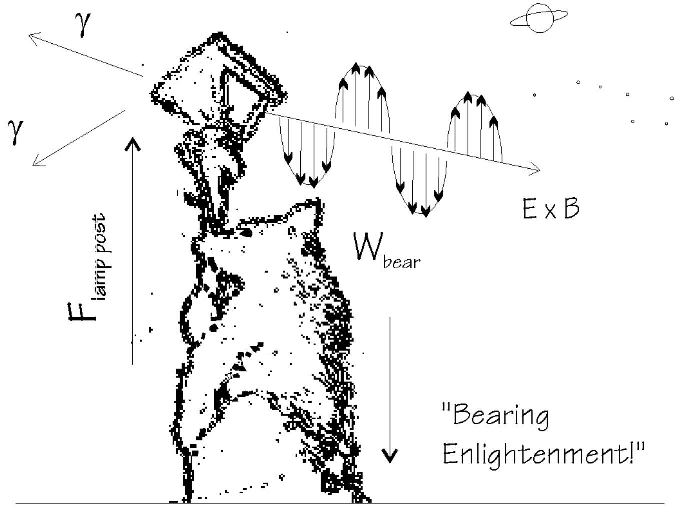
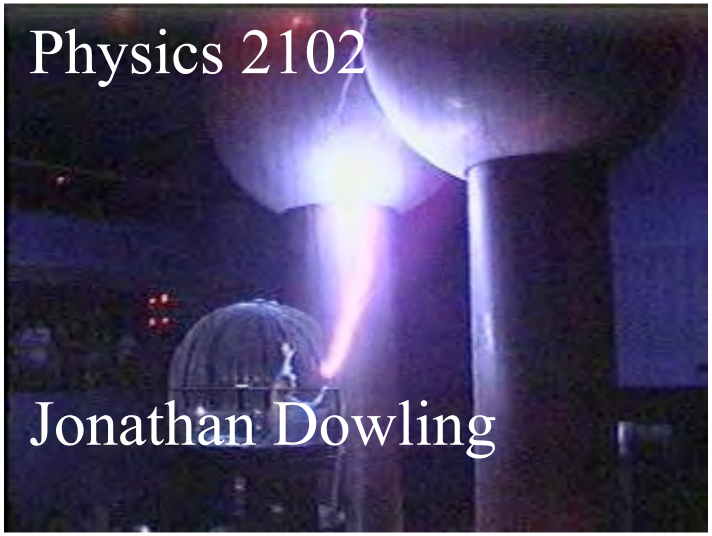
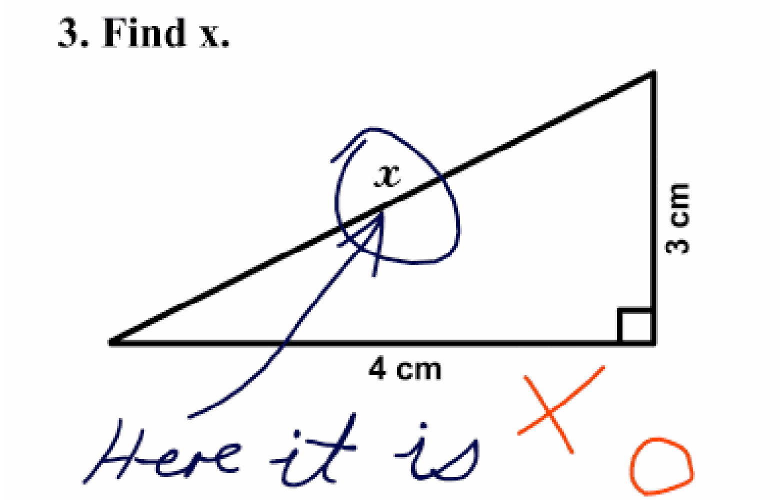




Physics 2102
Final Exam Review

One Brick Shy

"Well! I think this calls for a little celebration."



Ocular Trauma - by Wade Clarke ©2005

A few concepts: electric force, field and potential

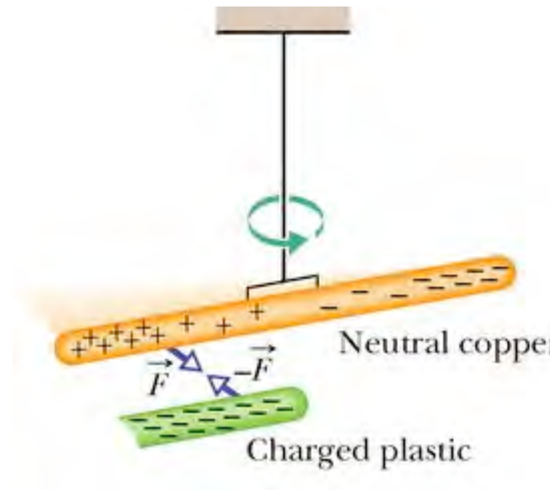
- Electric force:
  - What is the force on a charge produced by other charges?
  - What is the force on a charge when immersed in an electric field?
- Electric field:
  - What is the electric field produced by a system of charges? (Several point charges, or a continuous distribution)
- Electric potential:
  - What is the potential produced by a system of charges? (Several point charges, or a continuous distribution)

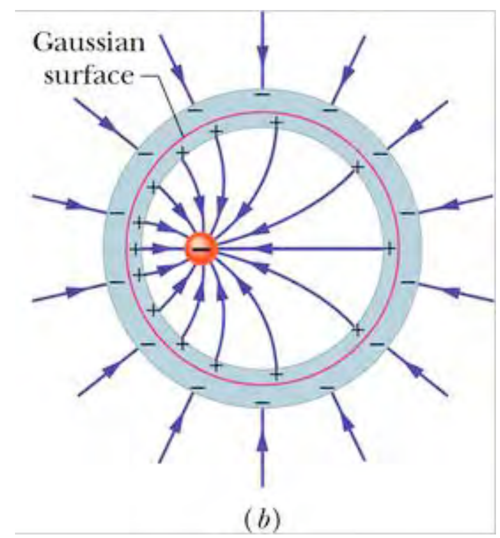
Plus a few other items…

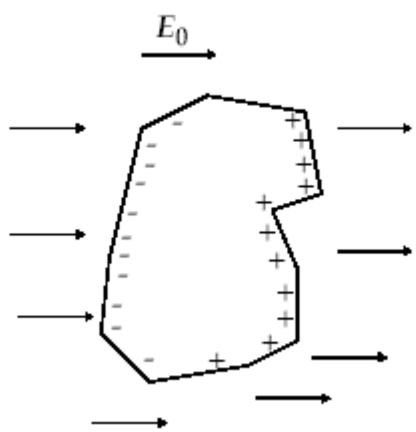
- Electric field lines, equipotential surfaces: lines go from +*ve* to –*ve* charges; lines are perpendicular to equipotentials; lines (and equipotentials) never cross each other…
- Gauss' law: $\Phi=q/\varepsilon_0$. Given the field, what is the charge enclosed? Given the charges, what is the flux? Use it to deduce formulas for electric field.
- Electric dipoles: field and potential produced BY a dipole, torque ON a dipole by an electric field, potential energy of a dipole
- Electric potential, work and potential energy: work to bring a charge somewhere is $W = -qV$ (signs!). Potential energy of a system = negative work done to build it.
- Conductors: field and potential inside conductors, and on the surface.
- Shell theorem: systems with spherical symmetry can be thought of as a single point charge (but how much charge?)
- Symmetry, and "infinite" systems.

Conductors and insulators

- Will two charged objects attract or repel?
- Can a charged object attract or repel an uncharged object?
- What is the electric field inside a conductor?
- What is the direction of the electric field on the surface of a conductor?
- What happens to a conductor when it is immersed in an electric field?







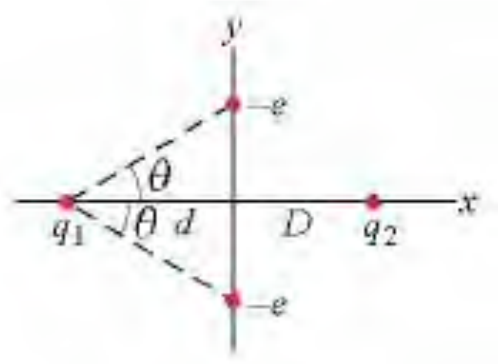
Electric forces and fields: point charges

- Coulomb's law: $F = k\frac{|q_1||q_2|}{r^2}$
- Electric field measured with a test charge: $\vec{E} = \frac{\vec{F}}{q_o}$
- Force on a charge in an electric field: $\vec{F} = q\vec{E}$
- Electric field of a point charge: $E = \frac{F}{q_o} = k\frac{|q|}{r^2}$

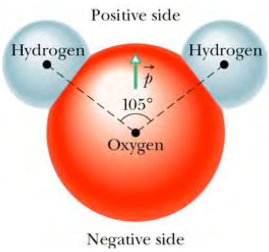
Figure 22N-14 shows an arrangement of four charged particles, with angle θ = 34° and distance d = 2.20 cm. The two negatively charged particles on the y axis are electrons that are fixed in place; the particle at the right has a charge $q_2 = +5e$

(a) Find distance D such that the net force on the particle at the left, due to the three other particles, is zero.

(b) If the two electrons were moved further from the x axis, would the required value of D be greater than, less than, or the same as in part (a)?

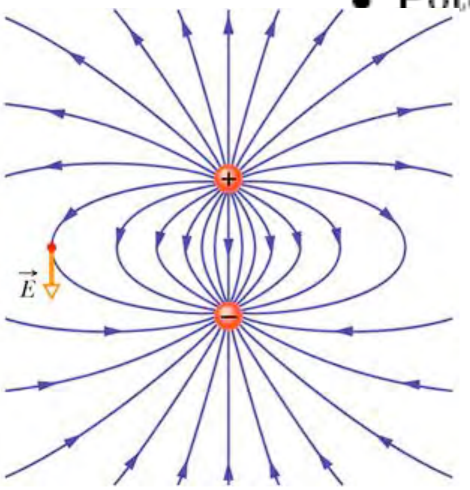


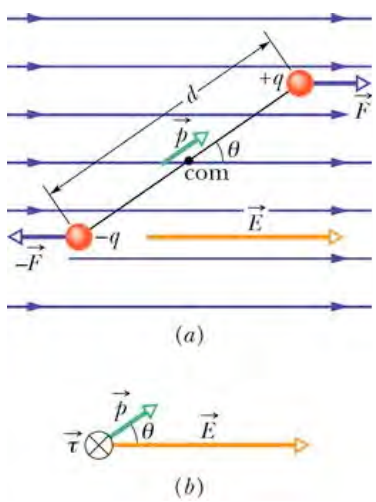
Other possible questions: what's the electric field produced by the charges XXX at point PPP ? what's the electric potential produced by the charges XXX at point PPP ? What's the potential energy of this system?

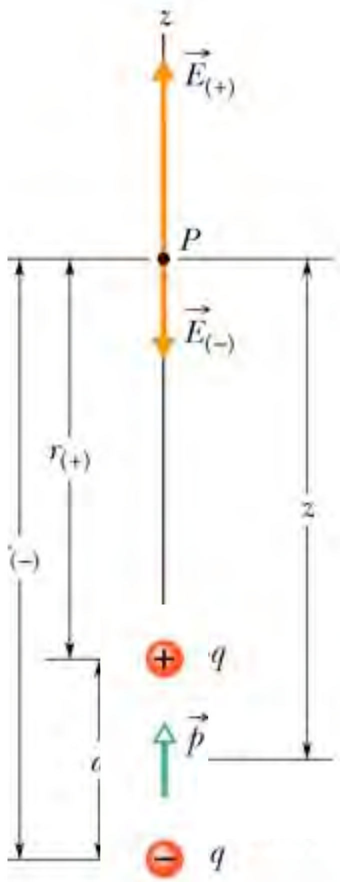


Electric dipoles

- Electric field of a dipole on axis: $E = \dfrac{2kp}{z^3}$
- Torque on a dipole in an electric field: $\vec{\tau} = \vec{p} \times \vec{E}$
- Potential energy of a dipole in electric field: $U = -\vec{p} \cdot \vec{E}$



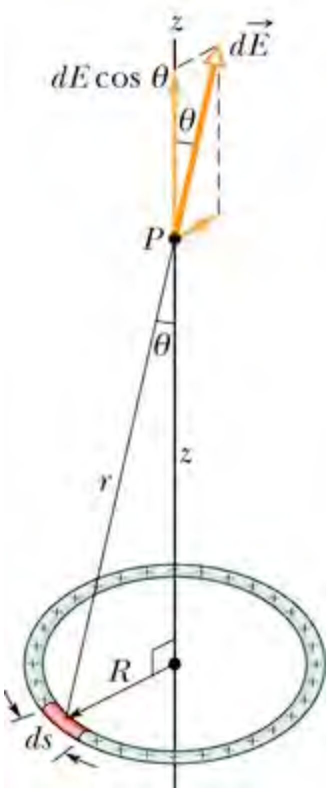


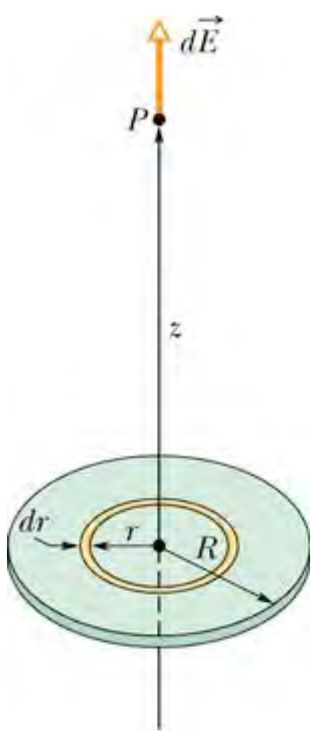


- What's the electric field at the center of the dipole? On axis? On the bisector? far away?
- What is the force on a dipole in a uniform field?
- What is the torque on a dipole in a uniform field?
- What is the potential energy of a dipole in a uniform field?

Electric fields of distributed charges

- Electric field of an infinite non-conducting plane with a charge density σ: $E = \dfrac{\sigma}{2\epsilon_o}$
- Electric field of an infinite line charge: $E = \dfrac{2k\lambda}{r}$



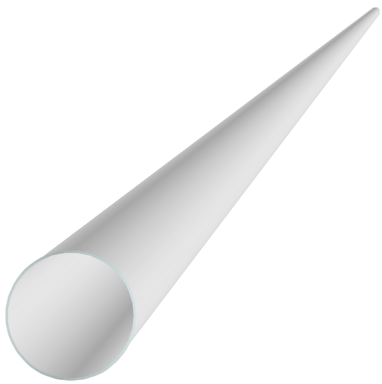


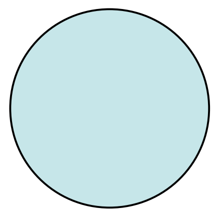
Possible problems, questions:

• What's the electric field at the center of a charged circle?
• What's the electric field at the center of ¼ of a charged circle?
• What's the electric field far from the ring? far from the disk?
• What's the electric field of an infinite disk?

Gauss' law

- Gauss' law: $\epsilon_o \oint \vec{E} \cdot d\vec{A} = q_{enc}$





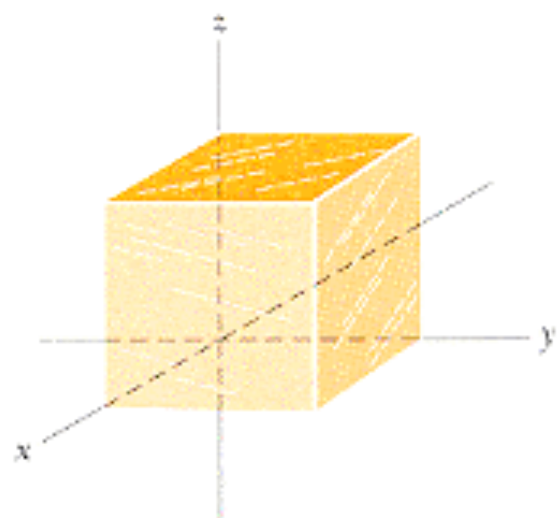
A long, non conducting, solid cylinder of radius 4.1 cm has a nonuniform volume charge density that is a function of the radial distance *r* from the axis of the cylinder, as given by $\rho = Ar^2$, with A = 2.3 μC/m^5.

(a) What is the magnitude of the electric field at a radial distance of 3.1 cm from the axis of the cylinder?

(b) What is the magnitude of the electric field at a radial distance of 5.1 cm from the axis of the cylinder?

