec{E} \qquad (20.8)$$

This equation shows that it's the charge-to-mass ratio, q/m, that determines a particle's response to an electric field. Electrons, nearly 2000 times less massive than protons but carrying the same charge, are readily accelerated by electric fields. Many practical devices, from X-ray machines to fluorescent lights, use electrons accelerated in electric fields.

When the electric field is uniform, problems involving the motion of charged particles reduce to the constant-acceleration problems of Chapter 2. An ink-jet printer is one application; a pair of oppositely charged plates creates a uniform field that "steers" charged ink droplets to the right place on the page (Fig. 20.19).

When the field isn't uniform, it's generally more difficult to calculate particle trajectories. An important exception is a particle moving perpendicular to a field that points radially. Under appropriate conditions, the result is uniform circular motion (see Section 5.3), as shown in the next example.

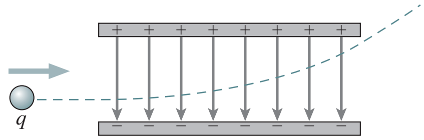


FIGURE 20.19 A pair of parallel charged plates creates a uniform electric field that deflects a charged particle. Can you tell the sign of the charge q?

EXAMPLE 20.8 Particle Motion: An Electrostatic Analyzer

Two oppositely charged curved metal plates establish an electric field given by $E = E_0(b/r)$, where E_0 and b are constants with the units of electric field and length, respectively. The field points toward the center of curvature, and r is the distance from the center. Find an expression for the speed v with which a proton entering vertically from below in Fig. 20.20 will leave the device moving horizontally.

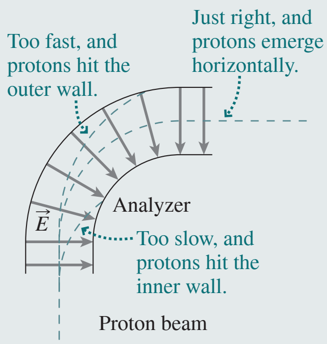


FIGURE 20.20 An electrostatic analyzer.

INTERPRET This problem is about charged-particle motion in an electric field that points radially. We're asked for the condition that will have a proton exiting the field region moving horizontally. Figure 20.20 shows that this requires its trajectory to be a circular arc.

DEVELOP Equation 20.8, $\vec{a} = (q/m)\vec{E}$, determines the acceleration of a charged particle in an electric field. Here we want uniform circular motion, so our plan is to write this equation with the given field and the acceleration v^2/r that we know applies in circular motion. Then we'll solve for v.

EVALUATE Under these conditions, Equation 20.8 becomes

$$a = \frac{v^2}{r} = \frac{eE}{m} = \frac{e}{m}E_0\frac{b}{r}$$

We then solve to get $v = \sqrt{eE_0b/m}$.

ASSESS Make sense? Strengthen the field by increasing E_0 or b, and the electric force becomes greater. For a given speed, that would result in more bending of the trajectory; to maintain the desired trajectory, we must therefore increase the speed. Note that the radius r canceled from our equations, showing that it doesn't matter where the protons enter the device. That's because the $1/r$ decrease in field strength matches the $1/r$ dependence of the acceleration. This device is called an electrostatic analyzer because it can sort charged particles by speed and charge-to-mass ratio. Spacecraft use such analyzers to characterize charged particles in interplanetary space.

GOT IT?

20.5 An electron, a proton, a deuteron (a neutron combined with a proton), a helium-3 nucleus (2 protons, 1 neutron), a helium-4 nucleus (2 protons, 2 neutrons), a carbon-13 nucleus (6 protons, 7 neutrons), and an oxygen-16 nucleus (8 protons, 8 neutrons) all find themselves in the same electric field. Rank in order their accelerations from lowest to highest under the assumption (only approximately correct) that the neutron and proton have the same mass and that the mass of a composite particle is the sum of the masses of its constituent neutrons and protons. Note any that have the same acceleration.

Dipoles in Electric Fields

Earlier in this chapter we calculated the field of an electric dipole, which consists of two opposite charges of equal magnitude. Here we study a dipole's response to electric fields. Since the dipole provides a model for many molecules, our results help explain molecular behavior.

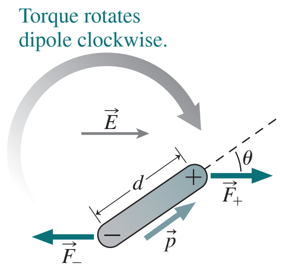


FIGURE 20.21 A dipole in a uniform electric field experiences a torque but no net force.

Figure 20.21 shows a dipole with charges $\pm q$ separated a distance d, located in a uniform electric field. The dipole moment vector $\vec{p}$ has magnitude qd and points from the negative to the positive charge (recall Fig. 20.14). Since the field is uniform, it's the same at both ends of the dipole. Since the dipole charges are equal in magnitude but opposite in sign, they experience equal but opposite forces $\pm q\vec{E}$—and therefore there's no net force on the dipole.

However, Fig. 20.21 shows that the dipole does experience a torque that tends to align it with the field. In Chapter 11 we described torque as the cross product of the position vector with the force: $\vec{\tau} = \vec{r} \times \vec{F}$, where the magnitude of the torque vector is $rF\sin\theta$ and its direction is given by the right-hand rule. Figure 20.21 thus shows that the torque about the center of the dipole due to the force on the positive charge has magnitude $\tau_+ = rF\sin\theta = (\frac{1}{2}d)(qE)\sin\theta$. The torque associated with the negative charge has the same magnitude, and both torques are in the same direction since both tend to rotate the dipole clockwise. Thus the net torque has magnitude $\tau = qdE\sin\theta$. Applying the right-hand rule shows that this torque is into the page. But qd is the magnitude of the dipole moment $\vec{p}$, and Fig. 20.21 shows that θ is the angle between the dipole moment vector and the electric field $\vec{E}$; therefore, we can write the torque vectorially as

$$\vec{\tau} = \vec{p} \times \vec{E} \qquad \text{(torque on a dipole)} \tag{20.9}$$

Because of this torque, the electric field does work on a dipole as it rotates. The electric force is conservative, so that work results in a change in potential energy. In Chapter 7 we defined potential-energy change as the negative of the work done by a conservative force: $\Delta U = -W$. Here we're dealing with rotational motion, and Equation 10.19 shows that the work done in a rotation from angular position θ_1 to θ_2 is given by $W = \int_{\theta_1}^{\theta_2}\tau\, d\theta$. Figure 20.21 shows that we're taking $\theta = 0$ when the dipole is aligned with the field. The figure also shows that the direction of increasing θ is counterclockwise or, in terms of rotational vectors, out of the page. The torque, in contrast, is clockwise or, vectorially, into the page. Thus the sign of the torque is opposite the angular change, so we need to write $\tau = -pE\sin\theta$ in the integral for the work. Let's now consider a dipole that's initially perpendicular to the field, so $\theta_1 = \pi/2$. Then the work done by the electric force as the dipole rotates to an arbitrary angle θ becomes

$$W = \int_{\pi/2}^{\theta} \tau\, d\theta = \int_{\pi/2}^{\theta} (-pE\sin\theta)\, d\theta = -pE[-\cos\theta]_{\pi/2}^{\theta} = pE\cos\theta$$

The potential-energy change is the negative of this work, and we note that $pE\cos\theta$ can be expressed as the dot product $\vec{p}\cdot\vec{E}$, so we can write the potential energy as

$$U = -\vec{p}\cdot\vec{E} \tag{20.10}$$

where $U = 0$ corresponds to the dipole at right angles to the field.

When the electric field isn't uniform, the charges at opposite ends of the dipole experience forces that differ in magnitude and/or aren't exactly opposite in direction. Then the dipole experiences a net force as well as a torque (Fig. 20.22). An important instance of this effect is the force on a dipole in the field of another dipole (Fig. 20.23). Because the dipole field falls off rapidly with distance and because the dipole responding to the field has closely spaced charges of equal magnitude but opposite sign, the dipole–dipole force is quite weak and falls extremely rapidly with distance. This weak force, which Fig. 20.23 shows to be attractive, is partly responsible for the van der Waals interaction between gas molecules that we mentioned in Chapter 17.

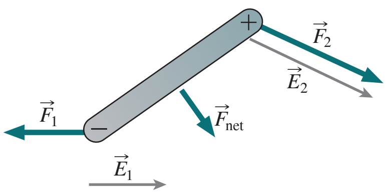


FIGURE 20.22 When the electric field differs in magnitude or direction at the two ends of the dipole, the dipole experiences a nonzero net force as well as a torque.

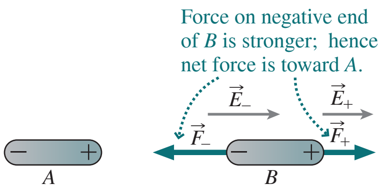


FIGURE 20.23 Dipole B aligns with the field of dipole A and then experiences a net force toward A.

Conductors, Insulators, and Dielectrics

Bulk matter contains vast numbers of point charges—namely, electrons and protons. In some matter—notably metals, ionic solutions, and ionized gases—individual charges are free to move throughout the material. In these **conductors**, the application of an electric field results in the ordered motion of electric charge that we call **electric current**. We'll consider conductors and current in later chapters.

Materials in which charge is not free to move are **insulators**, since they can't carry electric current. Insulators still contain charges—it's just that their charges are bound into neutral molecules. Some molecules, like water, have intrinsic dipole moments and therefore rotate in response to an applied electric field. Even if they don't have dipole moments, molecules may respond to an electric field by stretching and acquiring **induced dipole moments** (Fig. 20.24). In either case, the application of an electric field results in the alignment of molecular dipoles with the field (Fig. 20.25). The fields of the dipoles, pointing from their positive to their negative charges, then reduce the applied electric field within the material. We'll explore the consequences of this effect further in Chapter 23. Materials in which molecules either have intrinsic dipole moments or acquire induced moments are called **dielectrics**.

If the electric field applied to a dielectric becomes too great, individual charges are ripped free, and the material then acts like a conductor. Such **dielectric breakdown** can cause severe damage in materials and in electric equipment (Fig. 20.26). On a larger scale, lightning results from dielectric breakdown in air.

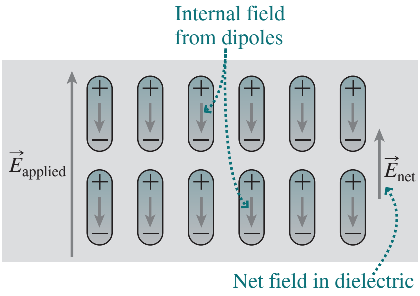


FIGURE 20.25 Alignment of molecular dipoles in a dielectric reduces the electric field within the dielectric.

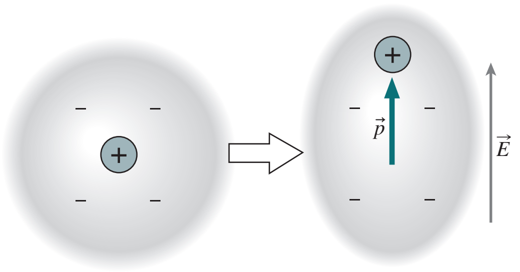


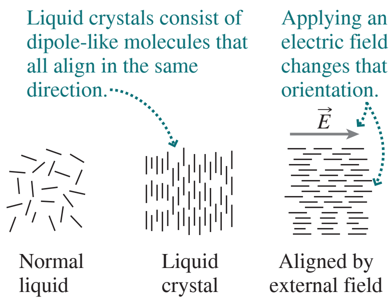
FIGURE 20.24 A molecule stretches in response to an electric field, acquiring a dipole moment.