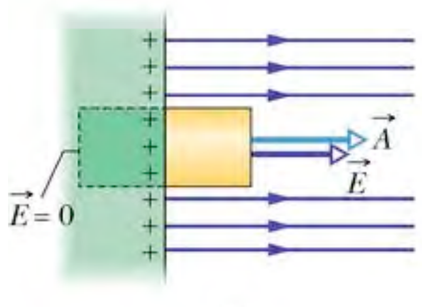
Gauss' law

At each point on the surface of the cube shown in Fig. 24-26, the electric field is in the z direction. The length of each edge of the cube is 2.3 m. On the top surface of the cube **E** = -38 **k** N/C, and on the bottom face of the cube **E** = +11 **k** N/C. Determine the net charge contained within the cube.
[-2.29e-09] C



Gauss' law: applications

- Electric field of an infinite non-conducting plane with a charge density σ: $E = \dfrac{\sigma}{2\epsilon_o}$
- Electric field of an infinite line charge: $E = \dfrac{2k\lambda}{r}$
- Electric field close to the surface of a conductor: $E = \dfrac{\sigma}{\epsilon_o}$



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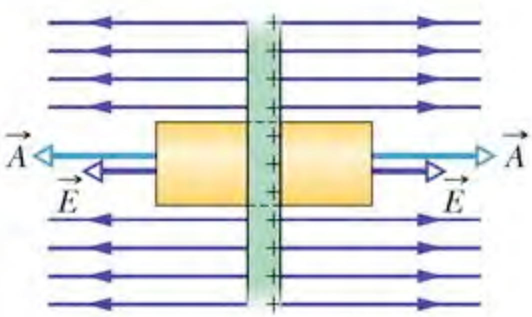

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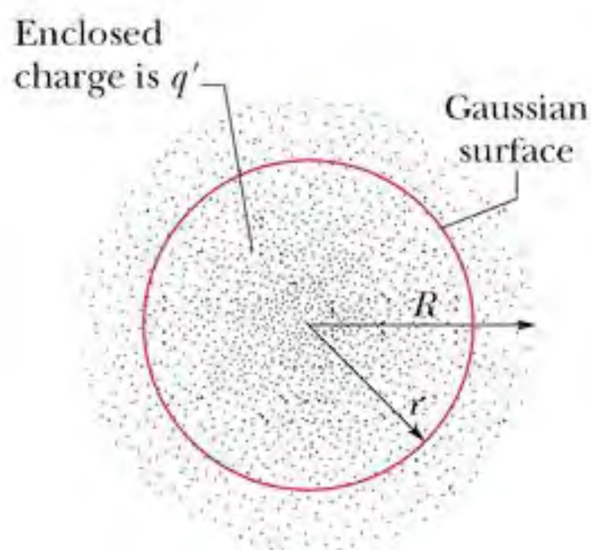
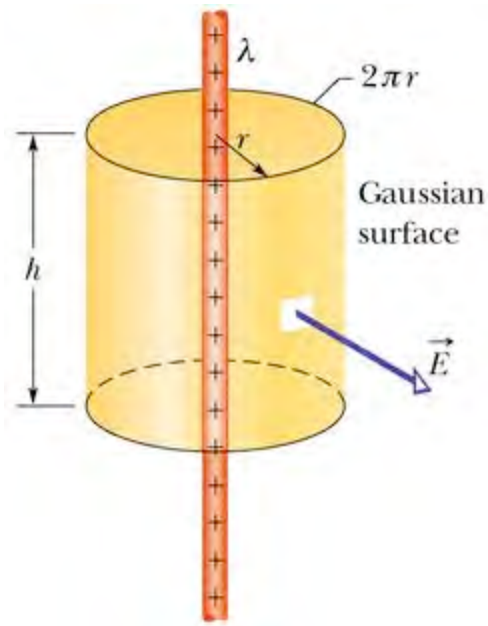




Electric potential, electric potential energy, work

- Work, potential energy, and electric potential:

$\Delta U = U_f - U_i = -W$ $\qquad$ $\vec{E} = -\vec{\nabla} V, \quad E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z}$

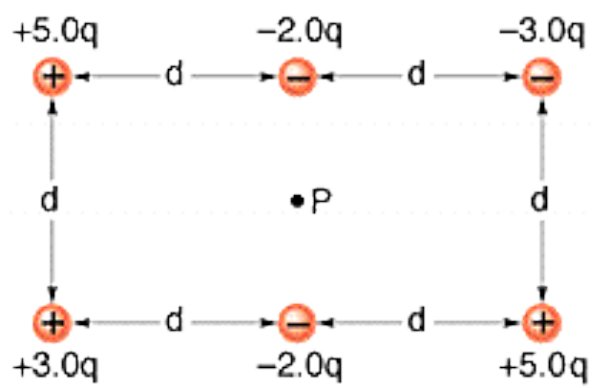
$U = -W_\infty$ $\qquad$ potential of a point charge q: $\quad V = k\frac{q}{r}$

$V = -\frac{W_\infty}{q}$ $\qquad$ potential of n point charges: $V = \sum_{i=1}^{n} V_i = k\sum_{i=1}^{n} \frac{q_i}{r_i}$

$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$

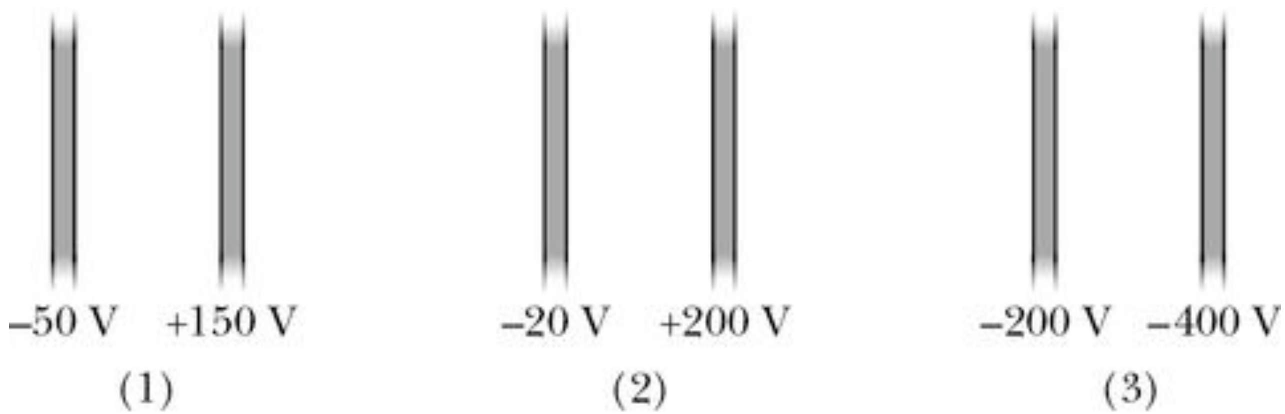
potential energy of two point charges: $U_{12} = W_{ext} = q_2 V_1 = q_1 V_2 = k\frac{q_1 q_2}{r_{12}}$

In Fig. 25-39, point *P* is at the center of the rectangle. With *V* = 0 at infinity, what is the net electric potential in terms of *q/d* at *P* due to the six charged particles?

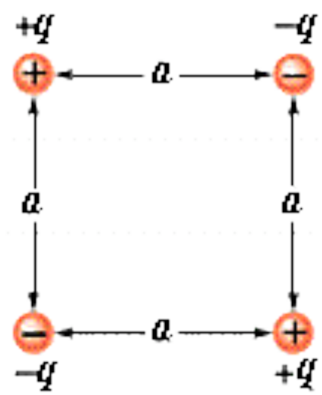


The figure shows conducting plates with area $A=1m^2$, and the potential on each plate. Assume you are far from the edges of the plates.

- What is the electric field between the plates in each case?
- What (and where) is the charge density on the plates in case (1)?
- What happens to an electron released midway between the plates in case (1)?

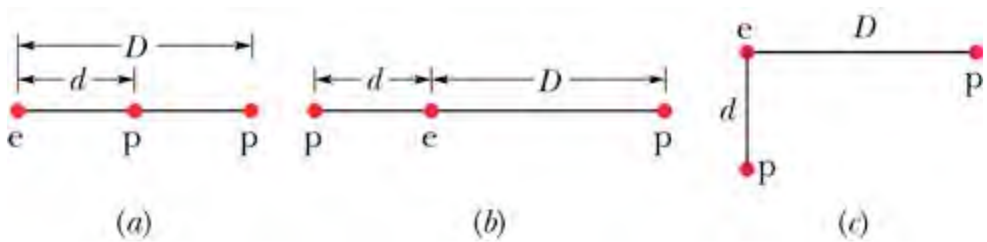


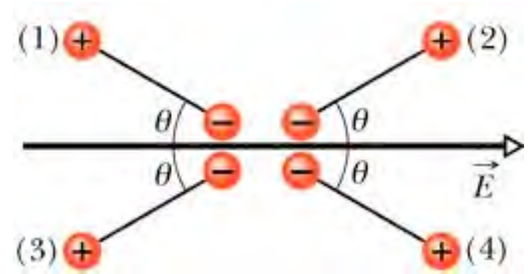
Derive an expression in terms of q^2/a for the work required to set up the four-charge configuration of Fig. 25-50, assuming the charges are initially infinitely far apart.



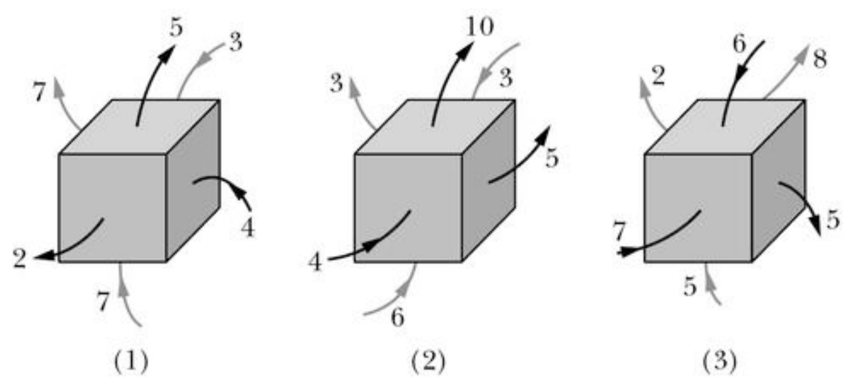
The electric potential at points in an *xy* plane is given by $V = (2.0\ \mathrm{V/m^2})x^2 - (4.0\ \mathrm{V/m^2})y^2$. What are the magnitude and direction of the electric field at point (3.0 m, 3.0 m)?

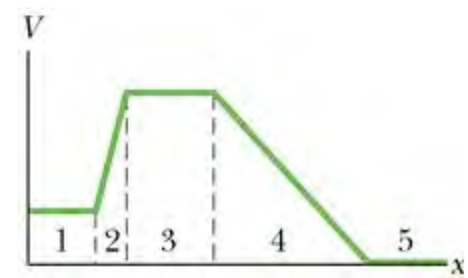
- Questions: from checkpoints and questions in the textbook!





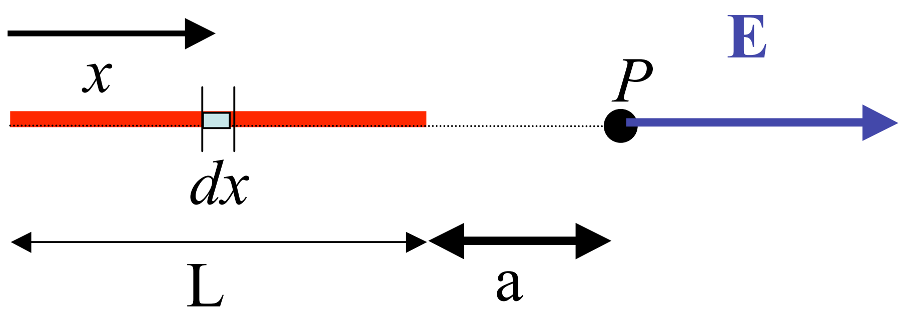
U = $-5U_0$, $-7U_{0,}$ $+3U_{0,}$ $+5U_0$





Problem

- Calculate electric field at point P.

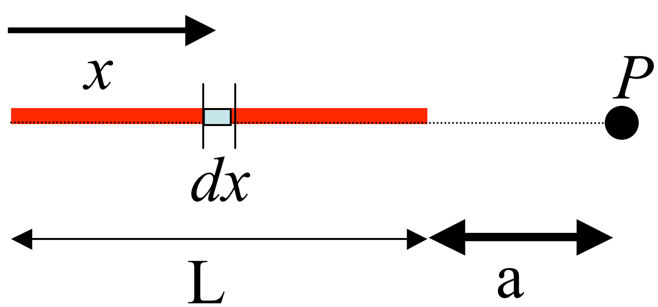


- Field very far away?

Potential of Continuous Charge Distribution

- Uniformly charged rod
- Total charge Q
- Length L
- What is V at position P shown?

$$\lambda = Q / L \qquad dq = \lambda\, dx$$

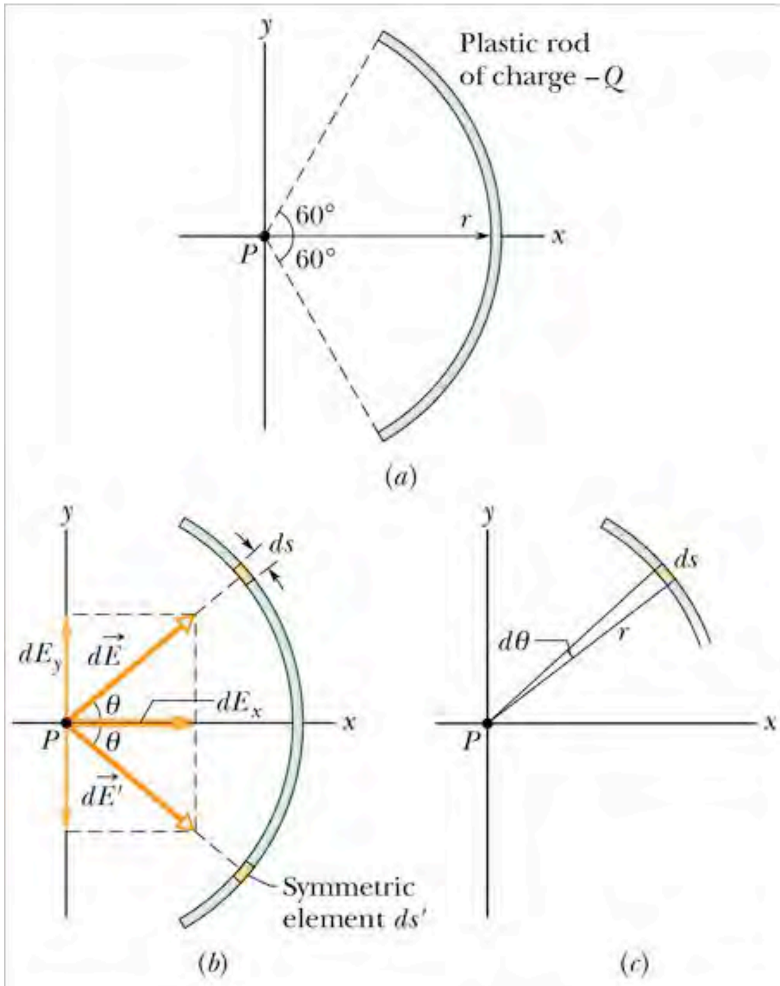


$$V = \int \frac{kdq}{r} = \int_0^L \frac{k\lambda dx}{(L+a-x)}$$

$$= k\lambda\left[-\ln(L+a-x)\right]_0^L$$

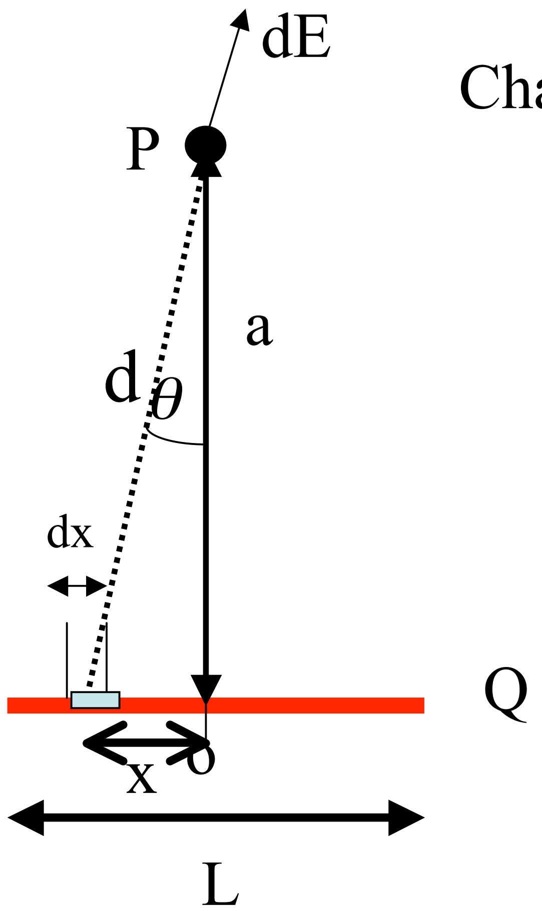
$$\boxed{V = k\lambda \ln\left[\frac{L+a}{a}\right]}$$

Problem



Field at center of arc?

Line Of Charge: Field on bisector



Distance $d = \sqrt{a^2 + x^2}$

Charge per unit length $\lambda = \dfrac{q}{L}$

$$dE = \frac{k(dq)}{d^2}$$

$$dE_y = dE\cos\theta = \frac{k(\lambda\, dx)a}{(a^2 + x^2)^{3/2}}$$

$$\cos\theta = \frac{a}{(a^2 + x^2)^{1/2}}$$

Line Of Charge: Field on bisector

$$E_y = k\lambda\, a \int_{-L/2}^{L/2} \frac{dx}{(a^2 + x^2)^{3/2}} = k\lambda\, a \left[\frac{x}{a^2 \sqrt{x^2 + a^2}} \right]_{-L/2}^{L/2}$$

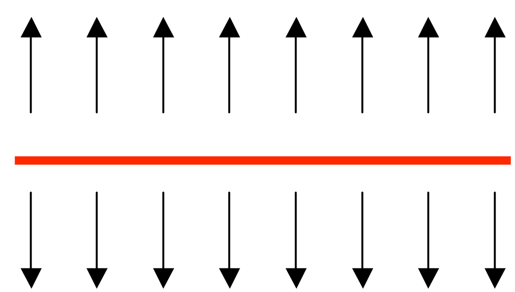
$$= \frac{2k\lambda L}{a\sqrt{4a^2 + L^2}}$$

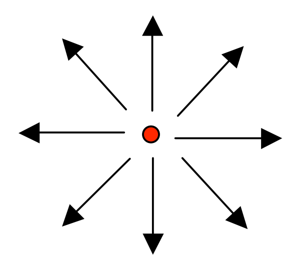
What is E very far away from the line *(L<<a)*?

$E_y \sim 2k\lambda L/a(2a) = k\lambda L/a^2 = kq/a^2$

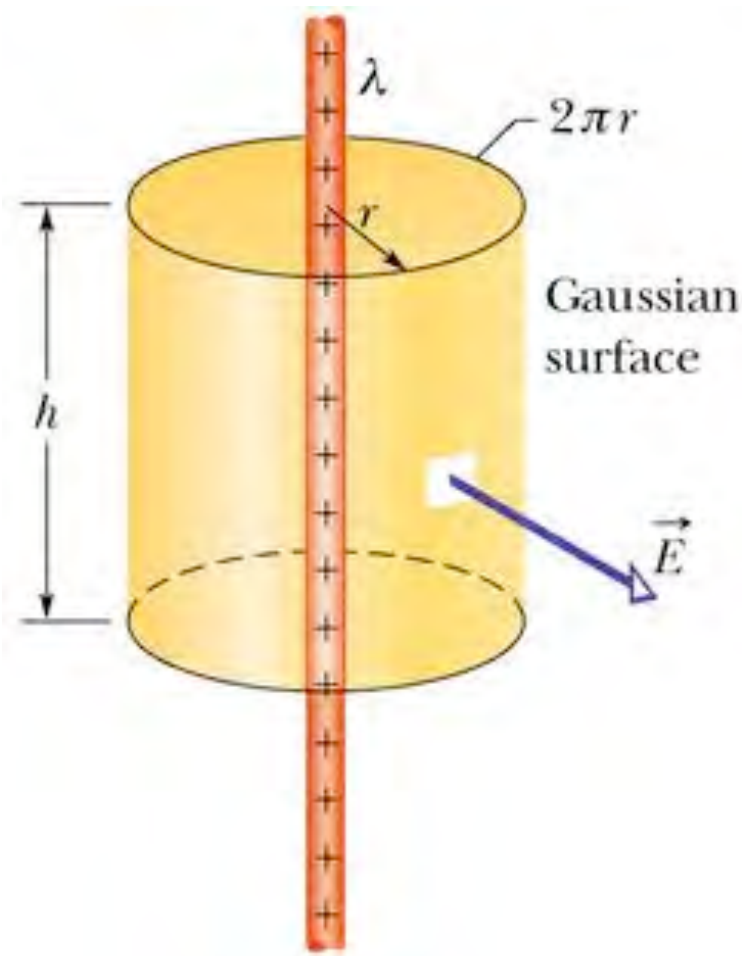
What is E if the line is infinitely long *(L >> a)*?

$$E_y = \frac{2k\lambda L}{a\sqrt{L^2}} = \frac{2k\lambda}{a}$$



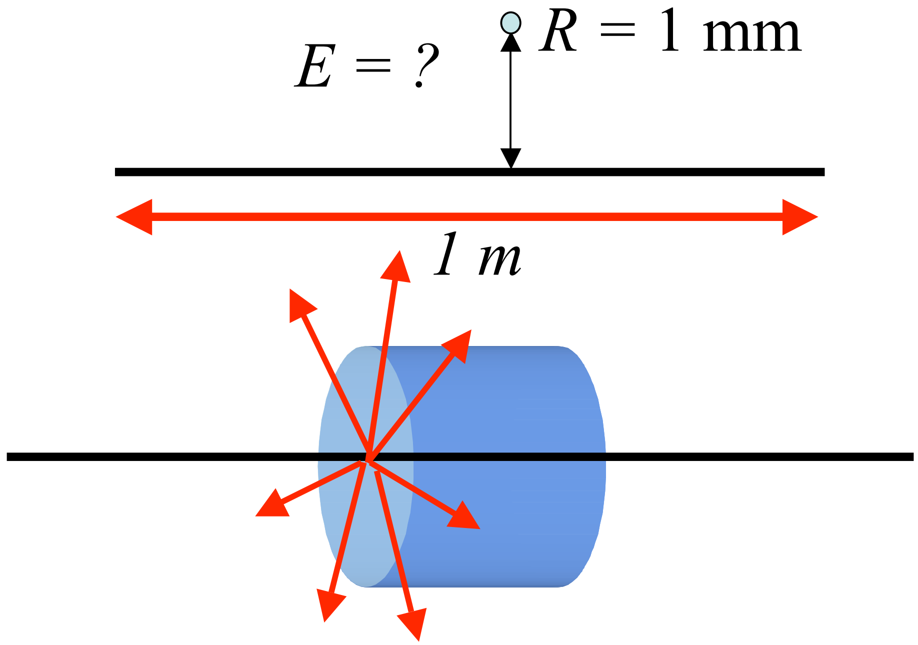


Problem: Gauss' Law to Find E



Gauss' Law: Cylindrical Symmetry

- Approximate as infinitely long line — E radiates outwards.
- Choose cylindrical surface of radius R, length L co-axial with line of charge.

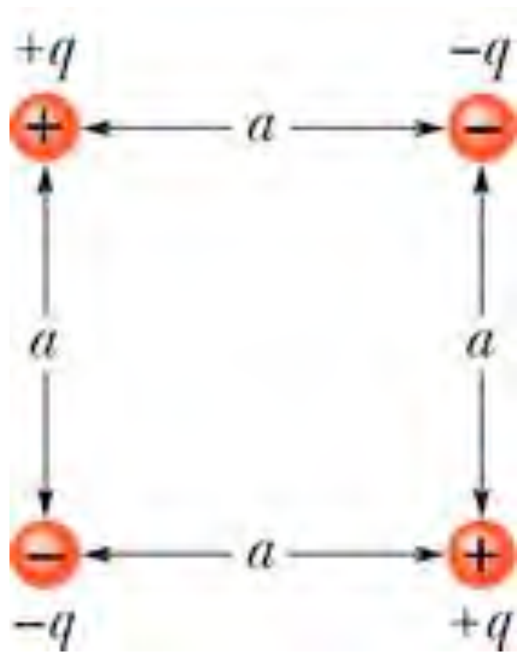


$$\Phi = |E|A = |E|2\pi RL$$

$$\Phi = \frac{q}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$$

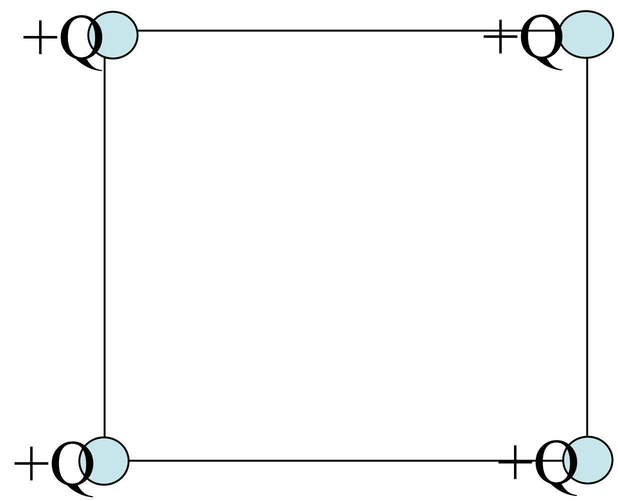
$$|E| = \frac{\lambda L}{2\pi\varepsilon_0 RL} = \frac{\lambda}{2\pi\varepsilon_0 R} = 2k\frac{\lambda}{R}$$

Potential Energy of a System of Charges



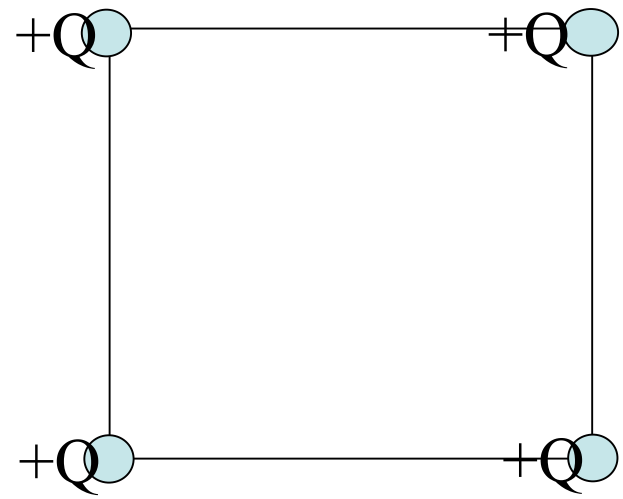
Potential Energy of A System of Charges

- 4 point charges (each $+Q$) are connected by strings, forming a square of side L
- If all four strings suddenly snap, what is the kinetic energy of each charge when they are very far apart?
- Use conservation of energy:
  - Final kinetic energy of all four charges = initial potential energy stored = energy required to assemble the system of charges



Do this from scratch! Don't memorize the formula in the book! We will change the numbers!!!

Potential Energy of A System of Charges: Solution



- No energy needed to bring in first charge: U_1=0
- Energy needed to bring in 2nd charge: $U_2 = QV_1 = \dfrac{kQ^2}{L}$
- Energy needed to bring in 3rd charge =

$$U_3 = QV = Q(V_1 + V_2) = \frac{kQ^2}{L} + \frac{kQ^2}{\sqrt{2}L}$$

- Energy needed to bring in 4th charge =

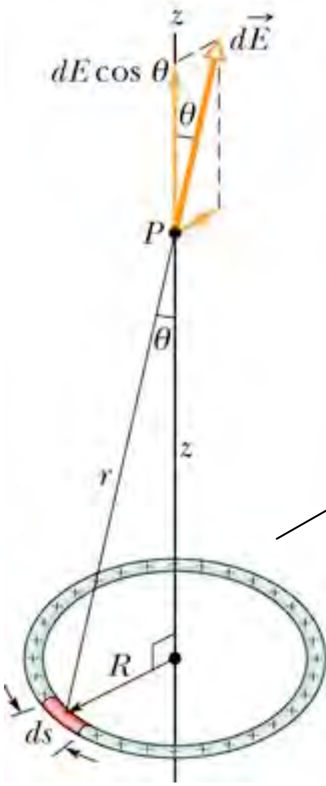
$$U_4 = QV = Q(V_1 + V_2 + V_3) = \frac{2kQ^2}{L} + \frac{kQ^2}{\sqrt{2}L}$$

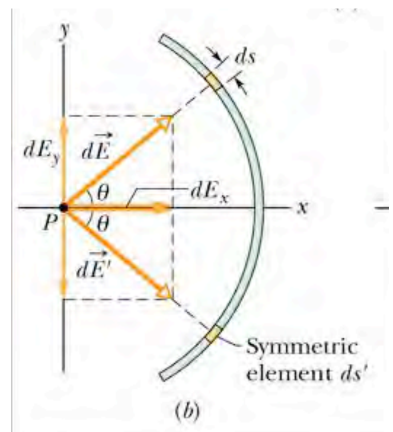
Total potential energy is sum of all the individual terms shown on left hand side = $\dfrac{kQ^2}{L}\left(4 + \sqrt{2}\right)$

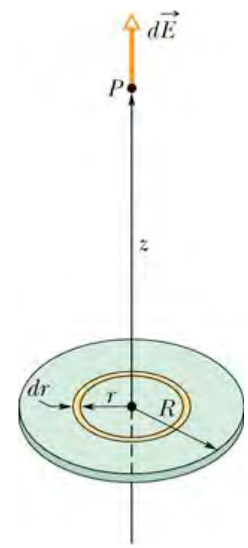
So, final kinetic energy of each charge = $\dfrac{kQ^2}{4L}\left(4 + \sqrt{2}\right)$

Electric fields: Example

Calculate the magnitude and direction of the electric field produced by a ring of charge *Q* and radius *R*, at a distance *z* on its





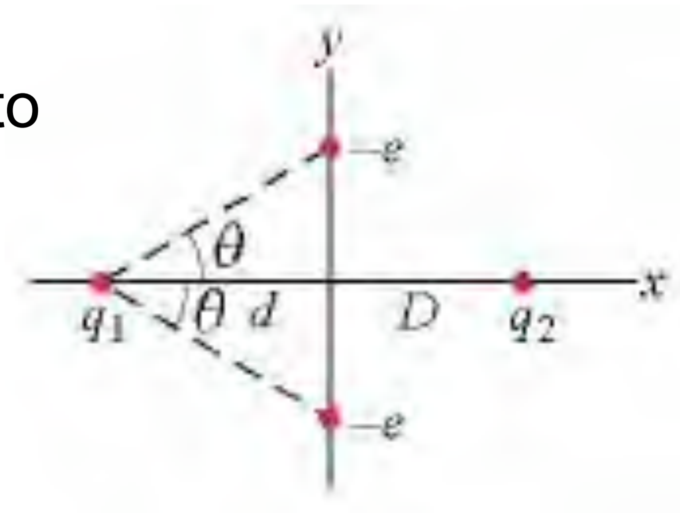


Sample Problem

Figure 22N-14 shows an arrangement of four charged particles, with angle θ = 34° and distance d = 2.20 cm. The two negatively charged particles on the y axis are electrons that are fixed in place; the particle at the right has a charge $q_2 = +5e$

(a)Find distance D such that the net force on the particle at the left, due to the three other particles, is zero.

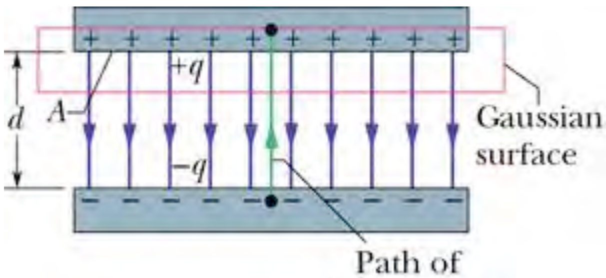
(b) If the two electrons were moved further from the x axis, would the required value of D be greater than, less than, or the same as in part (a)?

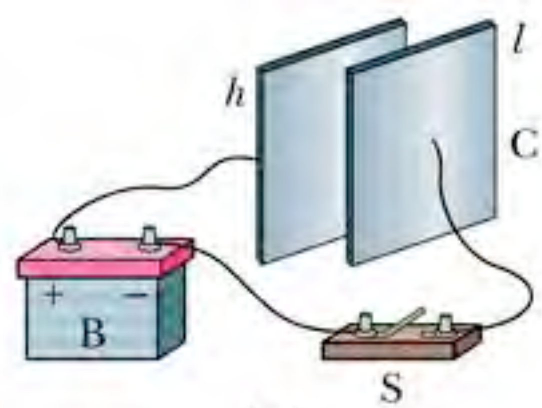


Exam 2

- (Ch 26) **Capacitors**: capacitance and capacitors; caps in parallel and in series, dielectrics; energy, field and potential in capacitors.
- (Ch 27) **Current and Resistance**: current, current density and drift velocity; resistance and resistivity; Ohm’s law.
- (Ch 28) **Circuits**: emf devices, loop and junction rules; resistances in series and parallel; DC single and multiloop circuits, power; RC circuits.