FIGURE 20.26 Dielectric breakdown in a solid piece of Plexiglas produced this striking fractal pattern that marks permanent changes in the material.

APPLICATION Microwave Cooking and Liquid Crystals

The torque on dipoles in electric fields forms the basis of two widespread contemporary technologies: the microwave oven and the liquid-crystal display (LCD).

A microwave oven works by generating an electric field whose direction changes several billion times per second. Water molecules, whose dipole moment is much greater than most others, attempt to align with the field. But the field is changing, so the molecules swing rapidly back and forth. As they jostle against each other, the energy they gain from the field is dissipated as heat that cooks the food.



Computer displays, TVs, cameras, cell phones, watches, and many other devices display visual images using liquid crystals. These unique materials combine the fluidity of a liquid with the order of a solid. The liquid crystal consists of long molecules whose chemical structure results in a dipole-like charge separation. In response to each others' electric fields, the molecules tend to align. As the figure shows, an external electric field can rotate the the dipoles that comprise the liquid crystal, thus, altering the material's optical properties. With optical components we'll study in Chapter 29, different sections of a liquid-crystal display can then be made to appear visible or invisible. Liquid-crystal displays consume very little power, but they generate no light of their own and therefore most have a built-in light source. This photo of an iPhone shows its high-resolution display; also shown is a microphoto of the liquid crystals.

Chapter 20 Summary

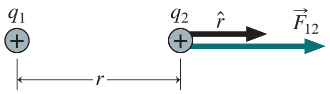
Big Idea

This chapter introduces several big ideas. First is **electric charge**, a fundamental property of matter that comes in positive and negative forms. Like charges repel and opposites attract; this is the **electric force**. It's convenient to define the **electric field** as the force per unit charge that a charge would experience if placed in the vicinity of other charges. Both force and field obey the **superposition principle**, meaning that the effects of several charges add vectorially.

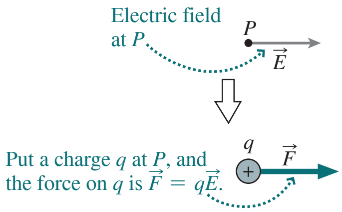
Key Concepts and Equations

Coulomb's law describes the electric force between point charges:

$$\vec{F}_{12} = \frac{kq_1q_2}{r^2}\hat{r}$$

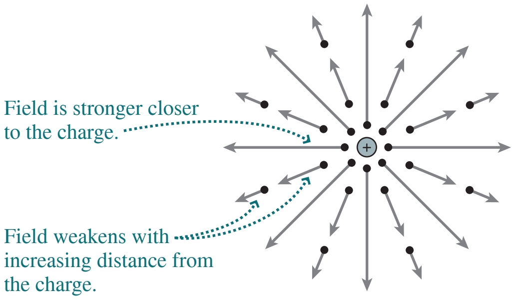


The electric field is the force per unit charge, $\vec{E} = \vec{F}/q$, and therefore the force a given charge q experiences in a field is $\vec{F} = q\vec{E}$.

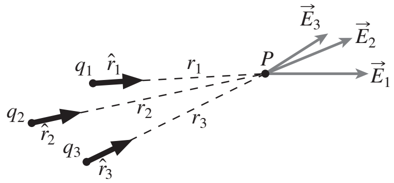


The field of a point charge follows from Coulomb's law:

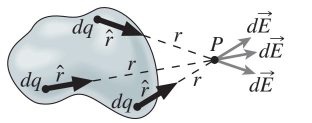
$$\vec{E} = \frac{kq}{r^2}\hat{r}$$



Fields of charge distributions are found by summing fields of individual point charges, or by integrating in the case of continuously distributed charge:



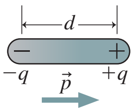
$$\vec{E}(P) = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \sum_i \frac{kq}{r_i^2}\hat{r}_i$$



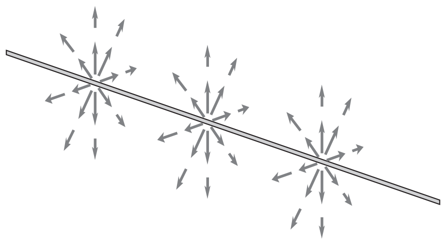
$$\vec{E}(P) = \int d\vec{E} = \int \frac{k\,dq}{r^2}\hat{r}$$

Applications

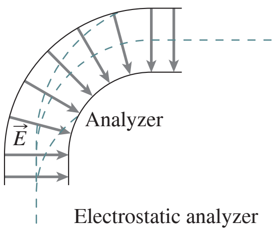
A **dipole** consists of equal but opposite charges $\pm q$ a distance d apart. For distances large compared with d, the dipole field drops as $1/r^3$, and the dipole is completely characterized by its **dipole moment** $p = qd$.



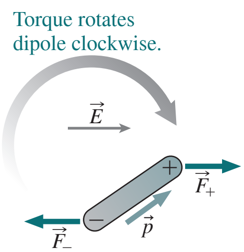
The field of an infinite line drops as $1/r$: $E = 2k\lambda/r$, with λ the charge per unit length. This is a good approximation to the field near an elongated structure like a wire.



Point charges respond to electric fields with acceleration proportional to the charge-to-mass ratio q/m.

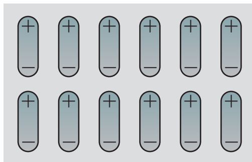


A dipole in an electric field experiences a torque that tends to align it with the field: $\vec{\tau} = \vec{p} \times \vec{E}$.



If the field is nonuniform, there's also a net force on the dipole.

Dielectrics are insulating materials whose molecules act like electric dipoles.



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BIO *Biology and/or medicine-related problems* **DATA** *Data problems* **ENV** *Environmental problems* **CH** *Challenge problems* **COMP** *Computer problems*

Learning Outcomes *After finishing this chapter you should be able to:*

LO 20.1 Describe electric charge as a fundamental property of matter.
For Thought and Discussion Questions 20.1, 20.2, 20.8; Exercises 20.11, 20.12, 20.13, 20.14, 20.15

LO 20.2 Use Coulomb's law to calculate the forces between charges.
Exercises 20.16, 20.17, 20.18, 20.19, 20.20; Problems 20.44, 20.57, 20.58

LO 20.3 Use the superposition principle to calculate forces involving multiple charges.
For Thought and Discussion Question 20.3; Problems 20.46, 20.47, 20.48, 20.49, 20.52

LO 20.4 Describe the concept of electric field.
For Thought and Discussion Questions 20.4, 20.5; Exercises 20.21, 20.22, 20.23, 20.24, 20.25, 20.26; Problem 20.50

LO 20.5 Determine the fields of electric charge distributions using superposition.
For Thought and Discussion Question 20.7; Exercises 20.27, 20.28; Problems 20.51, 20.56, 20.59

LO 20.6 Describe the electric dipole and the field it produces.
For Thought and Discussion Questions 20.6, 20.9; Problems 20.53, 20.54, 20.55, 20.67, 20.69, 20.70, 20.72

LO 20.7 Determine the fields of continuous charge distributions by integration.
Exercises 20.29, 20.30, 20.31; Problems 20.61, 20.64, 20.68, 20.71, 20.73, 20.74, 20.75, 20.76, 20.77, 20.78

LO 20.8 Determine the motion of charged particles in electric fields.
Exercises 20.32, 20.33, 20.34, 20.35; Problems 20.60, 20.63

LO 20.9 Determine forces and torques on electric dipoles in electric fields.
For Thought and Discussion Questions 20.9, 20.10; Problems 20.62, 20.65, 20.66

For Thought and Discussion

1. Conceptual Example 20.1 shows that the gravitational force between an electron and a proton is about 10^{-40} times weaker than the electric force between them. Since matter consists largely of electrons and protons, why is the gravitational force important at all?
2. A free neutron is unstable and soon decays to other particles, one of them a proton. Must there be others? If so, what electric properties must it or they have?
3. Where in Fig. 20.5 could you put a third charge so it would experience no net force? Would it be in stable or unstable equilibrium?
4. Equation 20.3 gives the electric field of a point charge. Does the direction of (a) $\hat{r}$ or (b) $\vec{E}$ depend on whether the charge is positive or negative?
5. Is the electric force on a charged particle always in the direction of the field? Explain.
6. Why does a dipole, which has no net charge, produce an electric field?
7. The ring in Example 20.6 carries total charge Q, and the point P is the same distance $r = \sqrt{x^2 + a^2}$ from all parts of the ring. So why isn't the electric field of the ring just kQ/r^2?
8. A spherical balloon is initially uncharged. If you spread positive charge uniformly over the balloon's surface, would it expand or contract? What would happen if you spread negative charge instead?
9. Why should there be a force between two dipoles, which each have zero net charge?
10. Dipoles A and B are both located in the field of a point charge Q, as shown in Fig. 20.27. Does either experience a net torque? A net force? If each dipole is released from rest, qualitatively describe its subsequent motion.

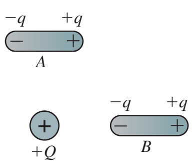


FIGURE 20.27 For Thought and Discussion 10

Exercises and Problems

Exercises

Section 20.1 Electric Charge

11. Suppose the electron and proton charges differed by one part in one billion. Estimate the net charge on your body, assuming it contains equal numbers of electrons and protons.
12. A typical lightning flash delivers about 25 C of negative charge from cloud to ground. How many electrons are involved?
13. Protons and neutrons are made from combinations of the two most common quarks, the u quark (charge $+\frac{2}{3}e$) and the d quark (charge $-\frac{1}{3}e$). How could three of these quarks combine to make (a) a proton and (b) a neutron?
14. Earth carries a net charge of about -5×10^5 C. How many more electrons are there than protons on Earth?
15. **BIO** As they fly, honeybees may acquire electric charges of about 180 pC. Electric forces between charged honeybees and spider webs can make the bees more vulnerable to capture by spiders. How many electrons would a honeybee have to lose to acquire a charge of +180 pC?

Section 20.2 Coulomb's Law

16. The electron and proton in a hydrogen atom are 52.9 pm apart. Find the magnitude of the electric force between them.
17. An electron at Earth's surface experiences a gravitational force $m_e g$. How far away can a proton be and still produce the same force on the electron? (Your answer should show why gravity is unimportant on the molecular scale!)
18. You break a piece of Styrofoam packing material, and it releases lots of little spheres whose electric charge makes them stick annoyingly to you. If two of the spheres carry equal charges and repel with a force of 20 mN when they're 17 mm apart, what's the magnitude of the charge on each?
19. A charge q is at the point $x = 5$ m, $y = 0$ m. Write expressions for the unit vectors you would use in Coulomb's law if you were finding the force that q exerts on other charges located at (a) $x = 5$ m, $y = 2.5$ m; (b) the origin; and (c) $x = 7$ m, $y = 3.5$ m. You're not given the sign of q. Why doesn't this matter?

20. A proton is at the origin and an electron is at the point $x = 0.41$ nm, $y = 0.36$ nm. Find the electric force on the proton.