Capacitors



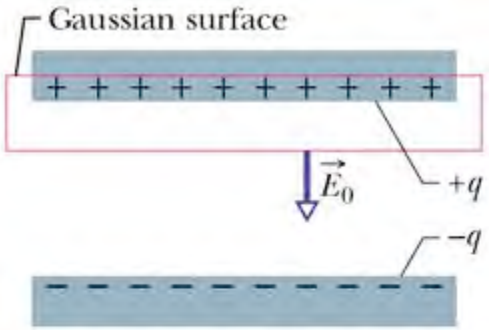


(a)

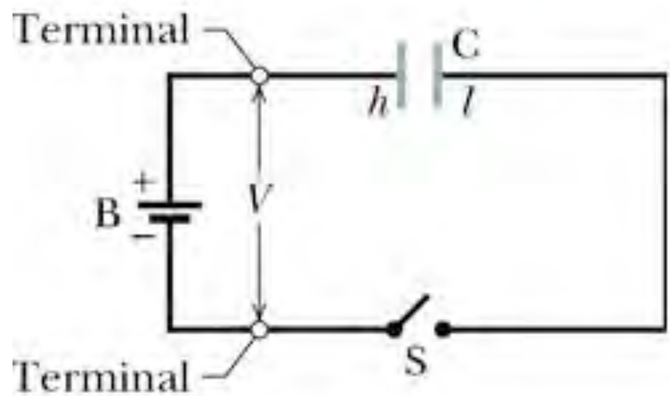
$$E = \sigma/\varepsilon_0 = q/A\varepsilon_0$$

$$E = V\,d$$

$$q = C\,V$$

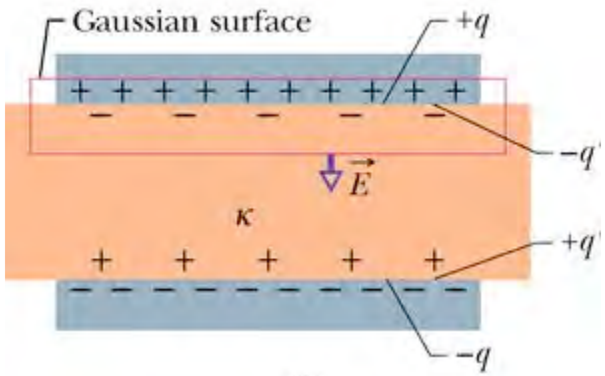


(a)



(b)

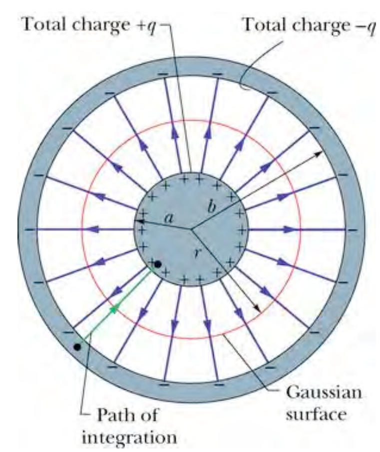
$$C = \varepsilon_0 A/d$$



(b)

$$C = \kappa\, \varepsilon_0 A/d$$

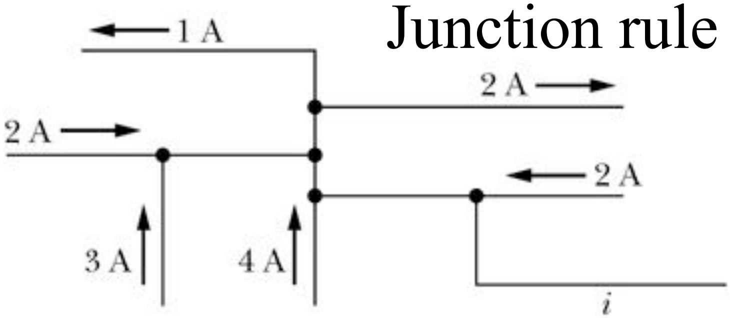
$$C=\varepsilon_0 ab/(b-a)$$



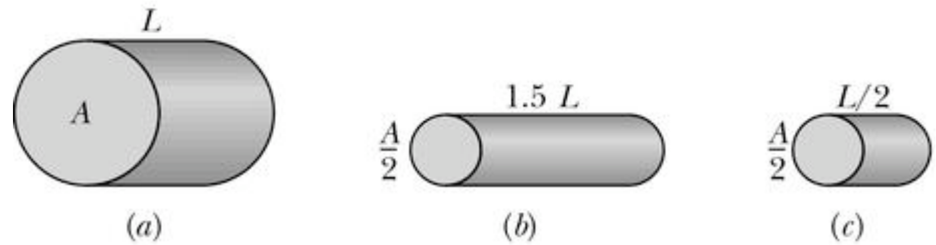
Current and resistance

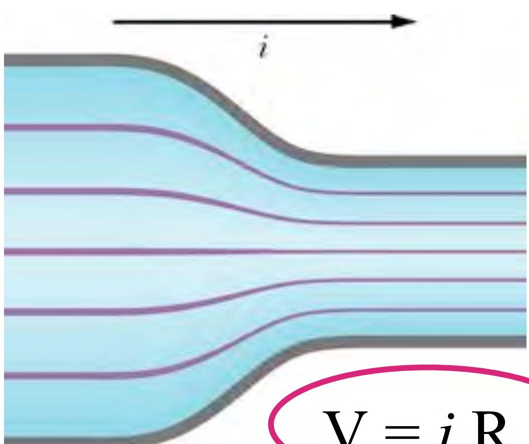
$i = dq/dt$

Junction rule



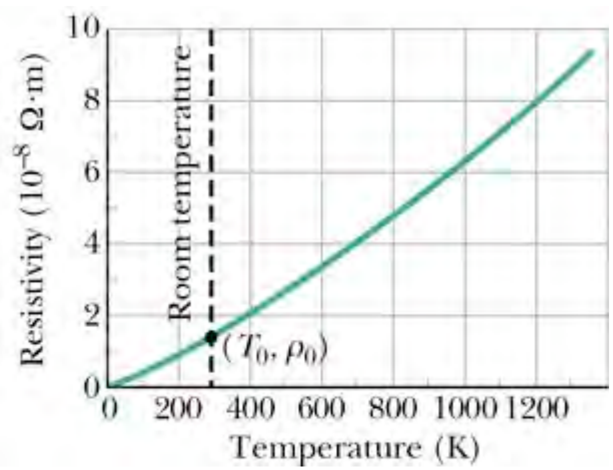
$R = \rho L/A$





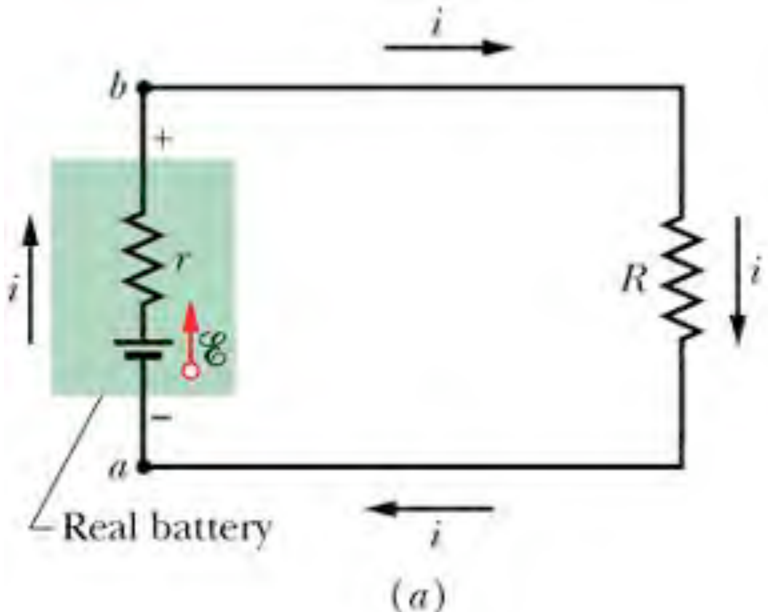
$V = i\,R$

$\mathbf{E} = \mathbf{J}\,\rho$

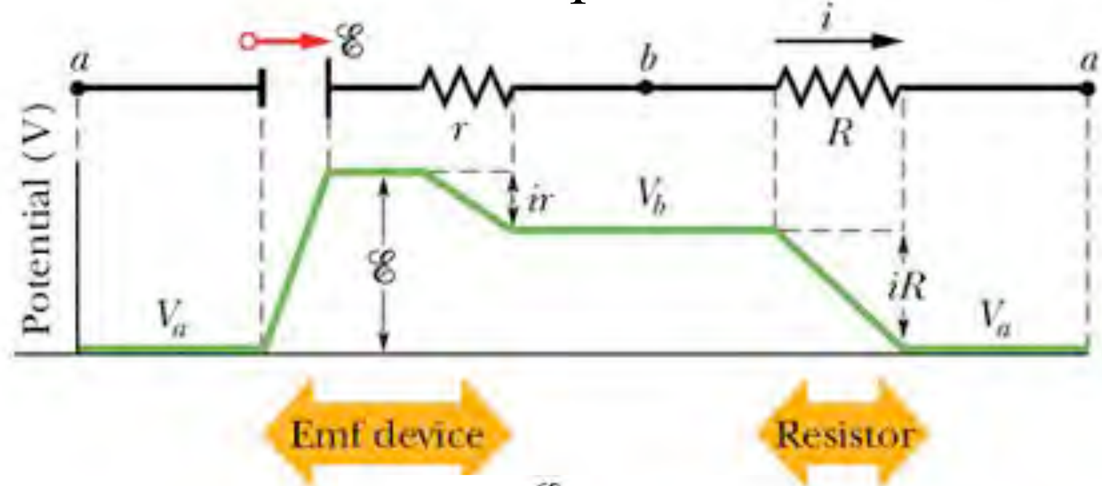


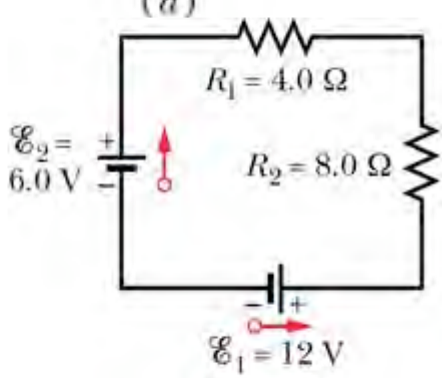
$\rho = \rho_0(1+\alpha(T-T_0))$

DC Circuits



Loop rule

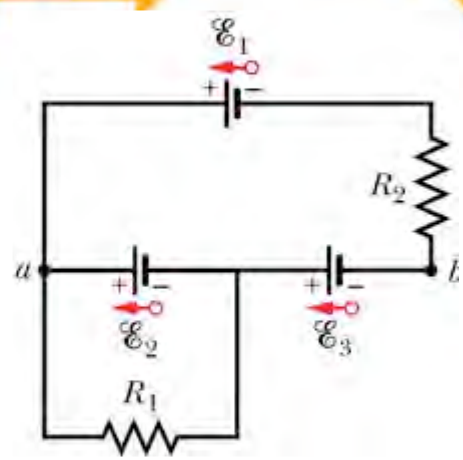




Single loop

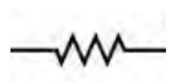
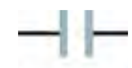
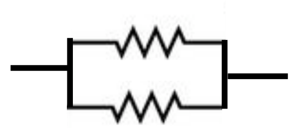
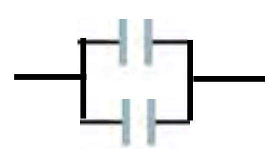
$$V = iR$$

$$P = iV$$

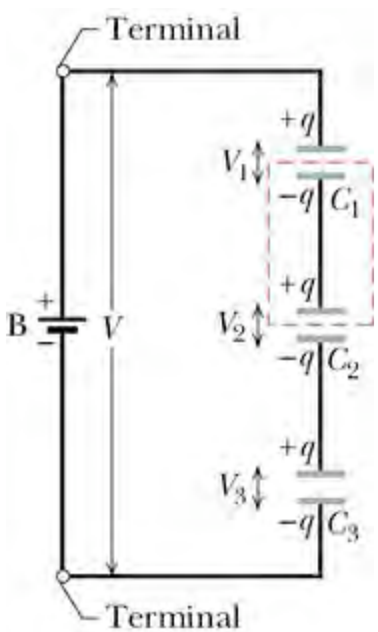


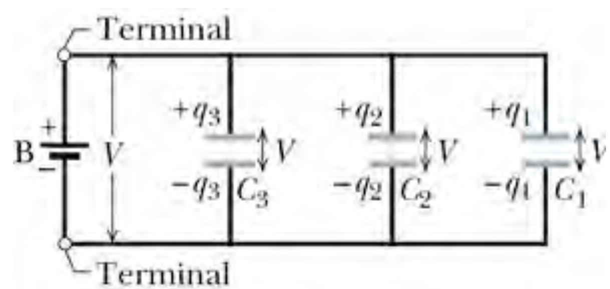
Multiloop

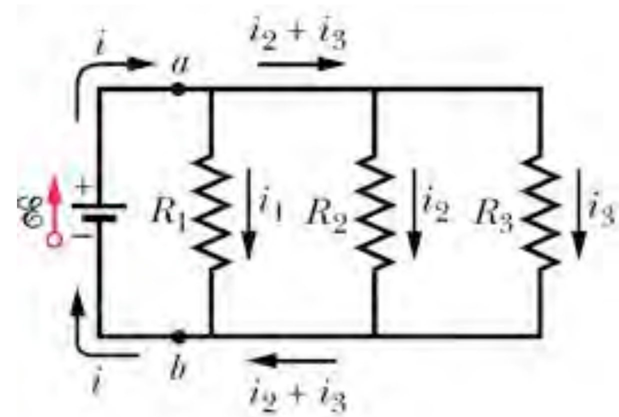
Resistors and Capacitors

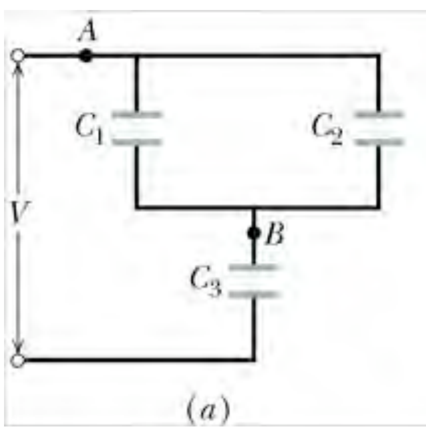
	Resistors	**Capacitors**
		
Key formula:	$V=iR$	$Q=CV$
In series:	same current	same charge
	$R_{eq}=\sum R_j$	$1/C_{eq}=\sum 1/C_j$
		
In parallel:	same voltage	same voltage
	$1/R_{eq}=\sum 1/R_j$	$C_{eq}=\sum C_j$
		

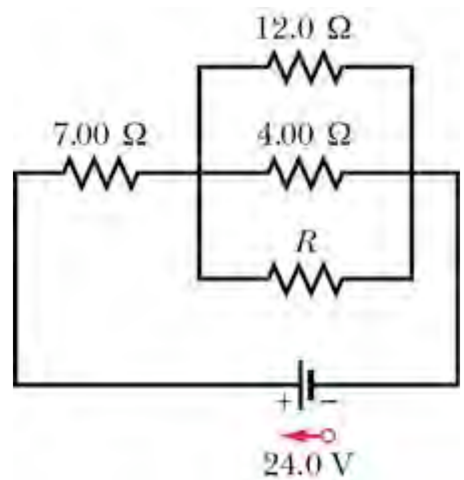
Capacitors and Resistors in Series and in Parallel

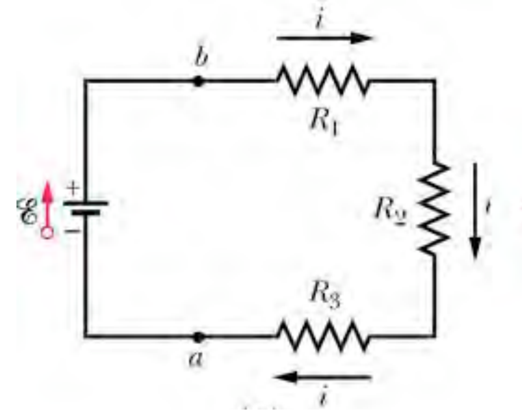












- What's the equivalent resistance (capacitance)?
- What's the current (charge) in each resistor (capacitor)?
- What's the potential across each resistor (capacitor)?
- What's the current (charge) delivered by the battery?