Section 20.3 The Electric Field

21. An electron experiences an electric force of 0.61 nN. What's the field strength at its location?
22. Find the magnitude of the electric force on a 6.0-μC charge in a 50-N/C electric field.
23. A 75-nC charge experiences a 144-mN force in a certain electric field. Find (a) the field strength and (b) the force that a 35-μC charge would experience in the same field.
24. BIO The electric field inside a cell membrane is 8.0 MN/C. What's the force on a singly charged ion in this field?
25. A -3.0-μC charge experiences a $9.0\hat{\imath}$-N electric force in a certain electric field. What force would a proton experience in the same field?
26. The electron in a hydrogen atom is 52.9 pm from the proton. At this distance, what's the strength of the electric field due to the proton?

Section 20.4 Fields of Charge Distributions

27. In Fig. 20.28, point P is midway between the two charges. Find the electric field in the plane of the page (a) 5.0 cm to the left of P, (b) 5.0 cm directly above P, and (c) at P.

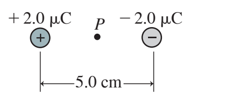


FIGURE 20.28 Exercise 27

28. The water molecule's dipole moment is 6.17×10^{-30} C·m. What would be the separation distance if the molecule consisted of charges $\pm e$? (The effective charge is actually less because H and O atoms share the electrons.)
29. The electric field 22 cm from a long wire carrying a uniform line charge density is 1.9 kN/C. What's the field strength 38 cm from the wire?
30. Find the line charge density on a long wire if the electric field 39 cm from the wire has magnitude 210 kN/C and points toward the wire.
31. Find the magnitude of the electric field due to a charged ring of radius a and total charge Q on the ring axis at distance a from the ring's center.

Section 20.5 Matter in Electric Fields

32. In his famous 1909 experiment that demonstrated quantization of electric charge, R. A. Millikan suspended small oil drops in an electric field. With field strength 20 MN/C, what mass drop can be suspended when the drop carries 10 elementary charges?
33. How strong an electric field is needed to accelerate electrons in an X-ray tube from rest to one-tenth the speed of light in a distance of 4.7 cm?
34. A proton moving to the right at 3.3×10^5 m/s enters a region where a 60-kN/C electric field points to the left. (a) How far will the proton get before it momentarily stops? (b) Describe its subsequent motion.
35. An electrostatic analyzer like that of Example 20.8 has $b = 7.5$ cm. What value of E_0 will enable the device to select protons moving at 84 m/s?

Example Variations

The following problems are based on two examples from the text. Each set of four problems is designed to help you make connections that enhance your understanding of physics and to build your confidence in solving problems that differ from ones you've seen before. The first problem in each set is essentially the example problem but with different numbers. The second problem presents the same scenario as the example but asks a different question. The third and fourth problems repeat this pattern but with entirely different scenarios.

36. **Example 20.2:** Charges on raindrops vary widely in both magnitude and sign. Consider a case where the two drops on the x-axis in Example 20.2 are 2.18 mm apart and have charge $q = 645$ nC, while the third drop is 12.3 mm up the y-axis and has charge $Q = -1.87$ μC. Find the electric force on the upper drop.
37. **Example 20.2:** Suppose that all three raindrops in Example 20.2 have equal charges and that their positions form an equilateral triangle with side 3.36 mm. If the electric force on the upper charge is $96.2\,\hat{\jmath}$ N, (a) what's the magnitude of the charge? (b) Can you determine the sign of the charge from the information given?
38. **Example 20.2:** (a) Repeat Example 20.2 to find the force on Q, now taking the right-hand charge on the x-axis to be $-q$. (b) For $y \gg a$, how does the force you found in (a) depend on the distance y?
39. **Example 20.2:** (a) Use calculus to show that the maximum force in the situation of Example 20.2 occurs when $y = a/\sqrt{2}$, and (b) find the magnitude of that maximum force.
40. **Example 20.7:** A 1.00-km length of power line carries a total charge of 264 mC distributed uniformly over its length. Find the magnitude of the electric field 54.3 cm from the axis of the power line, and not near either end (staying away from the ends means you can approximate the field as that of an infinitely long wire).
41. **Example 20.7:** A uniformly charged wire is 2.18 m long and 0.15 mm in diameter. You measure the electric field 1.20 cm from the wire's axis, not near either end, and you find it to be 455 kN/C, pointing toward the wire. Find the total charge on the wire.
42. **Example 20.7:** A thin rod of length L lies on the x-axis with its center at the origin, as shown in Fig. 20.29. The rod carries charge Q distributed uniformly over its length. (a) Modify the calculation of Example 20.7 to find an expression for the electric field at point A in Fig. 20.29, located on the positive y-axis an arbitrary distance y from the origin (but with y large enough to put point A outside the thin rod). (b) Show that your result reduces to the field of a point charge Q when $y \gg L$.

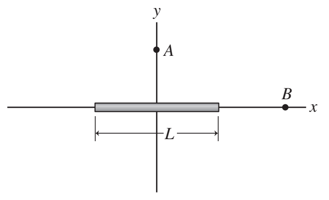


FIGURE 20.29 Problems 42 and 43

43. **Example 20.7:** A thin rod of length L lies on the x-axis with its center at the origin, as shown in Fig. 20.29. The rod carries charge Q distributed uniformly over its length. (a) Find an expression for the electric field at point B in Fig. 20.29, located on the positive x-axis a distance x from the origin, where $x > L/2$, so that point B is beyond the right end of the rod. (b) Show that your result reduces to the field of a point charge Q when $x \gg L$.

Problems

44. Two charges, of which one has a magnitude three times as large as the other's, are located 14.5 cm apart and experience an attractive force of 156 N. (a) What's the magnitude of the larger charge? (b) Can you determine the sign of the larger charge?
45. A proton is on the x-axis at $x = 1.3$ nm. An electron is on the y-axis at $y = 0.86$ nm. Find the net force the two exert on a helium nucleus (charge $+2e$) at the origin.
46. A charge $3q$ is at the origin, and a charge $-2q$ is on the positive x-axis at $x = a$. Where would you place a third charge so it would experience no net electric force?
47. A negative charge $-q$ lies midway between two positive charges $+Q$. What must Q be such that the electric force on all three charges is zero?
48. In Fig. 20.30, take $q_1 = 68$ μC, $q_2 = -34$ μC, and $q_3 = 15$ μC. Find the electric force on q_3.
49. In Fig. 20.30, take $q_1 = 21$ μC and $q_2 = 18$ μC. If the force on q_1 points in the $-x$-direction, find (a) q_3 and (b) the magnitude of the force on q_1.

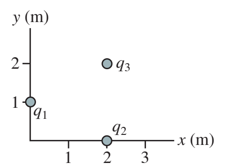


FIGURE 20.30 Problems 48 and 49

50. DNA fragments introduced into an electrophoresis apparatus (see
BIO DATA Application, page 390) generally carry negative charges equivalent to two extra electrons per base pair of nucleotides in the fragment. The table below shows the forces on several DNA fragments in an electrophoresis apparatus, as a function of the number of base pairs. Plot these data, establish a best-fit line, and use the resulting slope to determine the strength of the electric field in the electrophoresis apparatus.

Base pairs	400	800	1200	2000	3000	5000
Force (pN)	0.235	0.472	0.724	1.15	1.65	2.87

51. A proton is at the origin and an ion at $x = 8.0$ nm. If the electric field is zero at $x = -4.0$ nm, what's the ion's charge?
52. Four equal charges Q are at the corners of a square of side a. Find an expression for the magnitude of the force on each charge.
53. A dipole lies on the y-axis and consists of an electron at $y = 0.60$ nm and a proton at $y = -0.60$ nm. Find the electric field (a) midway between the two charges; (b) at the point $x = 2.0$ nm, $y = 0$ nm; and (c) at the point $x = -20$ nm, $y = 0$ nm.
54. Show that the field on the x-axis for the dipole of Example 20.5 is given by Equation 20.6b, for $|x| \gg a$.
55. You're 1.44 m from a charge distribution that is well under 1 cm in size. You measure an electric field strength of 296 N/C due to this distribution. You then move to a distance of 2.16 m from the distribution, where you measure a field strength of 87.7 N/C. What's the net charge of the distribution? *Hint*: Don't try to calculate the charge. Determine instead how the field decreases with distance, and from that infer the charge.
56. Three identical charges q form an equilateral triangle of side a, with two charges on the x-axis and one on the positive y-axis. (a) Find an expression for the electric field at points on the y-axis above the uppermost charge. (b) Show that your result reduces to the field of a point charge $3q$ for $y \gg a$.
57. Two identical small metal spheres initially carry charges q_1 and q_2. When they're 1.0 m apart, they experience a 2.5-N attractive force. Then they're brought together so charge moves from one to the other until they have the same net charge. They're again placed 1.0 m apart, and now they repel with a 2.5-N force. What were the original charges q_1 and q_2?
58. Two 32.0-μC charges are attached to opposite ends of a spring
COMP with spring constant $k = 135$ N/m and equilibrium length 49.3 cm. By how much does the spring stretch? *Hint*: You'll need to use a computer or advanced calculator to solve the cubic equation that arises in this problem.
59. A positive charge Q is located at the origin, and another charge q is at $x = a$, where $a > 0$. Given that the electric field is zero at $x = 2a$, find an expression for q in terms of Q.
60. An electron is moving in a circular path around a long, uniformly charged wire carrying 1.4 nC/m. What's the electron's speed?
61. Find the line charge density on a long wire if a 6.5-μg particle carrying 2.2 nC describes a circular orbit about the wire with speed 270 m/s.
62. A dipole with dipole moment 1.6 nC · m is oriented at 30° to a 5.0-MN/C electric field. Find (a) the magnitude of the torque on the dipole and (b) the work required to rotate the dipole until it's antiparallel to the field.
63. You have a job examining patent applications. You're presented with the device in Fig. 20.31, which its inventor claims will separate isotopes of a particular element. Atoms are first stripped completely of their electrons, then accelerated from rest through an electric field chosen to give the desired isotope exactly the right speed to pass through the electrostatic analyzer (see Example 20.8). Will the device work?

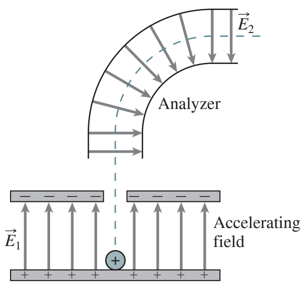


FIGURE 20.31 Problem 63

64. A 5.0-μm strand of DNA carries charge $+e$ per nm of length.
BIO Treating it as a charged line, what's the electric field strength 21 nm from the DNA, not near either end?
65. Heating in a microwave oven occurs as water molecules rotate back and forth to align their dipole moments with a rapidly changing electric field. Given water's dipole moment of 6.17×10^{30} C·m, what's the energy change when a water molecule, with its dipole moment initially opposite a 2.95-kN/C electric field, swings to align with the field?
66. A dipole with charges $\pm q$ and separation $2a$ is located a distance
CH x from a point charge $+Q$, oriented as shown in Fig. 20.32. Find expressions for the magnitude of (a) the net torque and (b) the net force on the dipole, both in the limit $x \gg a$. (c) What's the direction of the net force?