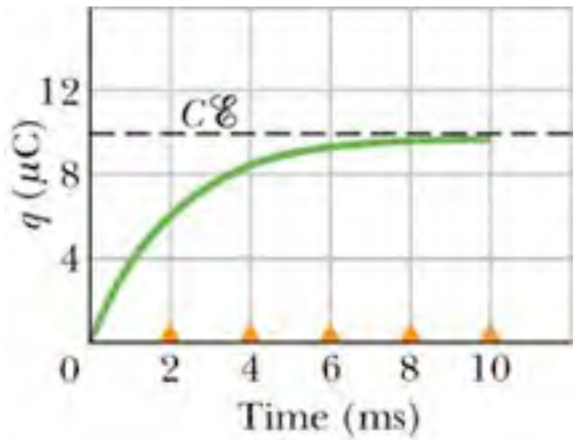
RC Circuits

Time constant: RC

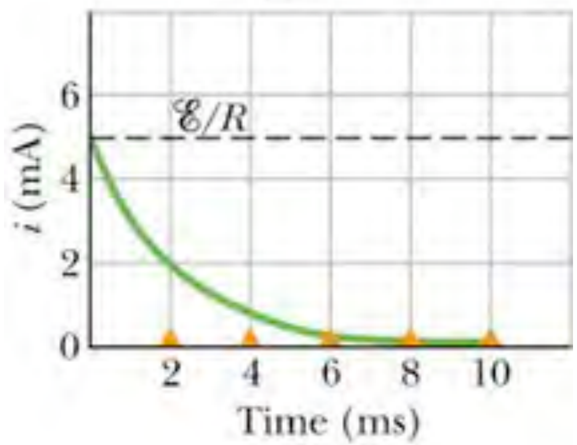
Charging: $q(t) = CE\left(1 - e^{-t/RC}\right)$

Discharging: $q(t) = q_0 e^{-t/RC}$

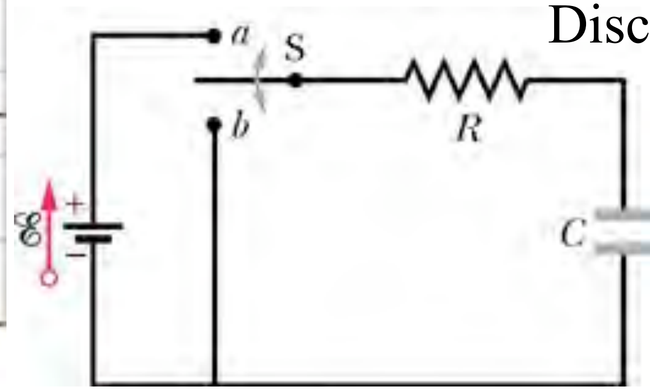
$i(t) = dq/dt$

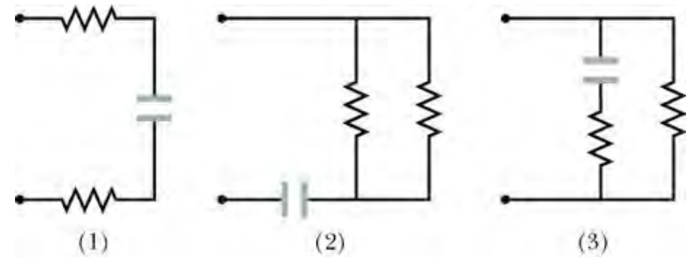


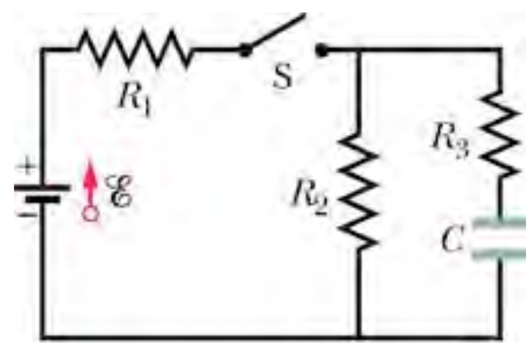
(a)



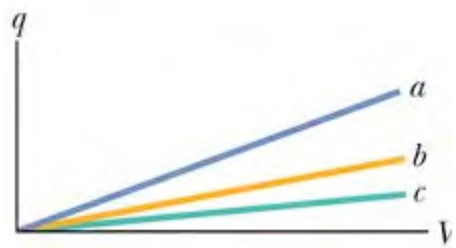
(b)

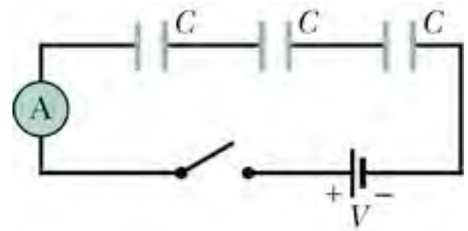


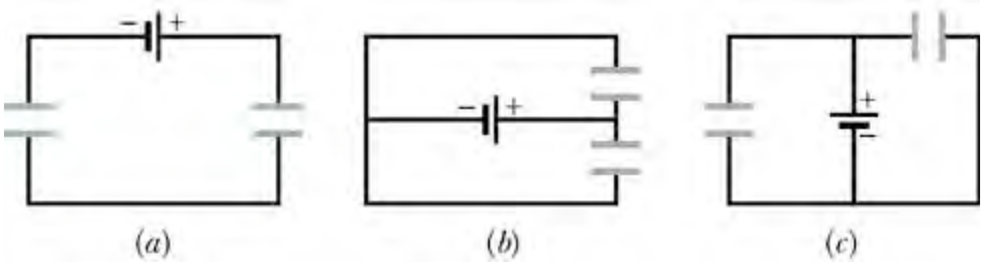


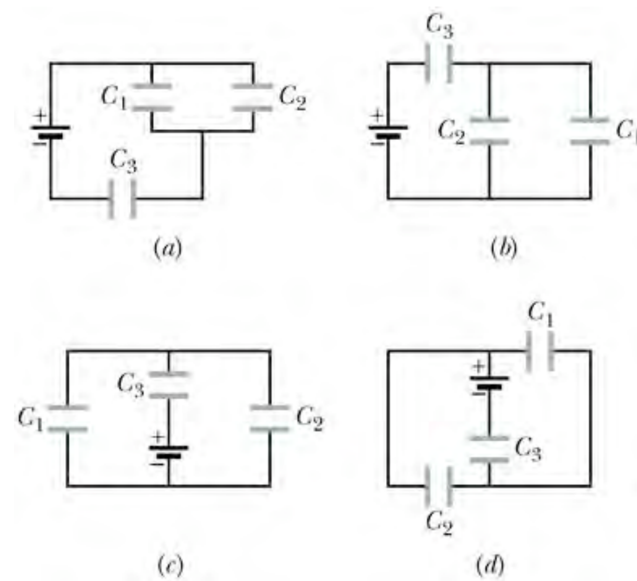


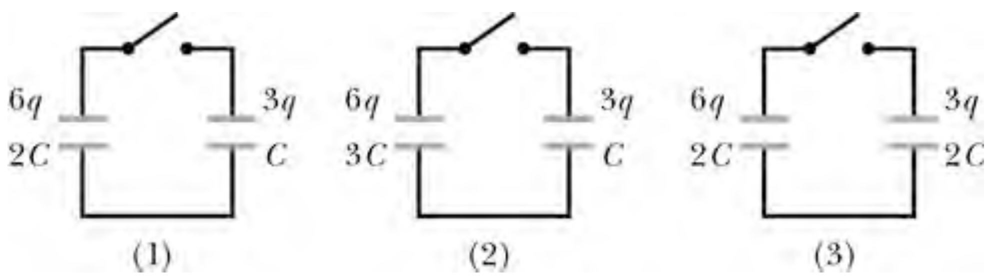
Capacitors: Checkpoints, Questions

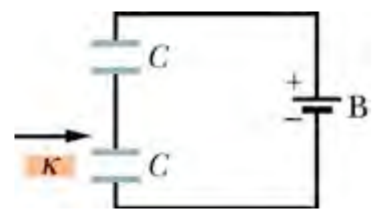






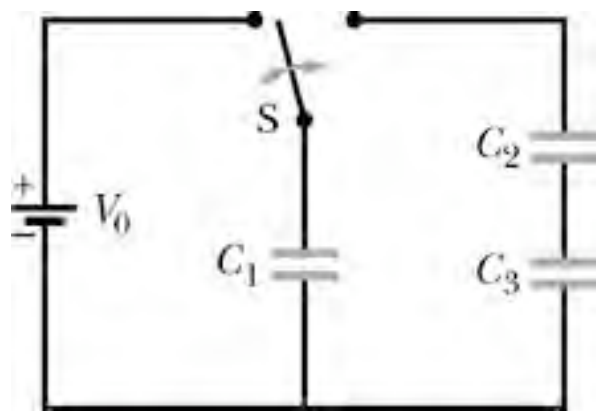






Problem 25-21

When switch S is thrown to the left, the plates of capacitor 1 acquire a potential V_0. Capacitors 2 and 3 are initially uncharged. The switch is now thrown to the right. What are the final charges q_1, q_2, and q_3 on the capacitors?



21. The charges on capacitors 2 and 3 are the same, so these capacitors may be replaced by an equivalent capacitance determined from

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{C_2 + C_3}{C_2 C_3}.$$

Thus, $C_{\text{eq}} = C_2C_3/(C_2 + C_3)$. The charge on the equivalent capacitor is the same as the charge on either of the two capacitors in the combination and the potential difference across the equivalent capacitor is given by q_2/C_{eq}. The potential difference across capacitor 1 is q_1/C_1, where q_1 is the charge on this capacitor. The potential difference across the combination of capacitors 2 and 3 must be the same as the potential difference across capacitor 1, so $q_1/C_1 = q_2/C_{\text{eq}}$. Now some of the charge originally on capacitor 1 flows to the combination of 2 and 3. If q_0 is the original charge, conservation of charge yields $q_1 + q_2 = q_0 = C_1 V_0$, where V_0 is the original potential difference across capacitor 1.

(a) Solving the two equations

$$\frac{q_1}{C_1}=\frac{q_2}{C_{\mathrm{eq}}} \qquad \text{and} \qquad q_1+q_2=C_1V_0$$

for q_1 and q_2, we obtain

$$q_1=\frac{C_1^2V_0}{C_{\mathrm{eq}}+C_1}=\frac{C_1^2V_0}{\dfrac{C_2C_3}{C_2+C_3}+C_1}=\frac{C_1^2\left(C_2+C_3\right)V_0}{C_1C_2+C_1C_3+C_2C_3}.$$

With V_0 = 12.0 V, C_1= 4.00 μF, C_2= 6.00 μF and C_3=3.00 μF, we find C_{eq} = 2.00 μF and q_1 = 32.0 μC.

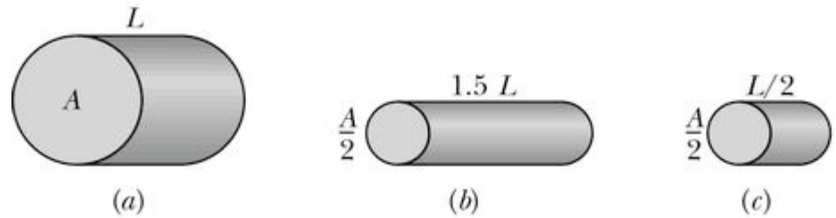
(b) The charge on capacitors 2 is

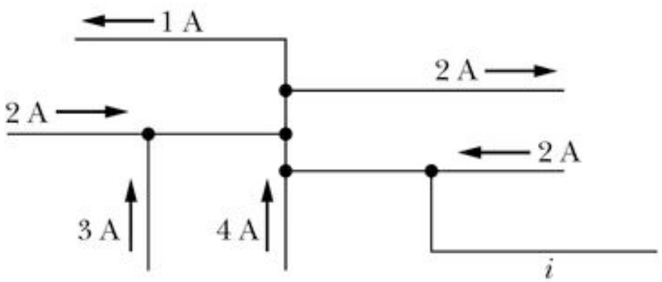
$$q_2=C_1V_0-q_1=(4.00\mu\mathrm{F})(12.0\mathrm{V})-32.0\mu\mathrm{F}=16.0\mu\mathrm{F}$$

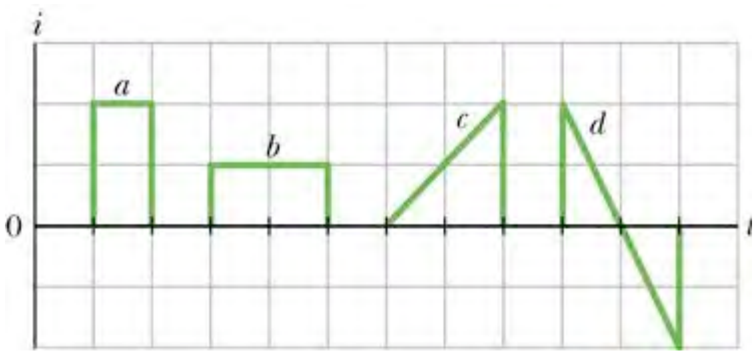
(c) The charge on capacitor 3 is the same as that on capacitor 2:

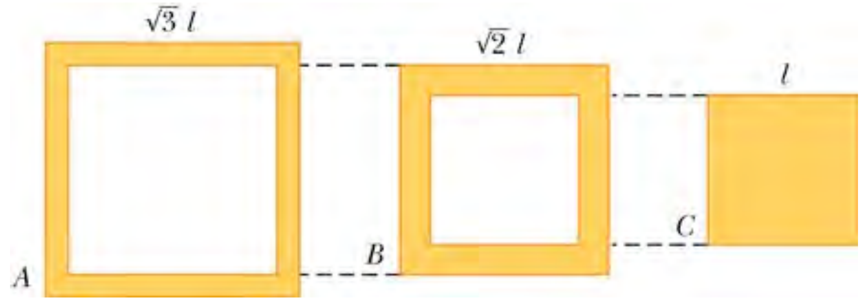
$$q_3=C_1V_0-q_1=(4.00\mu\mathrm{F})(12.0\mathrm{V})-32.0\mu\mathrm{F}=16.0\mu\mathrm{F}$$

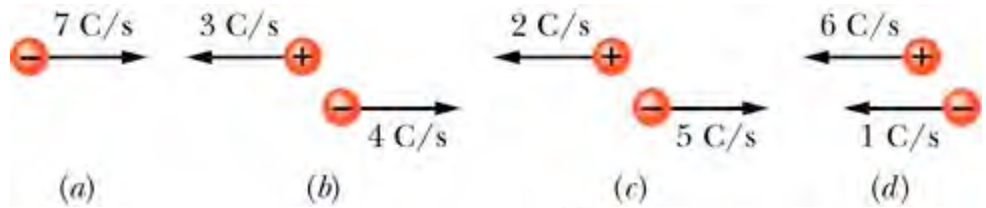
Current and Resistance: Checkpoints, Questions

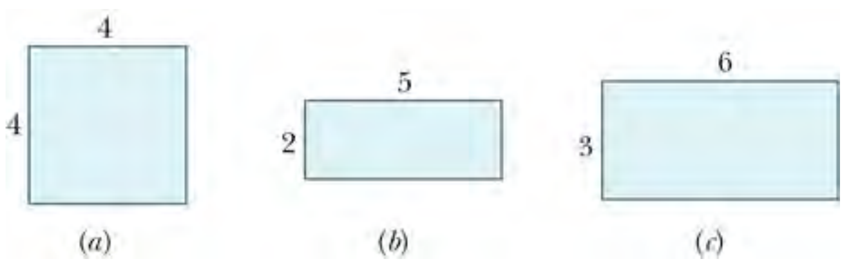


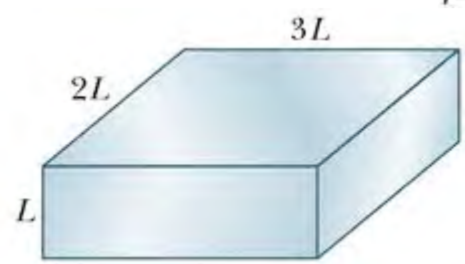


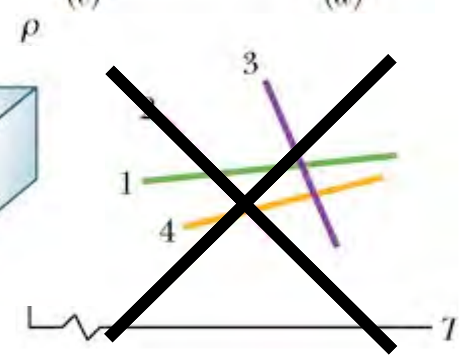












Problem 26-56

A cylindrical resistor of radius 5.0mm and length 2.0 cm is made of a material that has a resistivity of $3.5 \times 10^{-5}\ \Omega m$. What are the (a) current density and (b) the potential difference when the energy dissipation rate in the resistor is 1.0W?