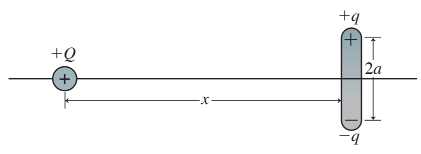


FIGURE 20.32 Problem 66

67. You're taking physical chemistry, and your professor is discussing molecular dipole moments. Water, he says, has a dipole moment of "1.85 debyes," while carbon monoxide's dipole moment is only "0.12 debye." Your physics professor wants these moments expressed in SI. She tells you that the atomic separation in these two covalent compounds is about the same, and asks what that indicates about the way shared charge is distributed. What do you tell her?
68. The electric field on the axis of a uniformly charged ring has magnitude 340 kN/C at a point 10 cm from the ring center. The magnitude 25 cm from the center is 110 kN/C; in both cases the field points away from the ring. Find (a) the ring's radius and (b) its charge.
69. An *electric quadrupole* consists of two oppositely
CH directed dipoles in close proximity. (a) Calculate the field of the quadrupole shown in Fig. 20.33 for points to the right of $x = a$ and (b) show that for $x \gg a$ the quadrupole field falls off as $1/x^4$.

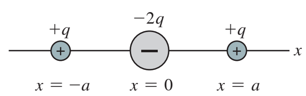


FIGURE 20.33 Problem 69

70. Four charges lie at the corners of a square of side a, with the cen-
CH ter of the square at the origin. The two charges with $y = a/2$ have magnitude Q and are positive. The two charges with $y = -a/2$ also have magnitude Q but are negative. (a) Find an expression for the magnitude of the electric field for points on the y-axis with $y > a/2$. (b) Show that, for $y \gg a$, your result exhibits the $1/y^3$ falloff you would expect for an electric dipole. (c) Compare the result of (b) with Equation 20.6b and write an expression for the magnitude of the dipole moment of this four-charge distribution. *Hint*: Be careful with your approximation in (b)! If you get 0 for your answer, then you've gone too far. You can neglect a^2 when compared with y^2, but you can't neglect a when compared with y or you'll be throwing out the charge separation that makes this distribution resemble a dipole at large distances.
71. A straight wire 12 m long carries 28 μC distributed uniformly over its length. (a) What's the line charge density on the wire? Find the electric field strength (b) 20 cm from the wire axis, not near either end, and (c) 450 m from the wire. Make suitable approximations in both cases.

72. CH Two thin rods, each of length a, lie along the x-axis as shown in Fig. 20.34. The left-hand rod, which extends from $x = -a$ to the origin, carries negative charge $-Q$ distributed uniformly over its length. The right-hand rod extends from the origin to $x = a$ and carries positive Q distributed uniformly over its length. (a) Find an expression for the magnitude of the electric field on the x-axis for $x > a$ (that is, beyond the right end of the right-hand rod). (b) Show that, for $x \gg a$, the field exhibits the $1/x^3$ falloff of an electric dipole. (c) Compare the result of (b) with Equation 20.6b and write an expression for the magnitude of the dipole moment of this two-rod structure.

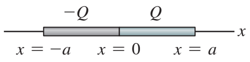


FIGURE 20.34 Problem 72

73. CH Figure 20.35 shows a thin, uniformly charged disk of radius R. Imagine the disk divided into rings of varying radii r, as suggested in the figure. (a) Show that the area of such a ring is very nearly $2\pi r\, dr$. (b) If the disk carries positive surface charge density σ, use the result of part (a) to write an expression for the charge dq on an infinitesimal ring. (c) Use the result of (b) along with the result of Example 20.6 to write the infinitesimal electric field dE of this ring at any point on the disk axis, taken to be the x-axis. (d) Integrate over all such rings to show that the net electric field on the axis has magnitude

$$E = 2\pi k\sigma\left(1 - \frac{|x|}{\sqrt{x^2 + R^2}}\right)$$

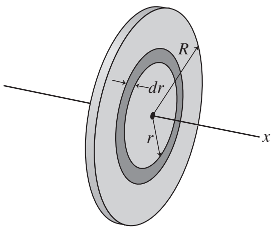


FIGURE 20.35 Problem 73

74. CH Use the result of Problem 73 to show that the field of an *infinite*, uniformly charged flat sheet is $2\pi k\sigma$, where σ is the surface charge density. (This result is independent of distance from the sheet.)

75. Use the binomial theorem to show that, for $x \gg R$, the result of Problem 73 reduces to the field of a point charge whose total charge is the charge density times the disk area.

76. CH A semicircular loop of radius a carries positive charge Q distributed uniformly. Find the electric field at the loop's center (point P in Fig. 20.36). (*Hint*: Divide the loop into charge elements dq as shown, write dq in terms of the angle $d\theta$, then integrate over θ.)

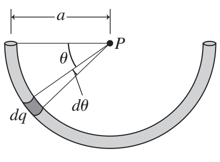


FIGURE 20.36 Problem 76

77. CH A thin rod carries charge Q distributed uniformly over its length L and is situated on the x-axis between $x = \pm L/2$. (a) Find the electric field at an arbitrary point (x, y). (You'll have to do separate integrals for the x- and y-components.) (b) Show that your result reduces to that of Example Variation 42 when $x = 0$ and to that of Example Variation 43 when $y = 0$ and $x > L/2$.

78. CH A thin rod extends along the x-axis from $x = 0$ to $x = L$ and carries line charge density $\lambda = \lambda_0(x/L)^2$, where λ_0 is a constant. Find the electric field at $x = -L$.

79. You're working on the design of an ink-jet printer. Ink drops of mass m, speed v, and charge q will enter a region of uniform electric field E between two charged plates (Fig. 20.37). The drops enter midway between the plates, and the electric field deflects them toward the correct place on the page. Find an expression for the maximum electric field for which drops can still get through without hitting either plate.

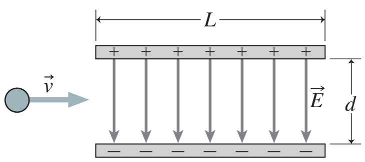


FIGURE 20.37 Problem 79

Passage Problems

BIO The human heart consists largely of elongated muscle cells, some 100 μm long and 15 μm in diameter. In its resting state, a cell contains two concentric layers of charge, which confine the electric field to the cell membrane (Fig. 20.38*a*). When the heart contracts, a wave of depolarization sweeps through, depleting charge and giving each cell a dipole moment (Fig. 20.38*b*). As a result, the entire organ acts like an electric dipole, producing an external field, which is indirectly detected by electrocardiography. Although the direction of the heart's dipole moment varies, Fig. 20.38*c* is typical. In answering the questions that follow, consider the heart in isolation—don't concern yourself with the effect of surrounding tissues on its electric field.

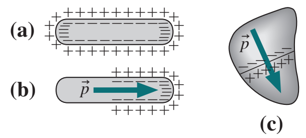


FIGURE 20.38 Heart cells (a) in the resting state and (b) partially depolarized, resulting in a dipole moment $\vec{p}$. (c) Typical orientation of the heart's dipole moment vector. Cells along the line are depolarizing.

80. At a distance r, far from the heart, the heart's electric field
    a. falls off as $1/r$.
    b. falls off as $1/r^2$.
    c. falls off as $1/r^3$.
    d. falls off as $1/r^4$.

81. At a given distance, far from the heart compared with its size, the electric field
    a. is weaker along an extension of the line shown in Fig. 20.38*c* than on a perpendicular line.
    b. is stronger along a an extension of the line shown in Fig. 20.38*c* than on a perpendicular line.
    c. has the same value at positions perpendicular and parallel to the line in Fig. 20.38*c*.

82. The difference between Figs. 20.38*a* and 20.38*b* that results in an external electric field in one case but not the other is that
    a. there's no net charge in Fig. 20.38*a* but there is a net charge in Fig. 20.38*b*.
    b. the total charge is greater in Fig. 20.38*a*.
    c. the charge is distributed in Fig. 20.38*b* so there's more negative charge to the left and more positive charge to the right.

83. At the instant shown in Fig. 20.38*c*, there's an electric field within the heart that points approximately
    a. in the direction of the dipole moment vector $\vec{p}$.
    b. opposite the dipole moment vector $\vec{p}$.
    c. perpendicular to the dipole moment vector $\vec{p}$.

Answers to Chapter Questions

Answer to Chapter Opening Question

The electric field in the atmosphere must be strong enough to rip electrons from air molecules, making the air an electrical conductor. This typically happens when E exceeds about 3 MV/m.

Answers to GOT IT? Questions

20.1 (c) *uud* because its net charge is $+e$ (*udd* is the neutron)
20.2 (1) $-\hat{\imath}$; (2) $-\frac{\sqrt{2}}{2}\hat{\imath} + \frac{\sqrt{2}}{2}\hat{\jmath}$; The unit vector always points away from the source charge regardless of the sign.
20.3 (a)
20.4 (1) drops by $1/2^2$, to 200 N/C; (2) drops by $1/2^3$, to 100 N/C
20.5 carbon-13, (oxygen-16, helium-4, deuteron—all the same), helium-3, proton, electron

Gauss's Law

Skills & Knowledge You'll Need

- Coulomb's law (Section 20.2)
- The dot product of two vectors (Section 6.2)

Learning Outcomes

After finishing this chapter you should be able to:

LO 21.1 Describe electric fields using field lines.

LO 21.2 Explain the concept of electric flux in relation to field lines.

LO 21.3 Describe Gauss's law and explain how it relates to Coulomb's law.

LO 21.4 Use Gauss's law to determine electric fields for charge distributions with sufficient symmetry.

LO 21.5 Approximate electric fields of general charge distributions using what you know about simpler distributions.

LO 21.6 Describe electrostatic equilibrium in conductors and use Gauss's law to determine electric fields associated with charged conductors.

Huge sparks jump to the operator's cage in the Hall of Electricity at the Boston Museum of Science, but the operator is unharmed. Why?

In this chapter we introduce an elegant way of describing electric fields that makes it much easier to calculate the fields of certain charge distributions. In the process we'll formulate one of the four fundamental laws of electromagnetism—a statement that embodies Coulomb's law but that gives deeper insights into the electric field.

21.1 Electric Field Lines

LO 21.1 *Describe electric fields using field lines.*

We can visualize electric fields by drawing **electric field lines**, continuous lines whose direction is everywhere the same as that of the electric field. To draw a field line, determine the field direction at some point. Move a small distance in the direction of the field, and evaluate the field direction at the new point. Extending the process in both directions from your starting point traces out an electric field line. You'll find that field lines begin on positive charges and either end on negative charges or extend to infinity. Drawing many field lines gives a picture of the overall field.

To explore the field of a positive point charge, as shown in Fig. 21.1*a*, start near the charge. The field points radially outward, so move a little way outward. The field is still radial. Repeat the process, and you'll trace a straight line that extends indefinitely. So the field lines of a positive point charge are straight lines that begin on the charge and extend radially to infinity (Fig. 21.1*b*).

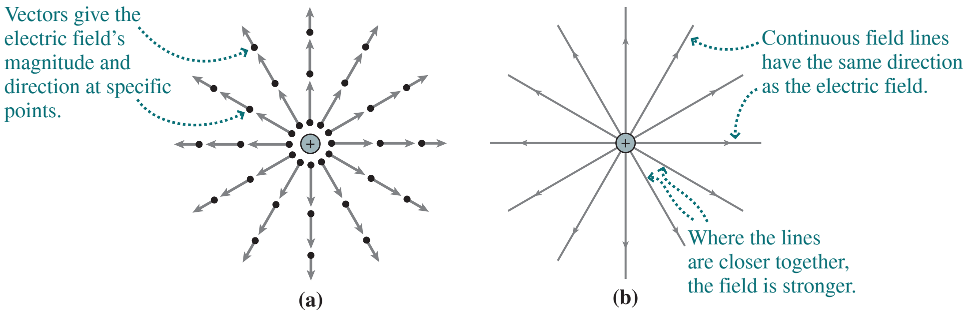


FIGURE 21.1 Vectors (a) and field lines (b) provide two ways to visualize the electric field.