56. (a) Since $P = i^2 R = J^2 A^2 R$, the current density is

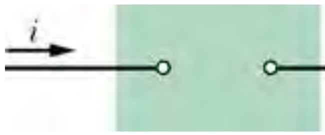
$$J = \frac{1}{A}\sqrt{\frac{P}{R}} = \frac{1}{A}\sqrt{\frac{P}{\rho L / A}} = \sqrt{\frac{P}{\rho L A}}$$

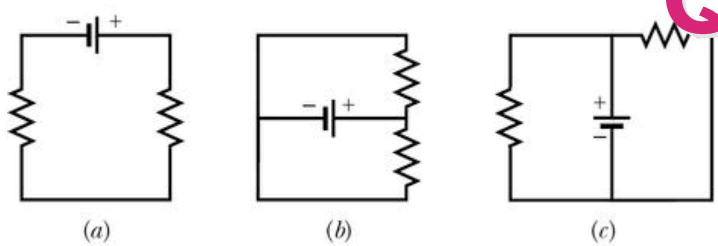
$$= \sqrt{\frac{1.0\,\mathrm{W}}{\pi\left(3.5\times10^{-5}\,\Omega\cdot\mathrm{m}\right)\left(2.0\times10^{-2}\,\mathrm{m}\right)\left(5.0\times10^{-3}\,\mathrm{m}\right)^2}} = 1.3\times10^{5}\,\mathrm{A/m^2}.$$

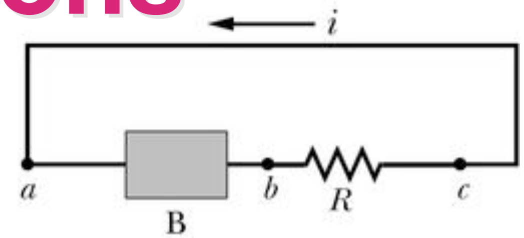
(b) From $P = iV = JAV$ we get

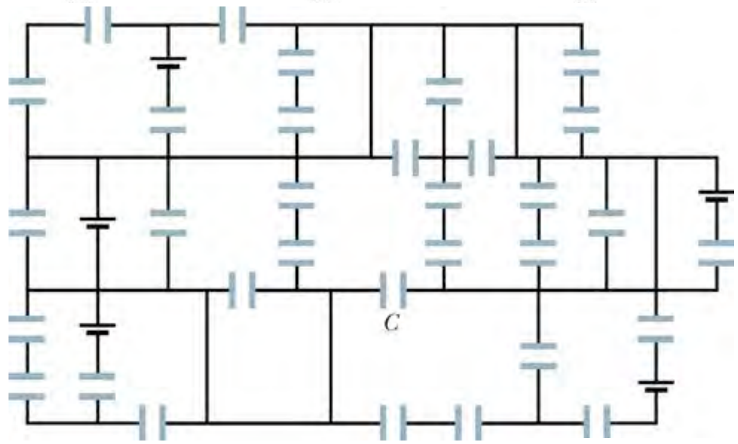
$$V = \frac{P}{AJ} = \frac{P}{\pi r^2 J} = \frac{1.0\,\mathrm{W}}{\pi\left(5.0\times10^{-3}\,\mathrm{m}\right)^2\left(1.3\times10^{5}\,\mathrm{A/m^2}\right)} = 9.4\times10^{-2}\,\mathrm{V}.$$

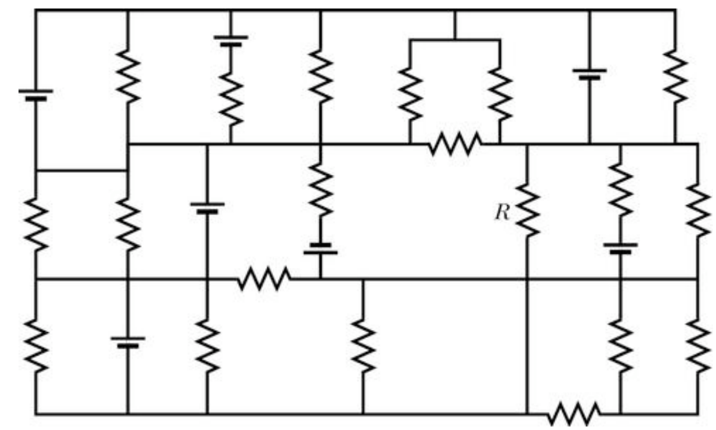
Circuits: Checkpoints, Questions

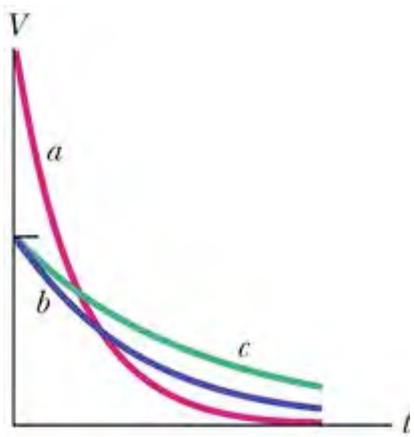


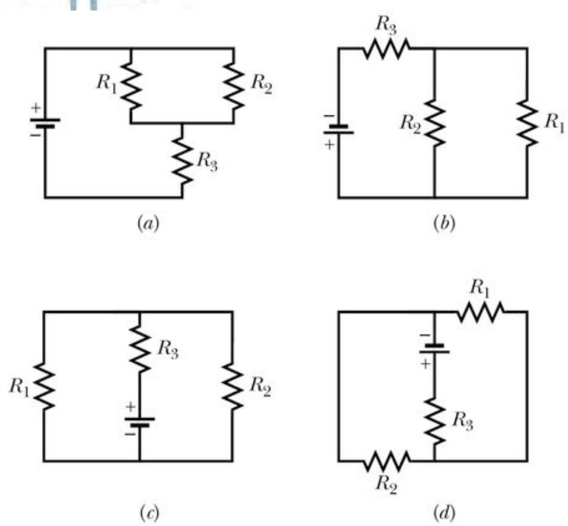


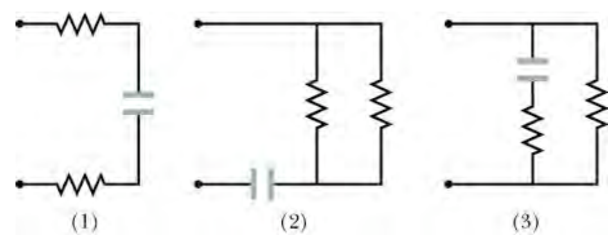










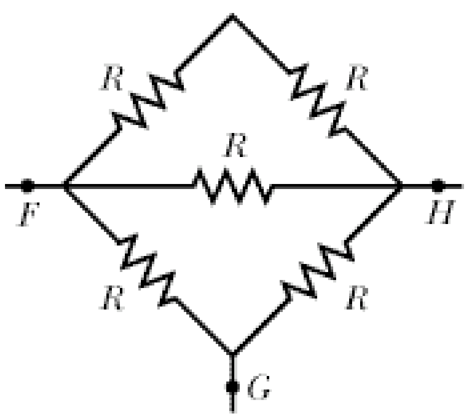


1. HRW7 27.P.018. [406649]
Figure 27-33 shows five 5.00 resistors.
(Hint: For each pair of points, imagine that a battery is connected across the pair.)

Fig. 27-33

(a) Find the equivalent resistance between points F and H.

(b) Find the equivalent resistance between points F and G.



18. (a) R_{eq} (FH) = (10.0 Ω)(10.0 Ω)(5.00 Ω)/[(10.0 Ω)(10.0 Ω) + 2(10.0 Ω)(5.00 Ω)] = 2.50 Ω.

(b) R_{eq} (FG) = (5.00 Ω) R/(R + 5.00 Ω), where

$$R = 5.00\ \Omega + (5.00\ \Omega)(10.0\ \Omega)/(5.00\ \Omega + 10.0\ \Omega) = 8.33\ \Omega.$$

So R_{eq} (FG) = (5.00 Ω)(8.33 Ω)/(5.00 Ω + 8.33 Ω) = 3.13 Ω.

5. HRW7 27.P.046. [406629]

In an RC series circuit, $\mathcal{E} = 17.0$ V, R = 1.50 MΩ, and C = 1.80 μF.

(a) Calculate the time constant.

(b) Find the maximum charge that will appear on the capacitor during charging.

(c) How long does it take for the charge to build up to 10.0 μC?

46. (a) $\tau = RC = (1.40 \times 10^6\ \Omega)(1.80 \times 10^{-6}\ \text{F}) = 2.52\ \text{s}.$

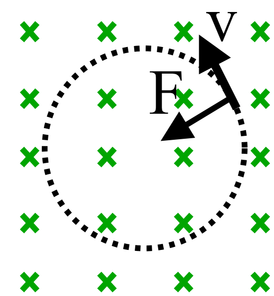
(b) $q_o = \varepsilon C = (12.0\ \text{V})(1.80\ \mu\text{F}) = 21.6\ \mu\text{C}.$

(c) The time t satisfies $q = q_0(1 - e^{-t/RC})$, or

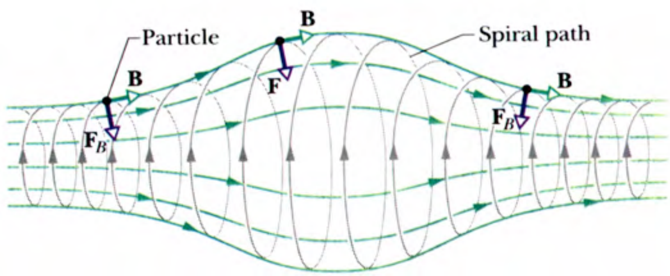
$$t = RC \ln\left(\frac{q_0}{q_0 - q}\right) = (2.52\,\text{s})\ln\left(\frac{21.6\,\mu\text{C}}{21.6\,\mu\text{C} - 16.0\,\mu\text{C}}\right) = 3.40\,\text{s}.$$

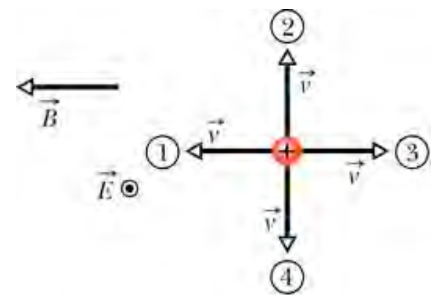
Magnetic Forces and Torques

$$\vec{F} = q\,\vec{v} \times \vec{B} + q\,\vec{E}$$

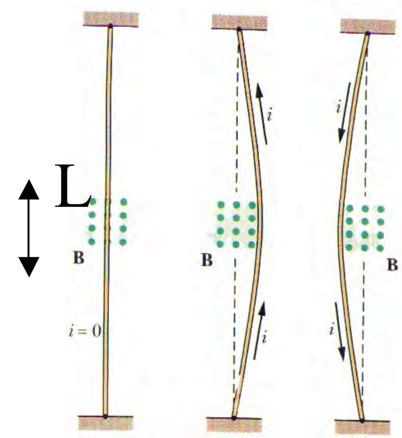


$$r = \frac{mv}{qB}$$

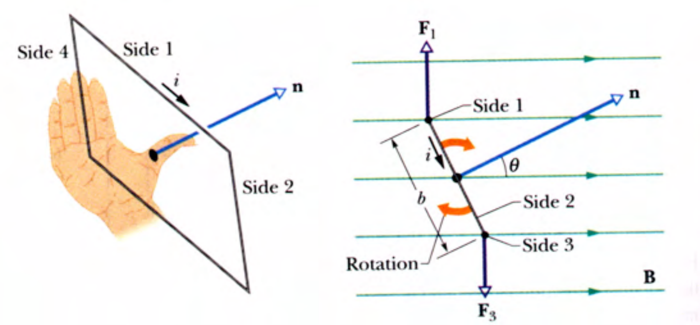




$$d\vec{F} = i\,d\vec{L} \times \vec{B}$$

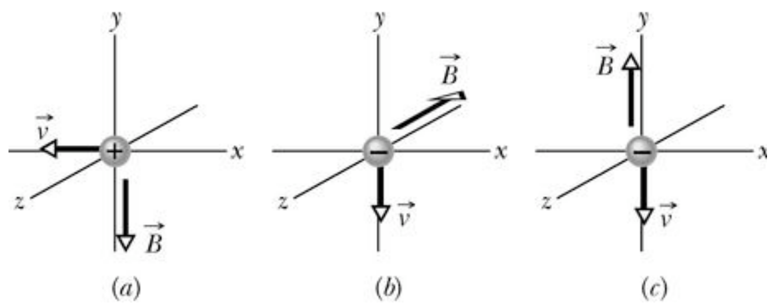


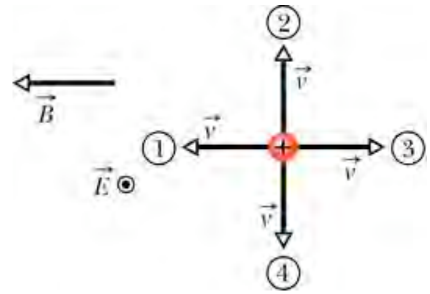
$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

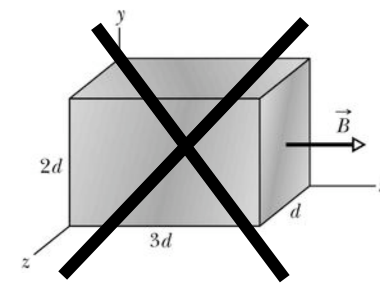


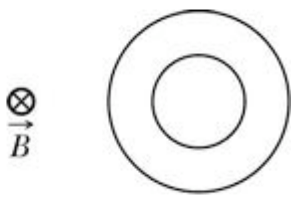
(b) (c)

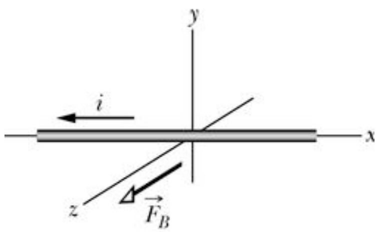
Ch 28: Checkpoints and Questions

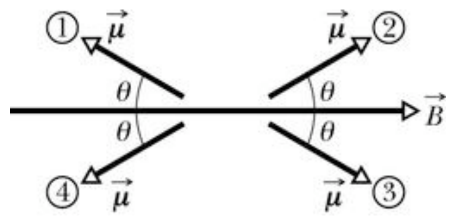


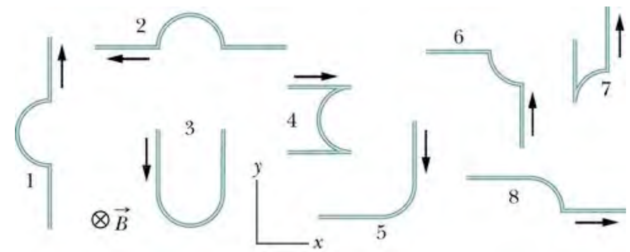


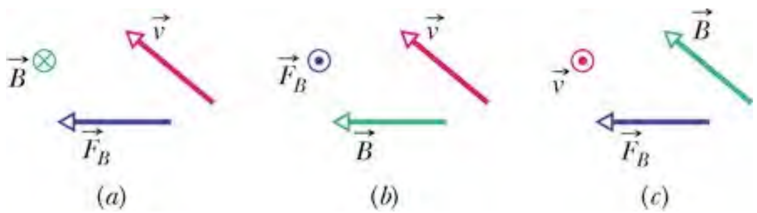


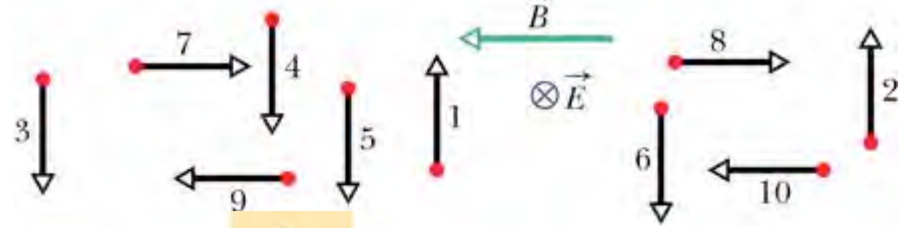


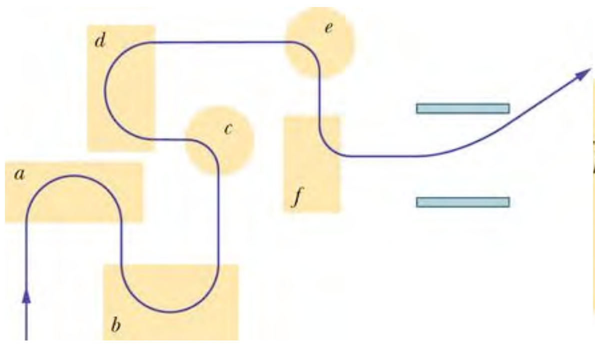


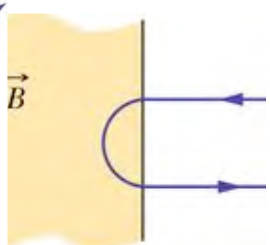


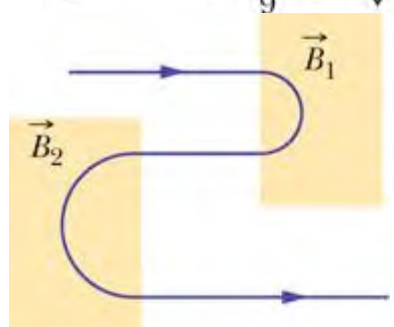


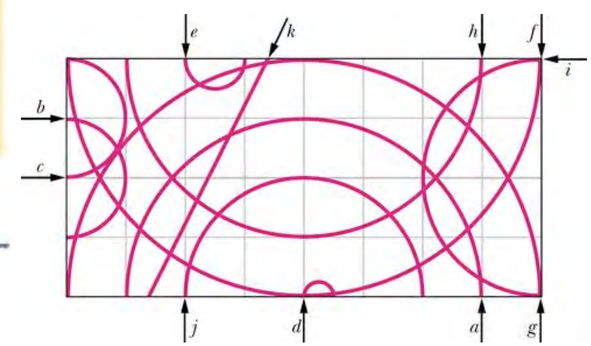












5. HRW7 28.P.024. [566302]

In the figure below, a charged particle moves into a region of uniform magnetic field , goes through half a circle, and then exits that region. The particle is either a proton or an electron (you must decide which). It spends 160 ns in the region.

(a) What is the magnitude of B?

(b) If the particle is sent back through the magnetic field (along the same initial path) but with 3.00 times its previous kinetic energy, how much time does it spend in the field during this trip?

24. We consider the point at which it enters the field-filled region, velocity vector pointing downward. The field points out of the page so that $\vec{v} \times \vec{B}$ points leftward, which indeed seems to be the direction it is "pushed"; therefore, $q > 0$ (it is a proton).

(a) Eq. 28-17 becomes $T = 2\pi m_p / e|\vec{B}|$, or

$$2\left(130\times10^{-9}\right)=\frac{2\pi\left(1.67\times10^{-27}\right)}{\left(1.60\times10^{-19}\right)|\vec{B}|}$$

which yields $\left|\vec{B}\right| = 0.252\,\mathrm{T}$.

(b) Doubling the kinetic energy implies multiplying the speed by $\sqrt{2}$. Since the period T does not depend on speed, then it remains the same (even though the radius increases by a factor of $\sqrt{2}$). Thus, $t = T/2 = 130$ ns, again.

- **Generating Magnetic Fields**

$\mu_0 = 4\pi \times 10^{-7} \frac{\mathrm{T} \cdot \mathrm{m}}{\mathrm{A}}$

Biot-Savart Law: $d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{s} \times \vec{r}}{r^3}$

Magnetic field of a long straight wire: $B = \frac{\mu_0}{4\pi} \frac{2i}{r}$

Magnetic field of a circular arc: $B = \frac{\mu_0}{4\pi} \frac{i}{r} \phi$

Force between parallel wires carrying currents: $F_{ab} = \frac{\mu_0 i_a i_b}{2\pi d} L$
(parallel currents attract, opposite repel)

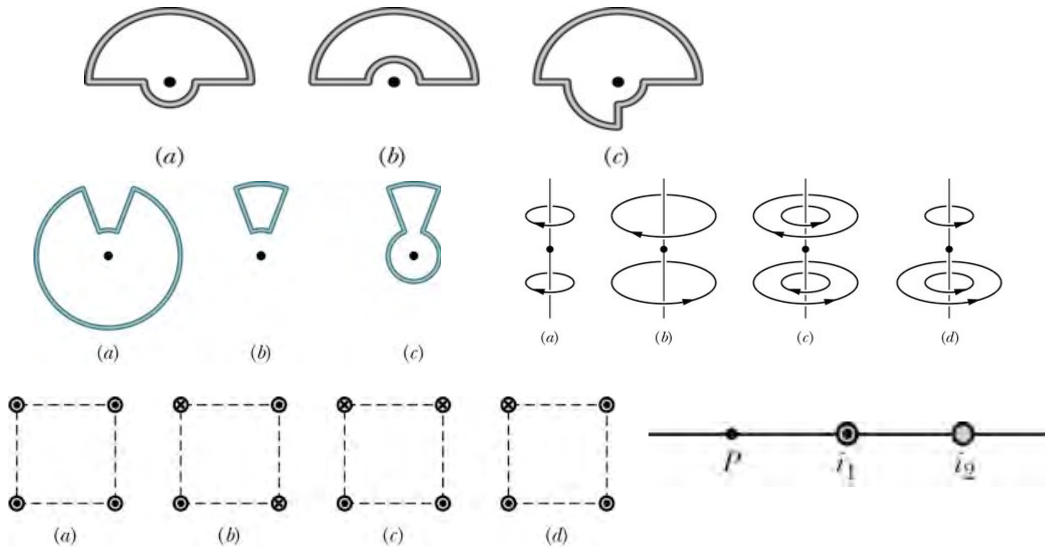
Ampere's law: $\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$

Magnetic field of a solenoid: $B = \mu_0 i n$

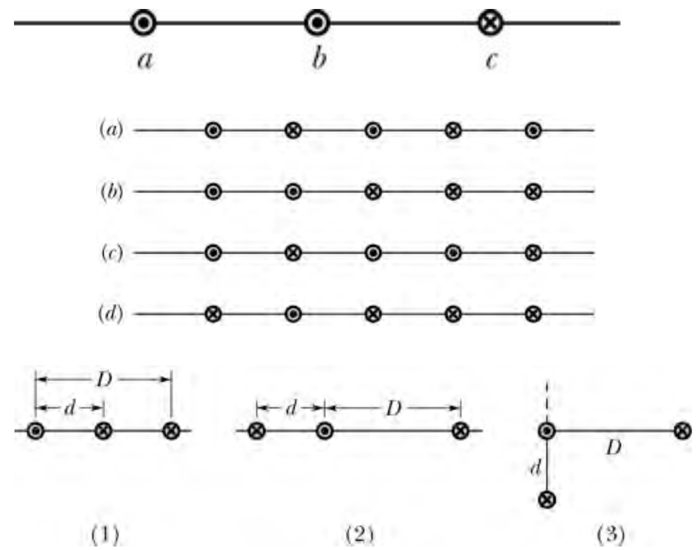
Magnetic field of a dipole (on axis): $\vec{B} = \frac{\mu_0 i}{2\pi} \frac{\vec{\mu}}{z^3}$

Checkpoints/Questions

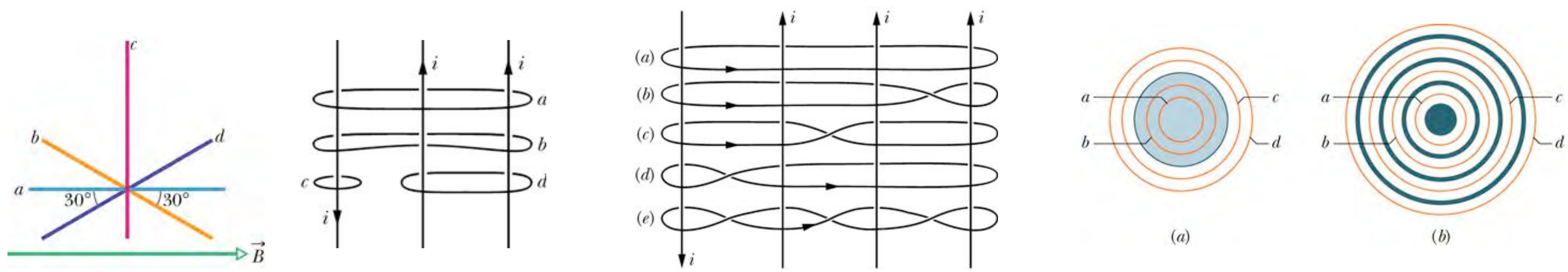
Magnetic field?



Force on each wire due to currents in the other wires?

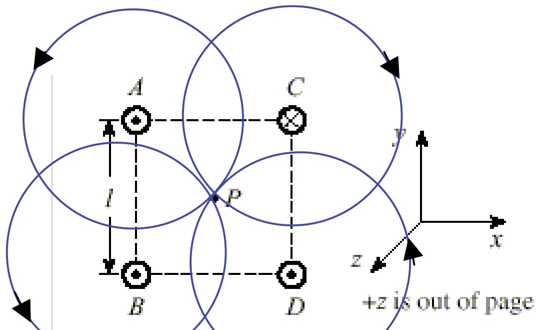


Magnitude of ∫B·ds?

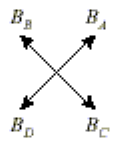


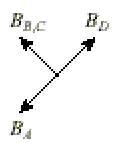
Four Parallel Conductors
30:03, calculus, numeric, > 1 min.
027
Four long, parallel conductors carry equal currents of 5 A. An end view of the conductors is shown in the figure. Each side of the square has length of $\ell = 0.2$ m.

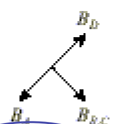
Note: The current direction is out of the page at points A, B, and D indicated by the dots and into the page at point C indicated by the cross.

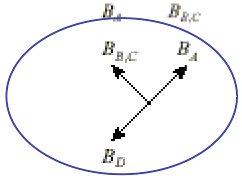


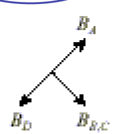
Which of the diagrams shown below correctly denotes the directions of the magnetic fields from each conductor at the point P?

1.


2.


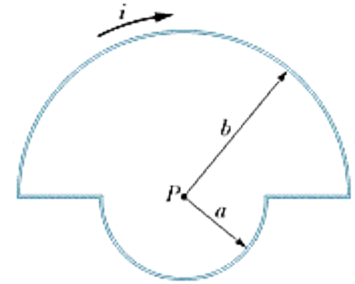
3.


4.


5.


The current in wires A,B,D is out of the page, current in C is into the page. Each wire produces a circular field line going through P, and the direction of the magnetic field for each is given by the right hand rule. So, the circles centers in A,B,D are counterclockwise, the circle centered at C is clockwise. When you draw the arrows at the point P, the fields from B and C are pointing in the same direction (up and left).

A length of wire is formed into a closed circuit with radii *a* and *b*, as shown in the Figure, and carries a current *i*.



(a) What are the magnitude and direction of **B** at point *P*?

$\frac{\mu_0 i}{4}\left(\frac{1}{a}+\frac{1}{b}\right)$ $\frac{\mu_0 i}{4}\left(\frac{1}{a}-\frac{1}{b}\right)$ $\frac{\mu_0 i}{4}\left(\frac{1}{b}-\frac{1}{a}\right)$ $\frac{\mu_0 i}{2}\left(\frac{1}{a}+\frac{1}{b}\right)$

$\boxed{B=\frac{\mu_0 i\phi}{4\pi R}}$

(b) Find the magnetic dipole moment of the circuit.

$\frac{1}{2}\left(\pi b^2-\pi a^2\right)i$ $\frac{1}{2}\left(\pi a^2-\pi b^2\right)i$ $\frac{1}{2}\left(\pi a^2+\pi b^2\right)i$ $\frac{1}{4}\left(\pi a^2+\pi b^2\right)i$

$\boxed{\mu = NiA}$

- **Inductors:**

  Magnetic Flux: $\Phi = \int \vec{B} \cdot d\vec{A}$

  Faraday's law: $\mathcal{E} = -\frac{d\Phi}{dt}$ $(= -N\frac{d\Phi}{dt}$ for a coil with N turns)

  Induced Electric Field: $\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi}{dt}$

  Motional emf: $\mathcal{E} = BLv$

  Definition of Self-Inductance: $L = \frac{N\Phi}{i}$

  Inductance of a solenoid: $L = \mu_0 n^2 Al$

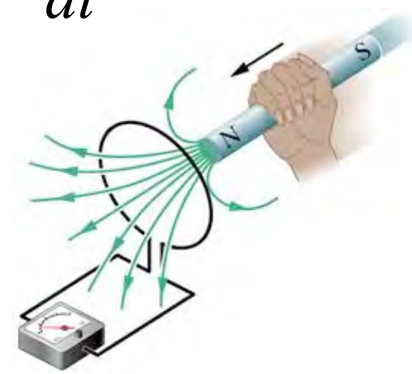
  EMF (Voltage) across an inductor: $\mathcal{E} = -L\frac{di}{dt}$

  RL Circuit: rise of current: $i = \frac{\mathcal{E}}{R}(1 - e^{-\frac{tR}{L}})$ decay of current: $i = i_0 e^{-\frac{tR}{L}}$

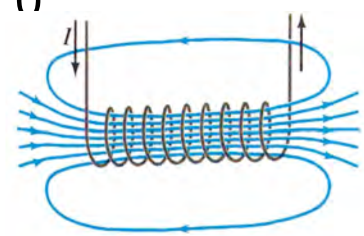
  Magnetic Energy: $U_B = \frac{1}{2}Li^2$ Magnetic energy density: $u_B = \frac{B^2}{2\mu_0}$

Induction and Inductance

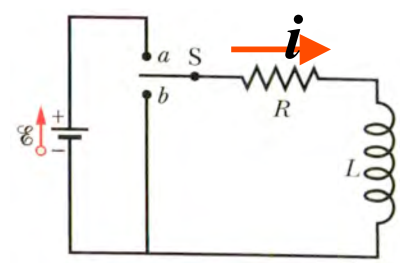
- Faraday's law: $EMF = -\frac{d\Phi_B}{dt}$ or $\oint_C \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$



- Inductance: $L=N\Phi/I$
  - For a solenoid: $L=\mu_0 n^2 A l=\mu_0 N^2 A/l$



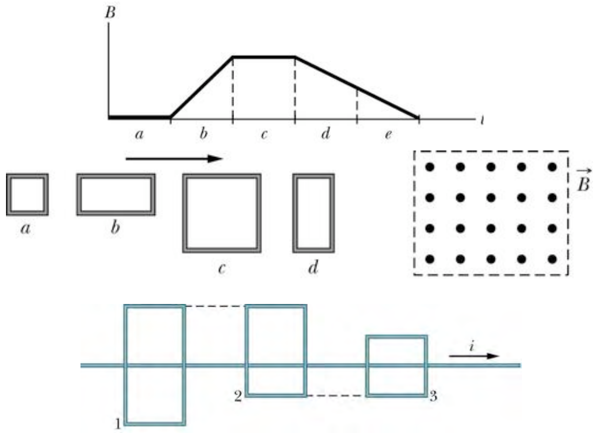
- Inductors: $E_L=-L\, di/dt$



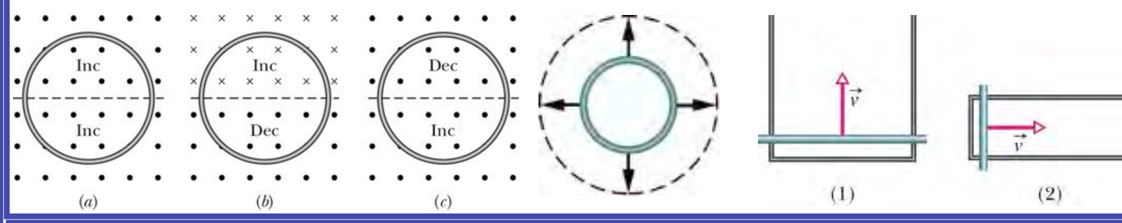
- RL circuits: $i(t)=(E/R)(1-e^{-tR/L})$ or $i(t)=i_0 e^{-tR/L}$
- Magnetic energy: $U=Li^2/2$; $u=B^2/2\mu_0$

Checkpoints/Questions

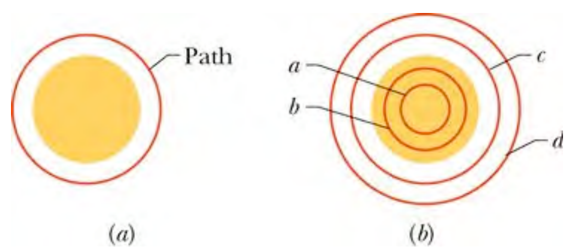
Magnitude of induced emf/current?



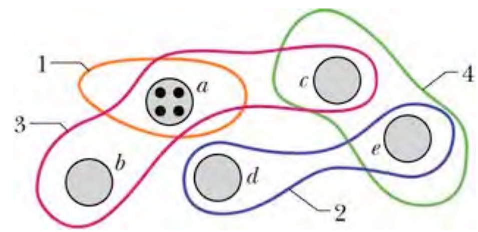
Magnitude/direction of induced current?
Magnitude/direction of magnetic field inducing current?



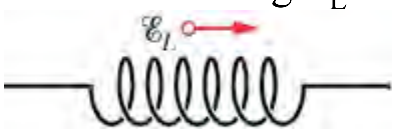
Given B, dB/dt, magnitude of electric field?