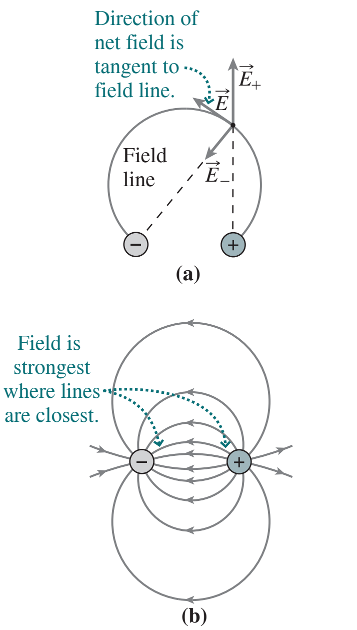


FIGURE 21.2 Field of an electric dipole. (a) At each point, the field-line direction is that of the *net* electric field, $\vec{E} = \vec{E}_+ + \vec{E}_-$ (b) Tracing many field lines shows the overall dipole field.

In Fig. 21.1*b* the field lines spread apart as they extend farther from the point charge. Coulomb's law shows that the field weakens farther from the charge, so in Fig. 21.1*b* the field is stronger where the lines are closer and weaker where they're farther apart. This is generally true, and lets us infer the field's relative magnitude as well as direction from field-line pictures.

To trace the field lines of a charge distribution, follow the net field—the vector sum of the field contributions from all charges in the distribution. Usually the field direction varies, so the line is curved. Figure 21.2*a* shows the details for one field line of a dipole. Figure 21.2*b* shows a number of dipole field lines; here you can see that the field is strongest near the individual charges and in the region between them. The electric field exists everywhere, so there are really infinitely many field lines. We can't draw them all, but in order to make field-line pictures somewhat precise, we associate a fixed number of field lines with a charge of a given magnitude. In Fig. 21.3, for example, eight field lines correspond to a charge of magnitude q. Study the figure, and you'll see how all the fields shown are consistent with this convention.

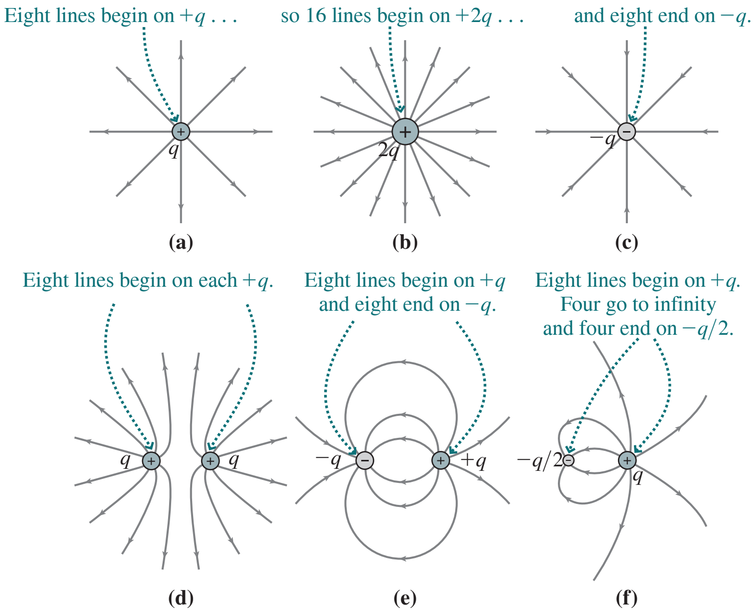
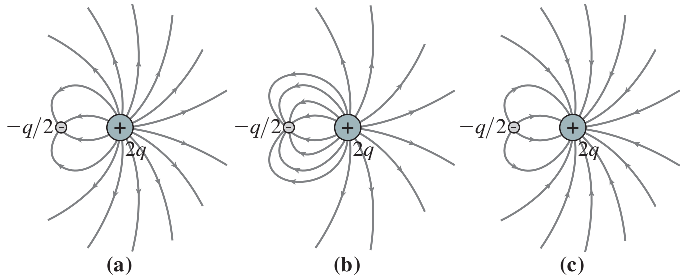


FIGURE 21.3 Field lines for six charge distributions, using the convention that eight lines correspond to a charge of magnitude q.

GOT IT?

21.1 Which figure represents the electric field of a charge distribution consisting of $+2q$ and $-q/2$, using the convention that eight lines correspond to a charge of magnitude q?



21.2 Electric Field and Electric Flux

LO 21.2 *Explain the concept of electric flux in relation to field lines.*

In Fig. 21.4 we've surrounded each charge distribution from Fig. 21.3 with several surfaces. Each surface is closed, so it's impossible to get from inside to outside without crossing the surface. (Figure 21.4 shows only the two-dimensional cross section of each surface.) How many field lines emerge from within each surface?

In Fig. 21.4*a* the answer for surfaces 1 and 2 is eight. For surface 3 one field line crosses twice going out and once going in; if we count the inward-going line as negative, then there's still a net of eight lines going out. *Any* closed surface surrounding $+q$ will have eight lines emerging from it, for the simple reason that eight lines begin on the charge and extend to infinity; to get there they all have to cross the closed surface.

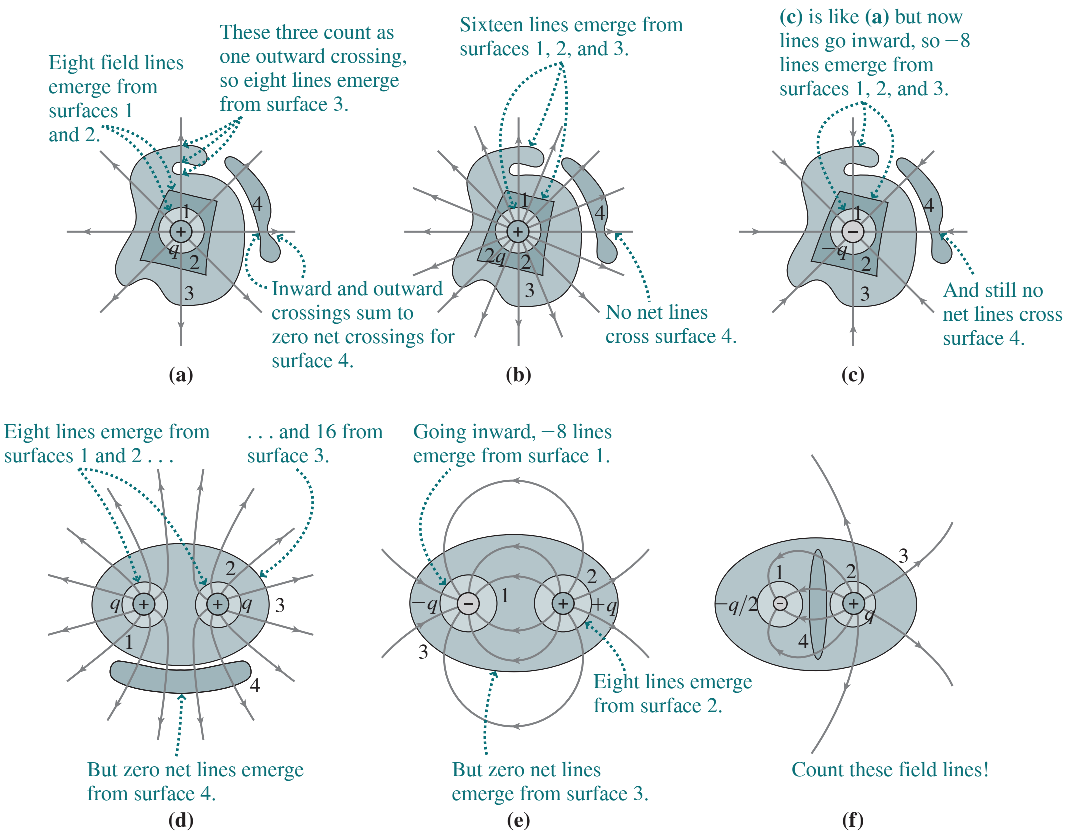


FIGURE 21.4 The number of field lines emerging from a closed surface depends only on the net charge enclosed.

What about surface 4? Two lines cross going outward and two inward, for a net of zero lines emerging. What's different is that surface 4 doesn't enclose the charge. You can convince yourself that *any* surface not enclosing the charge will have as many lines going in as out, for zero net field lines emerging.

Figure 21.4*b* is identical except that now 16 field lines emerge from any surface surrounding the charge: $+2q$ enclosed, so 16 field lines emerge. And Fig. 21.4*c* is similar, too, but with the negative charge, the field lines go inward and we count them as negative: $-q$ enclosed, so -8 field lines emerge. And, in all three cases, zero net lines emerge from surface 4, the one that doesn't enclose the charge: zero charge enclosed, so zero field lines emerging. The same is true even for surface 3 surrounding the dipole in Fig. 21.4*e*: Here there are two charges within the surface, but the *net charge enclosed* is zero, and sure enough, there are zero net field lines emerging. Study the rest of Fig. 21.4 and you can convince yourself that in all cases **the number of field lines emerging from any closed surface is proportional to the net charge enclosed**.

This statement is very general. It doesn't matter what shape the surface is or whether the enclosed charge is a single point charge or a lot of charges carrying the same net charge. Nor does it matter how the charges are arranged, as long as they're *enclosed* by the surface in question. The presence of charges *outside* the surface doesn't alter our conclusion about the number of field lines emerging—although it may alter the shapes of the field lines. We'll now rephrase our statement mathematically to obtain one of the four fundamental laws of electromagnetism. As we proceed, remember that the mathematics just reflects what's clear from Fig. 21.4: The number of field lines emerging from a closed surface depends only on the net charge enclosed.

The Flux Concept

There are many situations in physics and engineering where we deal with flows of matter or energy. Examples from Chapter 15 include the flows of blood through a vein, water in a pipe or river, or air over an airplane's wing. There, we measured flow in kg/s or m^3/s. Fluid flows such as these are also of interest to physiologists, hydrologists, oceanographers, meteorologists, and civil engineers. In Chapter 16 we considered heat flow, measured in watts, which is of interest to building engineers, biologists, climatologists, and many others. Coming soon, in Chapter 24, we'll consider electric current— the flow of electric charge. Each of these flows is a **flux**—a term that comes from the Latin *fluxus*, for "flow." In each case we can represent the flow as we did in Chapter 15 for fluids—by drawing lines that give the local direction of the flow. As Fig. 15.11 showed, those lines are closer where the flow speed is higher and vice versa. Those last two sentences should remind you of Section 21.1's recipe for drawing electric field lines, and how the proximity of field lines describes the strength of the field.

We can describe flows in more detail by giving the flow rate per unit area—which we could equally well call the flux per unit area. For a fluid, that would be measured in kilograms per second per square meter ($kg/s \cdot m^2$). If that quantity were uniform across the flow, then we could find the total flux by multiplying the flux per unit area by the area. For example, a river with cross-sectional area 25 m^2 and a flux per unit area of 100 $kg/s \cdot m^2$ would carry a total flux of 2500 kg/s. If the flow per unit area weren't uniform, then we'd have to consider small patches of area, calculate the flux over each, and sum the results. We'll quantify that process shortly.

Electric Flux

In the case of electric fields, there's nothing "flowing." But electric field lines are analogous to the streamlines we've used to describe fluid flow. Their direction gives the local direction of the electric field, and their proximity reflects the field strength. Furthermore, field lines begin or end only on electric charges; otherwise, they extend continuously to infinity. This property of electric fields—which you'll soon see is intimately related to Coulomb's law—is analogous to the conservation of matter in fluid flows. So we use the term **electric flux** to describe the electrical analog of the flux of a fluid. In the simplest case of a uniform electric field of magnitude E perpendicular to an area A, the flux of the electric field is $\Phi = EA$, in analogy with our finding the flux in a river by multiplying the flux per unit area by the river's cross-sectional area. Here we use Φ, the capital Greek phi, as our symbol for electric flux.

What good is electric flux? Figure 21.5 gives the answer, showing that electric flux carries the same information as the "number of field lines crossing a surface" that was so important in our discussion of Fig. 21.4. Consider first the flux through a flat surface. Comparison of Figs. 21.5*a* and *b* shows that the number of field lines crossing a given surface is larger when the field is stronger—a fact that's precisely captured in our formula $\Phi = EA$. Comparison of Figs. 21.5*b* and *c* shows further that, for a given field strength, the number of field lines crossing a given surface is larger when the surface is larger. Again, $\Phi = EA$ captures this situation. Finally, Fig. 21.5*d* shows that the number of field lines is reduced when the surface is tilted relative to the field. You can also see this effect in Fig. 21.6, which shows a water-flow analogy. Specifically, the flux is reduced by a factor $\cos\theta$, where θ is the angle between the electric field $\vec{E}$ and a vector $\vec{A}$ that's normal to the surface. So our flux expression generalizes to $\Phi = EA\cos\theta$. If we define the normal vector $\vec{A}$ as having magnitude equal to the surface area A, then we can write the electric flux through a flat surface in a uniform electric field using the dot product:

$$\Phi = \vec{E} \cdot \vec{A} \tag{21.1}$$

where the dot product, defined in Chapter 6, is the product of the two vector magnitudes with the cosine of the angle between them. Since the units of $\vec{E}$ are N/C, flux is measured in $\mathrm{N \cdot m^2/C}$.

The surfaces in Fig. 21.5 are *open* surfaces, meaning it's possible to get from one side to the other without passing through the surface. For open surfaces there's an ambiguity in the sign of Φ, since we could have taken $\vec{A}$ in either of the two directions along the perpendicular to the surface. But for *closed* surfaces, we unambiguously define the direction of $\vec{A}$ as the direction of the outward-pointing normal to the surface.

TIP **THE FLUX IS NOT THE FIELD** The flux Φ and field $\vec{E}$ are related but distinct quantities. The field is a vector defined at each point in space. The flux is a scalar and a global property, depending on how the field behaves over an extended surface rather than at a single point; it's a quantification of the number of field lines crossing a surface.

What if a surface is curved and/or the field varies with position? Then we divide the surface into patches, each small enough that it's essentially flat and that the field is essentially uniform over each (Fig. 21.7). If a patch has area dA, then Equation 21.1 gives the flux through it: $d\Phi = \vec{E} \cdot d\vec{A}$, where the vector $d\vec{A}$ is normal to the patch. The total flux through the surface is then the sum over all the patches. If we make the patches arbitrarily small, that sum becomes an integral, and the flux is

$$\Phi = \int_{\text{surface}} \vec{E} \cdot d\vec{A} \tag{21.2}$$

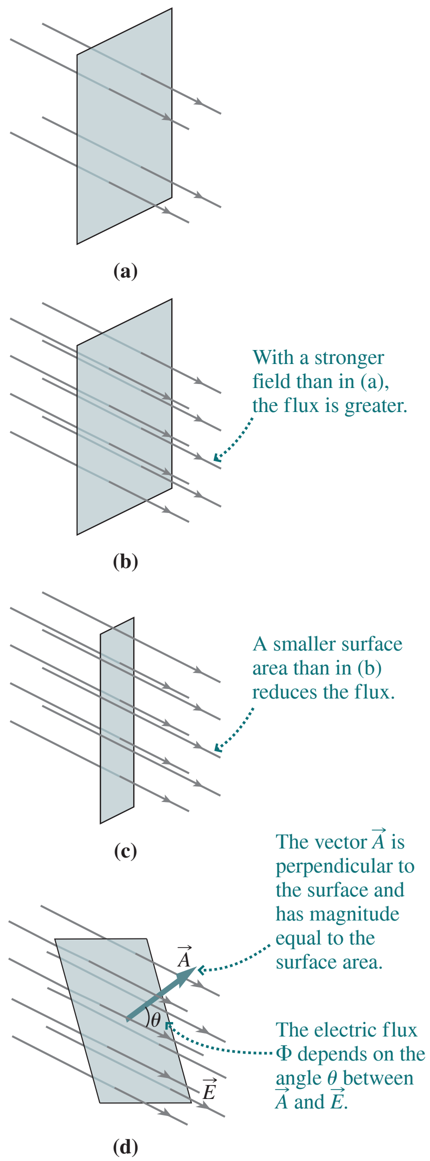


FIGURE 21.5 Electric flux through flat surfaces.

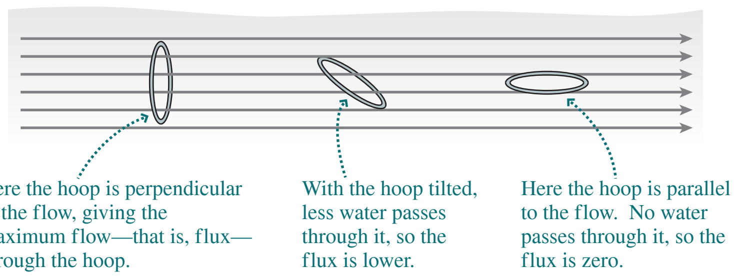


FIGURE 21.6 A water-flow analogy for electric flux, showing a circular hoop immersed in a flow of water.

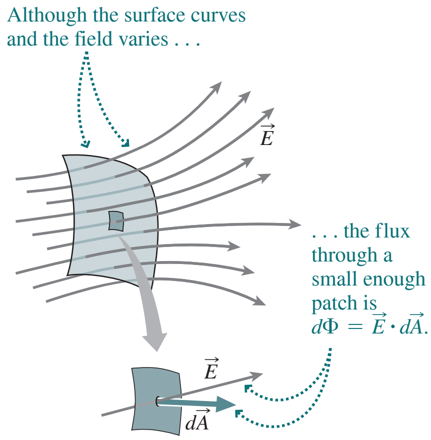


FIGURE 21.7 Finding the flux through a small area dA, so small it's essentially flat.

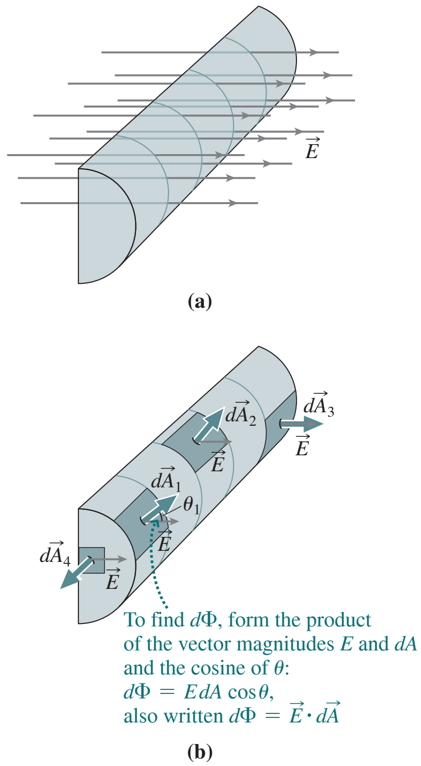


FIGURE 21.8 Meaning of the surface integral for electric flux, here for the case of a surface that isn't flat. (a) The surface is a half-cylinder in a uniform electric field $\vec{E}$. (b) The total flux through the surface is the sum of the fluxes $d\Phi = \vec{E}\cdot d\vec{A}$ over all the little patches constituting the surface, four of which are shown. In the limit that the patches become infinitesimally small, that sum becomes the integral in Equation 21.2.

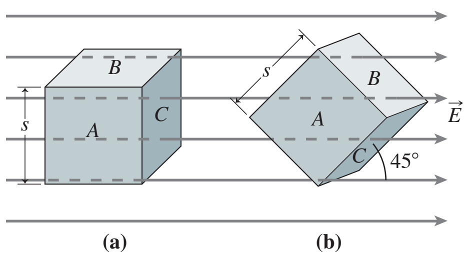
The limits of the integral range over the entire surface, picking up contributions from all the patches $d\vec{A}$. The result is a **surface integral**—in general, the integral of a vector field over a surface, with the orientation of field and surface taken into account. Tactics Box 21.1 shows the meaning of such a surface integral. Although a surface integral can be difficult to evaluate, we'll find it most useful in situations where its evaluation is almost trivial.

Tactics 21.1 SURFACE INTEGRALS

The math in Equation 21.2 looks complicated, with two vectors and a dot product inside the integral and limits that specify an entire surface. Figure 21.8 helps decipher this math. The figure shows a surface, which could be arbitrary but in this case is half a cylinder including its semicircular end caps. This surface is immersed in an electric field $\vec{E}$ (Fig. 21.8*a*), which in this case is uniform but need not be. Figure 21.8*b* shows several small patches of the surface area. Each patch is described by a vector $d\vec{A}$ with magnitude equal to the area of the corresponding small patch and direction given by the normal to that patch, similar to the patch we showed in Fig. 21.7. At each patch we consider the direction of the normal vector $d\vec{A}$ relative to the electric field $\vec{E}$ at that patch, and we form the quantity $E\,dA\cos\theta$ or, written more compactly, $\vec{E}\cdot d\vec{A}$. What is this quantity? It's the flux, $d\Phi$, through the small patch. Although it's formed from two vectors, $d\Phi$ itself is a scalar. So to find the total flux through our surface, all we have to do is add up all those little $d\Phi$s. And that—in the limit that the $d\Phi$s become infinitesimally small and infinitely many—is the meaning of the surface integral in Equation 21.2. As you work through this chapter, you'll see many instances where we evaluate such surface integrals. We'll be able to choose surfaces that make the actual evaluation straightforward—but to appreciate that evaluation you'll need to understand the meaning of the surface integral as we've outlined it here.

GOT IT?

21.2 The figure shows a cube of side s in a uniform electric field $\vec{E}$. (1) What's the flux through each of the three cube faces A, B, and C with the cube oriented as in (a)? (2) Repeat for the orientation in (b), with the cube rotated 45°.



21.3 Gauss's Law

LO 21.3 *Describe Gauss's law and explain how it relates to Coulomb's law.*

We've seen that the number of field lines emerging from a closed surface is proportional to the charge enclosed. Now that we've developed electric flux to express more rigorously the notion "number of field lines," we can state that **the electric flux through any closed surface is proportional to the net charge enclosed by that surface.** Writing the same thing mathematically gives $\Phi = \oint \vec{E}\cdot d\vec{A} \propto q_{\text{enclosed}}$, where the circle indicates that the integral is over a *closed* surface.