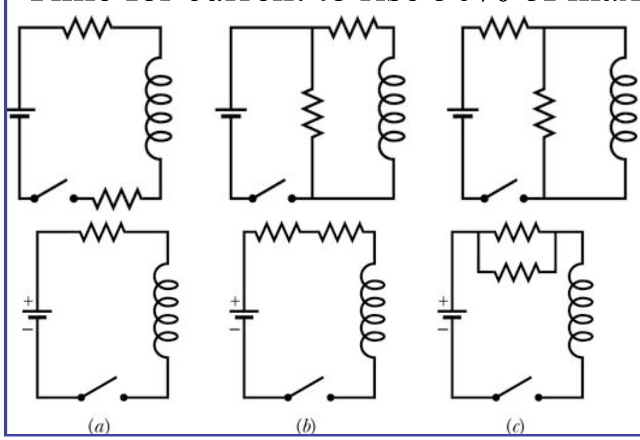
Given $|\int E\cdot ds|$, direction of magnetic field?



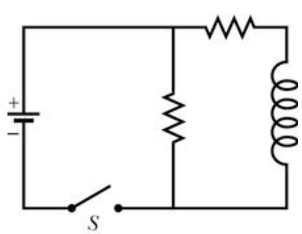
Current inducing E_L?



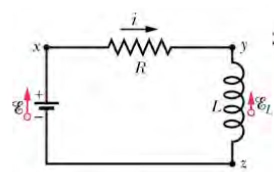
Current through the battery?
Time for current to rise 50% of max value?

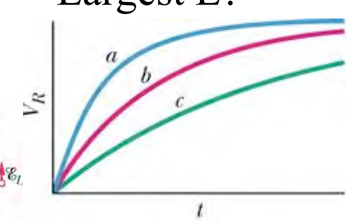


Largest current?

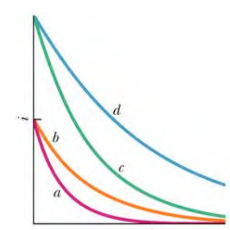


Largest L?





R,L or
2R,L or
R, 2L or
2R,2L?

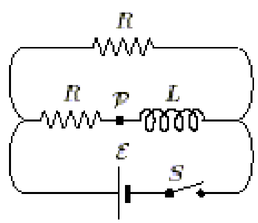


AP EM 1993 MC 59 61

32:02, calculus, multiple choice, > 1 min.

002

Consider the circuit shown.



What is the instantaneous current at point $\mathcal{P}$ immediately after the switch is closed?

1. $I_{\mathcal{P}}(0) = 0$
2. $I_{\mathcal{P}}(0) = \dfrac{\mathcal{E}}{R}$
3. $I_{\mathcal{P}}(0) = \dfrac{\mathcal{E}\,L}{2\,R}$
4. $I_{\mathcal{P}}(0) = \dfrac{\mathcal{E}}{R\,L}$
5. $I_{\mathcal{P}}(0) = \dfrac{\mathcal{E}}{2\,R}$

003

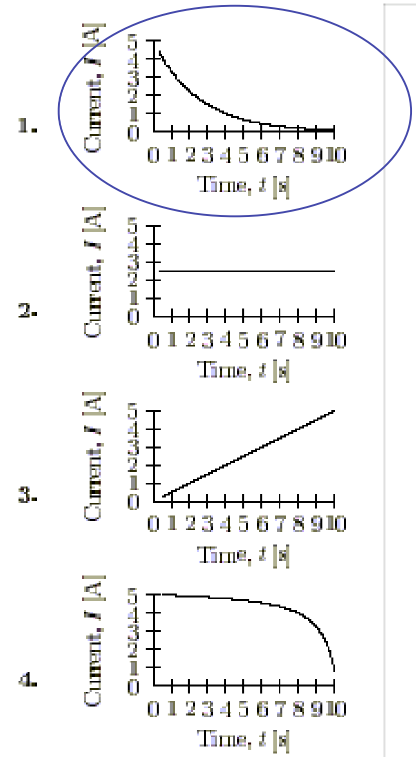
When the switch has been closed for a long time, what is the energy stored in the inductor?

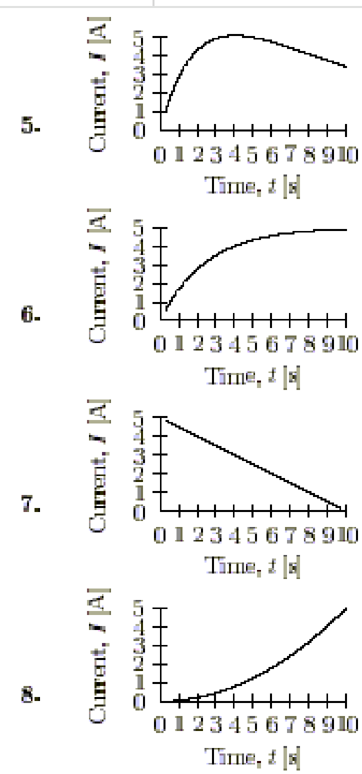
1. $U_L = \dfrac{L\,\mathcal{E}^2}{2\,R^2}$
2. $U_L = \dfrac{\mathcal{E}^2\,R^2}{4\,L}$
3. $U_L = \dfrac{L\,R^2}{2\,\mathcal{E}^2}$
4. $U_L = \dfrac{L\,\mathcal{E}}{2\,R}$
5. $U_L = \dfrac{L\,\mathcal{E}^2}{4\,R^2}$

004

After the switch has been closed for a long time, it is opened at time $t = 0$.

Which of the following graphs best represents the subsequent current i at point $\mathcal{P}$ as a function of time t?





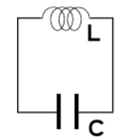
When the switch is closed, the inductor begins to get charged, and the current is $i = (E/R)(1 - e^{-tR/L})$.

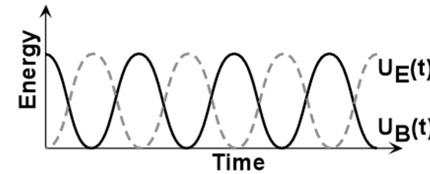
When the switch is opened, the inductors begins to discharge.

The current in this case is then $i = (E/R)\, e^{-t2R/L}$

- **LC circuits:**

  Electric energy in capacitor: $U_E = \dfrac{q^2}{2C} = \dfrac{CV^2}{2}$





  Magnetic energy in an inductor: $U_B = \dfrac{Li^2}{2}$

  LC circuit oscillations: $q = Q\sin(\omega t + \phi)$ $\quad (i = \dfrac{dq}{dt} \quad q = Cv) \quad \omega = \dfrac{1}{\sqrt{LC}}$

- **Circuits with generators**

  $\mathcal{E} = \mathcal{E}_m \sin\omega_d t \qquad i = I\sin(\omega_d t - \phi)$

single circuit elements:	resistor	inductor	capacitor
resistance or reactance:	R	$X_L = \omega_d L$	$X_C = \frac{1}{\omega_d C}$
phase constant ϕ:	0^o	$+90^o$	-90^o
voltage drop:	$V_R = IR$	$V_L = IX_L$	$V_C = IX_C$

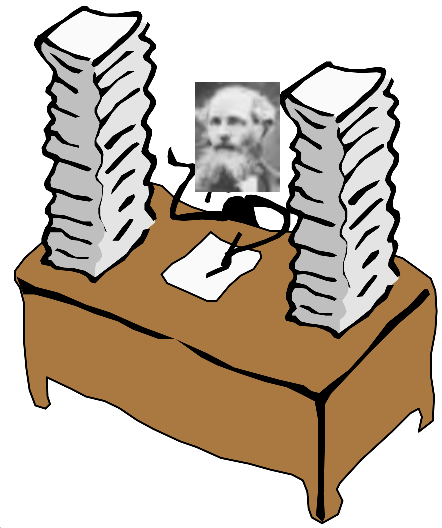
  RLC in series with AC generator: $I = \dfrac{\mathcal{E}_m}{Z}$

  Impedance: $Z = \sqrt{R^2 + (X_L - X_C)^2}$

  phase constant: $\tan\phi = \dfrac{X_L - X_C}{R}$

  power: $P_{av} = I_{rms}^2 R = \mathcal{E}_{rms} I_{rms} \cos\phi$

Maxwell's Equations:



Gauss' $$\oint_S E \bullet dA = q / \varepsilon_0$$

nameless $$\oint_S B \bullet dA = 0$$

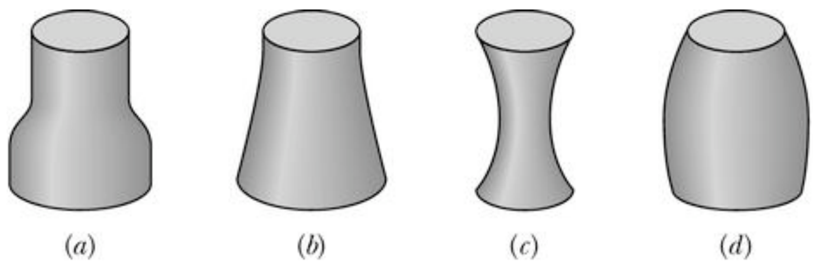
Ampere-Maxwell's $$\oint_C B \bullet ds = \mu_0 \varepsilon_0 \frac{d}{dt}\int_S E \bullet dA + \mu_0 i$$

Displacement current

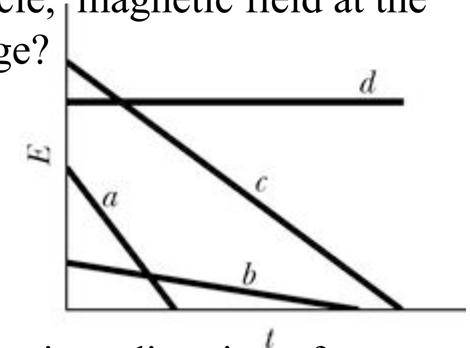
Faraday's $$\oint_C E \bullet ds = -\frac{d}{dt}\int_S B \bullet dA$$

Checkpoints/Questions

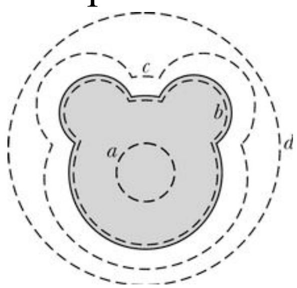
Given A, B at top and bottom, flux through curved sides?



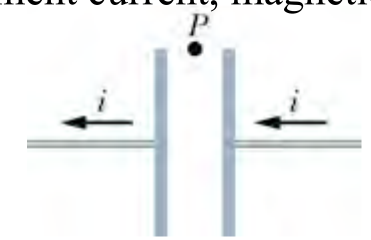
Given a uniform E(t) in a circle, magnetic field at the edge?



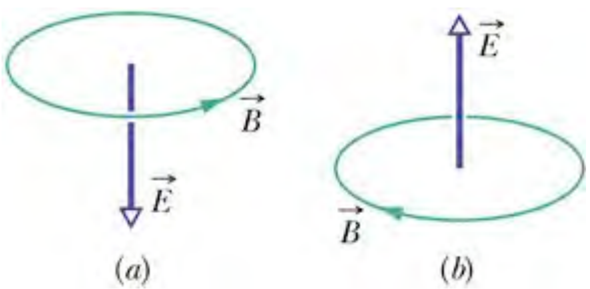
Rank integration paths according to $\int B \cdot ds$



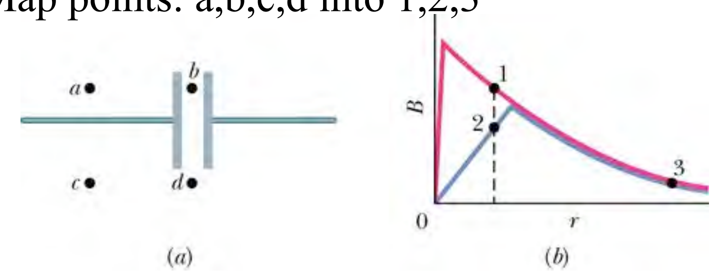
Discharging capacitor: direction of displacement current, magnetic field?



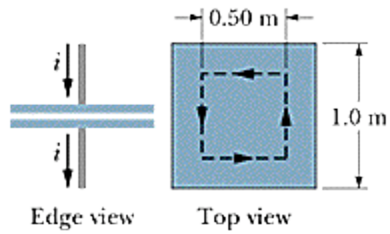
B= Magnetic field induced by E.
Is E increasing or decreasing?



Charging capacitor: Which curve is for the B in the wire, which for B in the capacitor gap?
Map points: a,b,c,d into 1,2,3



A parallel-plate capacitor has square plates 1.0 m on a side as in Fig. 32-35. A current of 3.2 A charges the capacitor, producing a uniform electric field **E** between the plates, with **E** perpendicular to the plates.



(a) What is the displacement current through the region between the plates?

(b) What is *dE*/*dt* in this region?

(c) What is the displacement current through the square dashed path between the plates?

(d) What is $\oint \mathbf{B} \cdot d\mathbf{s}$ around this square dashed path?

- **Electromagnetic Waves:** Wave traveling in +x direction: $E = E_m \sin(kx - \omega t) \quad B = B_m \sin(kx - \omega t)$, where $\vec{E} \perp \vec{B}$, and the direction of travel is $\vec{E} \times \vec{B}$

  $E/B = c, \quad f\lambda = c, \quad \lambda = \frac{2\pi}{k},$ Velocity of light in vacuum $= c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

  Energy flow: $\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} \quad I = \frac{1}{c\mu_0}E_{rms}^2 \quad E_{rms} = E_m/\sqrt{2} \quad I = \frac{P_s}{4\pi r^2}$

  Radiation force and pressure: total absorption : $F_r = \frac{IA}{c} \quad p_r = \frac{I}{c}$ total reflection : $F_r = \frac{2IA}{c} \quad p_r = \frac{2I}{c}$

- **Polarizing Sheets:** unpolarized$->$ polarized : $I = \frac{1}{2}I_0$, polarized$->$ polarized : $I = I_0 \cos^2\theta$
- **Reflection/refraction:** (Angles are measured from the normal to the interface)

  Law of reflection: $\theta_i = \theta_r$ Law of refraction: $n_2 \sin\theta_2 = n_1 \sin\theta_1$

  Total internal reflection (critical angle): $\theta_c = \sin^{-1}\frac{n_2}{n_1}$

- **Images from mirrors and thin lenses**

  Thin lens and mirror formula: $\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$

  Spherical mirror: $f = r/2$

  Magnification equation: $|m| = \frac{h'}{h}$, $m = -\frac{i}{p}$, $m > 0$ means image upright, $m < 0$ means image inverted.

  Total magnification of a two-lens system: $M = m_1 m_2$

- **Interference:**

  Constructive interference: Phase difference= $(m)2\pi$ (path length difference $m\lambda$)

  Destructive interference: Phase difference = $(m+\frac{1}{2})2\pi$ (path length difference $(m+\frac{1}{2})\lambda$) $\quad m=0,1,2,...$

  Index of refraction: $\lambda_n = \frac{\lambda}{n}$, $\quad v = \frac{c}{n}$, v = velocity of light in a medium.

  Phase difference (in wavelengths) when traveling through two different media: $N_2 - N_1 = \frac{L}{\lambda}(n_2 - n_1)$

- **Two-slit interference:** bright fringes : $\; d\sin\theta = (m)\lambda$, $\qquad$ dark fringes : $\; d\sin\theta = (m+\frac{1}{2})\lambda \quad m=0,1,2,...$

  Intensity in Two-Slit Interfence: $I = 4I_0\cos^2\frac{1}{2}\phi$, $\quad \phi = \frac{2\pi d}{\lambda}\sin\theta$

- **Thin-Film Interference** $\quad$ Thin film in air: Constructive interference when $2L = \left(m+\frac{1}{2}\right)\frac{\lambda}{n_2}$

  Destructive interference when $2L = (m)\frac{\lambda}{n_2}$, $\quad m=0,1,2,...$

- **Diffraction :**

  Single Slit diffraction: minima : $\; a\sin\theta = m\lambda$, $\quad m=1,2,...$ $\qquad$ intensity : $\; I(\theta) = I_m\left(\frac{\sin\alpha}{\alpha}\right)^2$, $\quad \alpha = \frac{\pi a}{\lambda}\sin\theta$

  Circular-Aperture diffraction: 1st minimum : $\; \sin\theta = 1.22\frac{\lambda}{d}$

  Rayleigh's criterion for resolvability: $\sin\theta_R = 1.22\frac{\lambda}{d}$

  Double Slit diffraction: $I(\theta) = I_m(\cos^2\beta)\left(\frac{\sin\alpha}{\alpha}\right)^2$, $\quad \alpha = \frac{\pi a}{\lambda}\sin\theta \quad \beta = \frac{\pi d}{\lambda}\sin\theta$