To evaluate the proportionality between flux and charge, consider a positive point charge q and a spherical surface of radius r centered on the charge (Fig. 21.9). The flux through this surface is given by Equation 21.2:

$$\Phi = \oint \vec{E}\cdot d\vec{A} = \oint E\,dA\cos\theta$$

But Fig. 21.9 shows that the surface normal $d\vec{A}$ and the electric field $\vec{E}$ are parallel at any point on the sphere, so $\cos\theta = 1$. Since the electric field varies as $1/r^2$, its magnitude is the same everywhere at the fixed radius r of our sphere. Thus, we can take E outside the integral, giving

$$\Phi = \oint_{\text{sphere}} E\,dA = E\oint_{\text{sphere}} dA = E(4\pi r^2)$$

where the last step follows because $\oint dA$ is just the surface area of the sphere, namely $4\pi r^2$. (Here's the first of many cases where the evaluation of a surface integral is straightforward

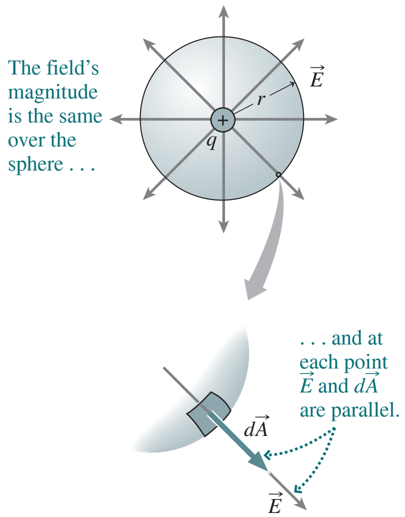


FIGURE 21.9 The electric field of a point charge, shown with a spherical surface centered on the charge.

because of the symmetry of the electric field and of the surface we're integrating over.) Now, the electric field of a point charge is given by Equation 20.3: $E = kq/r^2$. So we have $\Phi = E(4\pi r^2) = (kq/r^2)(4\pi r^2) = 4\pi kq$. Since the point charge q is the only charge inside our spherical surface, the proportionality constant between flux and enclosed charge is $4\pi k$.

Before proceeding, we introduce the so-called permittivity constant ϵ_0, defined as $\epsilon_0 = 1/4\pi k$, where k is the Coulomb constant. The value of ϵ_0 is approximately $8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$. There's no physics here, just a new constant that conveys the same information as k. That there are two redundant constants is a historical artifact, and we switch now from k to ϵ_0 because doing so makes subsequent formulas simpler. In terms of ϵ_0, the proportionality $4\pi k$ between flux and enclosed charge becomes $1/\epsilon_0$. So our statement that the flux through any closed surface is proportional to the net charge enclosed becomes

$\vec{E}$ is the electric field at the surface.

$d\vec{A}$ is an infinitesimal element of surface area. It's a vector with direction perpendicular to the surface.

The circle indicates integration over a *closed* surface.

q_{enclosed} is the net charge enclosed by the surface.

$$\oint \underbrace{\vec{E} \cdot d\vec{A}} = \frac{q_{\text{enclosed}}}{\epsilon_0} \qquad \text{(Gauss's law)} \tag{21.3}$$

As usual, the dot product is the scalar $E\,dA\cos\theta$.

ϵ_0 is a constant related to the Coulomb constant k.

Here the integral is taken over *any closed surface*, and q_{enclosed} is the charge enclosed *by that surface*.

Equation 21.3 is **Gauss's law**, one of four fundamental relations that govern the behavior of electromagnetic fields throughout the universe. Whether you journey into a star in some remote galaxy, down among the strands of a DNA molecule, or into the microprocessor chip at the heart of your computer, you'll find that the flux of the electric field through any closed surface depends only on the enclosed charge. In nearly 200 years of experiments, no electric field has ever been found to violate Gauss's law.

Gauss's law, though clothed in the mathematical finery of a surface integral, is just a more rigorous way of saying what's clear in Fig. 21.4: The number of field lines emerging from a closed surface is proportional to the net charge enclosed.

Gauss and Coulomb

Gauss's law and Coulomb's law look completely different, but they're closely related. Figure 21.10 shows that their relationship involves the inverse-square law. Gauss's law tells us that the flux through the two surfaces in the figure is the same and is equal to q/ϵ_0. But why? Because, as our arguments leading to Gauss's law show, the flux through a spherical surface of radius r centered on a point charge is the product of the surface area $4\pi r^2$ and the electric field E at the surface. But Coulomb's law says that the electric field drops off as $1/r^2$. As r increases, the surface area grows as r^2, but the $1/r^2$ decrease in field strength just compensates, giving a constant value for the flux. If the inverse-square law (e.g., Coulomb's law) didn't hold, then the flux wouldn't be constant and Gauss's law wouldn't hold either.

It's also the inverse-square law that makes electric field-line pictures useful for visualizing fields. Field lines begin or end only on charges; otherwise, they go off to infinity. As the field lines of a point charge spread in three dimensions, the number crossing any spherical surface (or any closed surface) remains the same. But larger spheres have larger surface areas, in proportion to r^2—and that means the density of field lines drops as $1/r^2$, accurately reflecting the field strength. Once again, the inverse-square law (Coulomb) and the relation between flux and enclosed charge (Gauss) are intimately connected. Incidentally, field-line pictures printed in a book or drawn on a blackboard generally can't be quantitatively correct because they don't show the spreading of field lines in all three dimensions.

We've been talking here only about isolated point charges, but Gauss's law applies to *all* electric fields, no matter how complicated the charge distributions that produce them. That's because the superposition principle allows us to add vectorially the electric fields

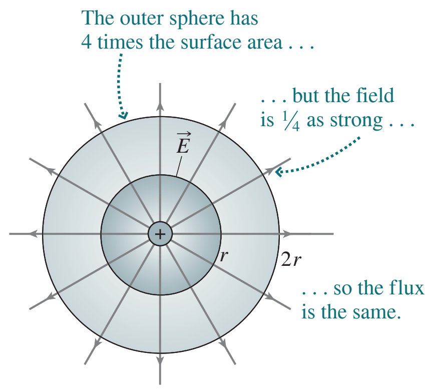


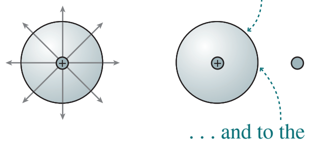
FIGURE 21.10 Gauss's law follows from the inverse-square aspect of Coulomb's law.

described individually by Coulomb's law for the point-charge field. So our argument leading to Gauss's law still applies when the field $\vec{E}$ is a superposition of point-charge fields.

For static charge distributions, Gauss's and Coulomb's laws are completely equivalent. But with moving charges only Gauss's law remains exact. So Gauss's law is more fundamental, and we count it among the four basic laws of electromagnetism.

GOT IT?

21.3 A spherical surface surrounds an isolated positive charge, as shown. (1) If a second charge is placed outside the surface, which of the following will be true of the total flux through the surface? (a) It doesn't change; (b) it increases; (c) it decreases; (d) it increases or decreases depending on the sign of the second charge. (2) Repeat for the magnitude of the electric field on the surface at the point between the charges.



21.4 Using Gauss's Law

LO 21.4 *Use Gauss's law to determine electric fields for charge distributions with sufficient symmetry.*

Gauss's law is a universal statement about electric fields; it's true for *any* surface enclosing *any* charge distribution. For charge distributions with sufficient symmetry—symmetric about a point, a line, or a plane—Gauss's law also provides a powerful alternative to Coulomb's law that makes electric-field calculations much easier. For such distributions it's possible to evaluate the flux integral on the left-hand side of Gauss's law (Equation 21.3) without actually knowing the field. We can then solve for E in terms of the enclosed charge. We begin with a general strategy for applying Gauss's law to symmetric charge distributions, followed by examples of the three symmetries.

PROBLEM-SOLVING STRATEGY 21.1 Gauss's Law

INTERPRET Check that your charge distribution has sufficient symmetry to use Gauss's law to find the electric field, and identify the symmetry. Is it spherical, line, or plane? If the charge distribution doesn't exhibit one of these symmetries, then Gauss's law—though always true—won't help you find the field.

DEVELOP Draw a diagram showing the charge distribution, and use the symmetry to infer the direction of the electric field. Then draw an appropriate **Gaussian surface**—an imaginary closed surface that will let you evaluate the flux integral in Gauss's law. The field should have constant magnitude over the surface and should be perpendicular to the surface. Sketch some field lines; the symmetry should indicate their direction. With line and plane symmetry, there may be parts of the surface where the field is perpendicular and parts where it's parallel; we'll show in examples how to handle these situations. If you can't find a suitable Gaussian surface, there probably isn't sufficient symmetry to use Gauss's law to calculate the field.

EVALUATE

- Evaluate the flux $\Phi = \oint \vec{E} \cdot d\vec{A}$ over your Gaussian surface. Since you've found a surface to which the field is perpendicular, $\vec{E}$ and the surface normal vector $d\vec{A}$ are parallel, so $\cos\theta = 1$ and the dot product becomes $E\,dA$. With the field strength E constant over the surface, it can come out of the integral, leaving $\oint dA$. That's just the surface area A. So the flux will be EA. By carefully choosing your Gaussian surface, you've turned that messy integral in Equation 21.2 into a mere multiplication!

If $\vec{E}$ is parallel to some parts of the area—as happens in line and plane symmetry— then $\vec{E} \perp d\vec{A}$, so $\vec{E} \cdot d\vec{A} = 0$ and there's no contribution to the flux from those areas.

- Evaluate the *enclosed* charge. This may or may not be the same as the total charge, depending on whether the position at which you're evaluating the field lies outside or inside the charge distribution.
- Evaluate the field E by invoking Gauss's law, equating the flux to $q_{\text{enclosed}}/\epsilon_0$, and solving for E. This is the field magnitude; the direction should be evident from symmetry.

ASSESS Does your answer make sense? Does the field behave as you would expect given what you know of simpler charge distributions—point charges, line charges, or charged sheets, depending on the symmetry?

You'll quickly get the hang of this strategy because Gauss's law is useful for finding the field only in the three common symmetries. That means you need to evaluate the integral for the flux just once for a given symmetry.

Spherical Symmetry

A charge distribution has spherical symmetry when the charge density depends only on the radial distance r from the center of the distribution—also called the *point of symmetry*. A point charge is one example, as are the three charge distributions we'll now treat in Examples 21.1 through 21.3. In fact, *any* spherical charge distribution has spherical symmetry, provided the charge density varies solely with distance from the center. The only electric field consistent with spherical symmetry is a field that points in the radial direction, either away from or toward the point of symmetry. We'll start with the simplest case, a charged spherical shell.

EXAMPLE 21.1 Gauss's Law: A Hollow Spherical Shell

A thin, hollow spherical shell of radius R has total charge Q distributed uniformly over its surface. Find the electric field (a) outside and (b) inside the shell.

INTERPRET This is a spherically symmetric charge distribution, so we can use Gauss's law to find the field.

DEVELOP With spherical symmetry, the field lines have to point radially outward for a positive charge and inward for a negative one. We've sketched the charged shell and some field lines outside it in Fig. 21.11. With spherical symmetry, the appropriate Gaussian surfaces are also spheres centered on the center of the shell. Since we're asked for the field both outside and inside the shell, we've sketched one such surface outside the shell and one inside.

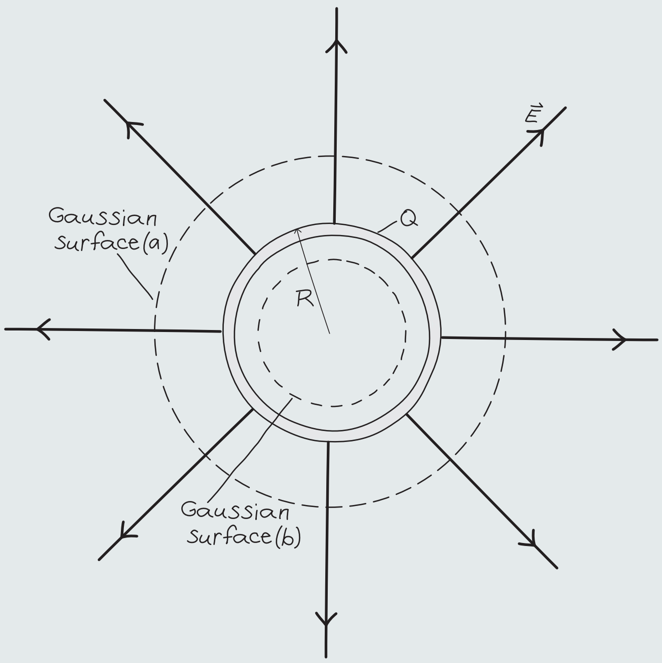


FIGURE 21.11 Finding the field of a charged shell.

EVALUATE

- We first evaluate the flux integral $\Phi = \oint \vec{E} \cdot d\vec{A}$ that appears on the left-hand side of Gauss's law. With our choice of a spherical Gaussian surface, the field is everywhere perpendicular to the surface, or parallel to the normal vector $d\vec{A}$ (recall Fig. 21.9). So $\cos\theta = 1$, and the dot product becomes just $E\,dA$. With our Gaussian surface being a sphere centered on the point of symmetry, the field magnitude E is the same all over the Gaussian surface. So E comes outside the integral, leaving $\Phi = E\oint dA$. The remaining integral is the sum of all the little areas dA over the spherical Gaussian surface—and that's the surface area of a sphere, $A = 4\pi r^2$. The flux then becomes $\Phi = 4\pi r^2 E$. Note that this result doesn't depend on whether the surface is outside or inside the shell or on the particular value of its radius r; it follows from the symmetry and our choice of a spherical Gaussian surface alone. So it holds for both Gaussian surfaces shown in Fig. 21.11, outside and inside the shell.

(*continued*)

- Now we evaluate the *enclosed* charge. That *is* different for the two surfaces, so we'll deal first with (a), the field outside the shell. For this we consider the Gaussian surface marked (a); it's outside the shell, so it encloses the entire charge Q. Substituting our expression for the flux, $\Phi = 4\pi r^2 E$, into Gauss's law, $\Phi = \oint \vec{E}\cdot d\vec{A} = q_{\text{enclosed}}/\epsilon_0$, gives $4\pi r^2 E = Q/\epsilon_0$. We can then solve for E to get

$$E = \frac{Q}{4\pi\epsilon_0 r^2} \quad \text{(field outside any spherical charge distribution)} \qquad (21.4)$$

- Now let's consider the interior of the shell. With all the charge on the shell's surface, there's no charge inside. So Gaussian surface (b) doesn't enclose any charge. But, as we argued previously, the flux is still $4\pi r^2 E$, so Gauss's law becomes $4\pi r^2 E = 0$. Solving for E gives $E = 0$. We could have drawn Gaussian surface (b) anywhere inside the shell, including right up to its surface, so there's no electric field anywhere inside the shell.

ASSESS

- Consider first the field outside the shell (Equation 21.4). Since $1/4\pi\epsilon_0$ is the Coulomb constant k, this field can be written $E = kQ/r^2$—precisely the field of a point charge! In fact, our argument leading to this result didn't depend on the charge being on a spherical shell; as long as a charge distribution is spherically symmetric and we're evaluating the field *outside* the distribution, we would find the same result. That's why we've made the result a numbered equation and labeled it "field outside any spherical charge distribution." Outside *any* spherical charge distribution, then, its field is *exactly* the same as that of a point charge with the same total charge, located at the center of symmetry. Incidentally, this result also holds for gravity because it, too, obeys an inverse-square law. That's why we can treat planets as though they were point masses located at their centers.
- Now the interior field: How can it be exactly zero everywhere inside the shell? Figure 21.12 shows that this remarkable result is a consequence of the inverse-square law.

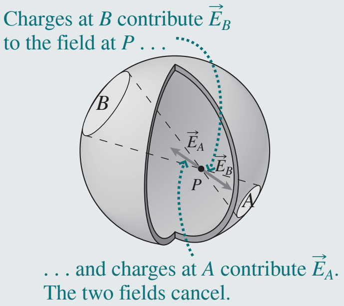


FIGURE 21.12 At any point P inside a charged shell, the field from the relatively few nearby charges at A is exactly canceled by the field from the more numerous but more distant charges at B. The result is zero field everywhere inside the shell.

TIP **SYMMETRY MATTERS!** We used the fact that there's no charge inside an empty spherical shell to conclude that the field inside the shell is zero. But be careful: That conclusion follows only when there's enough symmetry, as Fig. 21.13 shows. Here, the Gaussian surface encloses zero net charge, so the flux through the surface is zero. But the electric field both on and inside the surface isn't zero.

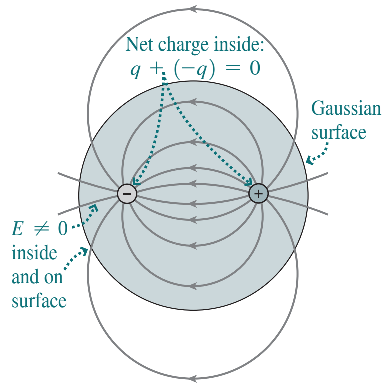


FIGURE 21.13 A spherical Gaussian surface surrounds a dipole. We have $q_{\text{enclosed}} = 0$, but $E \neq 0$ inside the surface because the dipole isn't spherically symmetric.

GOT IT? **21.4** A spherical shell carries charge Q distributed uniformly over its surface. If the charge on the shell doubles, what happens to the electric field (1) inside and (2) outside the shell?

Now let's consider a slightly more complicated charge distribution: a spherical shell with a point charge at the center.

EXAMPLE 21.2 Gauss's Law: A Point Charge within a Shell

Worked Example with Variation Problems

A positive point charge $+q$ is at the center of a spherical shell of radius R carrying charge $-2q$, distributed uniformly over its surface. Find expressions for the field strength inside and outside the shell.

INTERPRET This problem is about a charge distribution with spherical symmetry.

DEVELOP We want to find the fields both inside and outside the distribution, so we show two spherical Gaussian surfaces in our sketch, Fig. 21.14. We've also drawn some field lines, which will help in assessing our answer.

EVALUATE
- We already know that the flux through a spherical Gaussian surface is $\Phi = 4\pi r^2 E$ when we have spherical symmetry, so that's the flux for both surfaces.
- The outer surface encloses the charge $+q$ at the center and $-2q$ on the shell, so the net charge enclosed is $q_{\text{enclosed}} = -q$ for $r > R$. The inner surface encloses only the point charge $+q$, so $q_{\text{enclosed}} = +q$ for $r < R$.
- Now we find the field by equating the flux $\Phi = 4\pi r^2 E$ to $q_{\text{enclosed}}/\epsilon_0$. The result for $r > R$ is $E = -q/4\pi\epsilon_0 r^2$, where the minus sign appears because the enclosed charge is $-q$. For $r < R$ with enclosed charge $+q$, the result is $E = q/4\pi\epsilon_0 r^2$.

ASSESS Make sense? We've seen that the field outside a spherically symmetric charge distribution is that of an equivalent point charge at the center. Here the net charge is $-q$, and our result for $r > R$ is indeed the field of a point charge $-q$. Another way to see this is to apply superposition: The field outside due to the shell alone is that of a point charge $-2q$; that adds to the field of the central point charge $+q$ to give, again, the field of a point charge $-q$ in the region outside the shell. In Example 21.1 we found that the shell produces no field in its interior, so here superposition leaves us with the field of the central point charge alone, just as our result shows. Sketching the field lines, as we've done in Fig. 21.14, also shows how our results make sense.

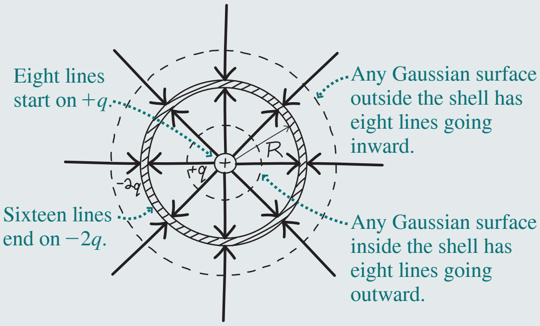


FIGURE 21.14 A shell carrying charge $-2q$ surrounds a point charge $+q$. Note that the number of field lines beginning on the point charge and the number ending on the shell correctly reflect their respective charges.

What if we have charge distributed throughout a volume? As long as the distribution is spherically symmetric, we can find the electric field using Gauss's law just as we did in Examples 21.1 and 21.2. The only difference comes when we evaluate the enclosed charge. Example 21.3 considers this situation.

EXAMPLE 21.3 Gauss's Law: A Uniformly Charged Sphere

A charge Q is distributed uniformly throughout a sphere of radius R. Find the electric field at all points, (a) outside and (b) inside the sphere.

INTERPRET The charge distribution has spherical symmetry, so we can use Gauss's law to find the field.

DEVELOP Figure 21.15 is a sketch of the spherical charge distribution. As you've already seen, the field in any case of spherical symmetry must be radial, so we've sketched some field lines on the figure. As in Examples 21.1 and 21.2, Gaussian surfaces appropriate to spherical symmetry are themselves spheres. Since we're asked for the field both outside and inside the charge distribution, we've drawn two such Gaussian surfaces, one outside and one inside the distribution.

EVALUATE
- We already know the answer to (a), the field outside the charge distribution. That's because, as we found in Example 21.1, the field outside *any* spherically symmetric charge distribution is exactly that of a point charge located at the center. So the field outside the charged sphere has magnitude $E_{\text{out}} = Q/4\pi\epsilon_0 r^2$ and points radially

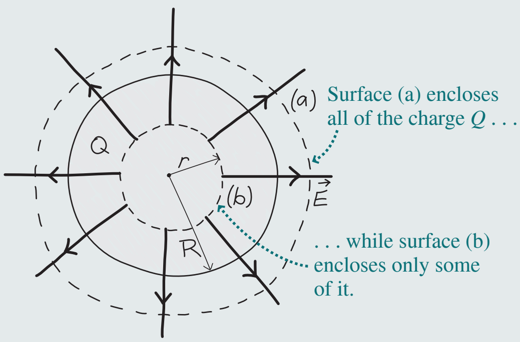


FIGURE 21.15 Finding the field of a uniformly charged sphere. The field also exists within the inner Gaussian surface, but we haven't shown it.

(*continued*)