

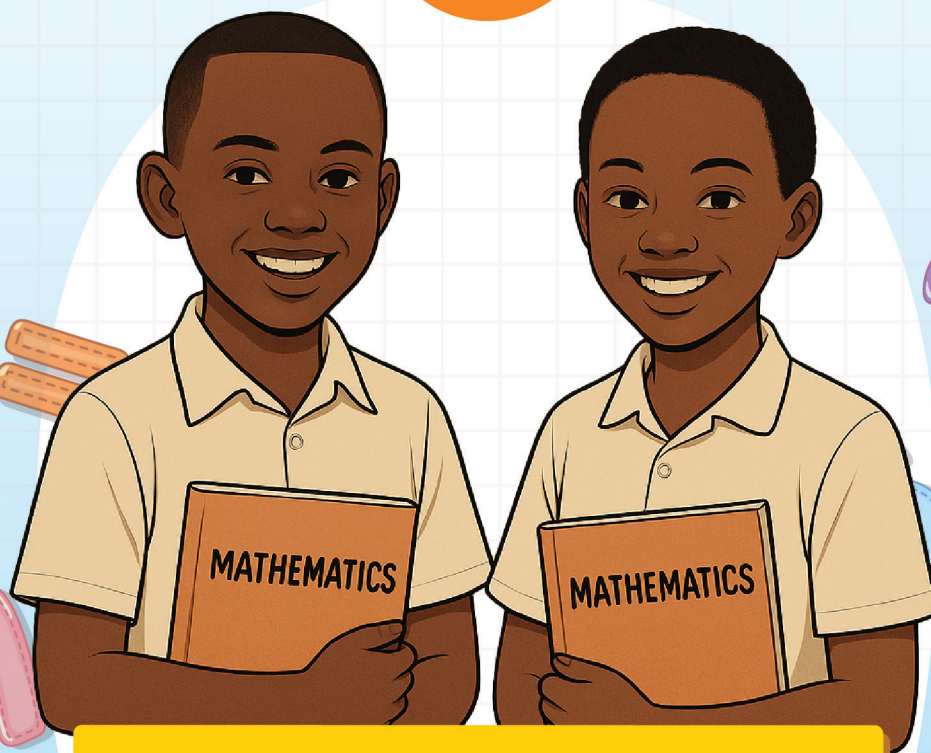


Mathematics



PRIMARY

5

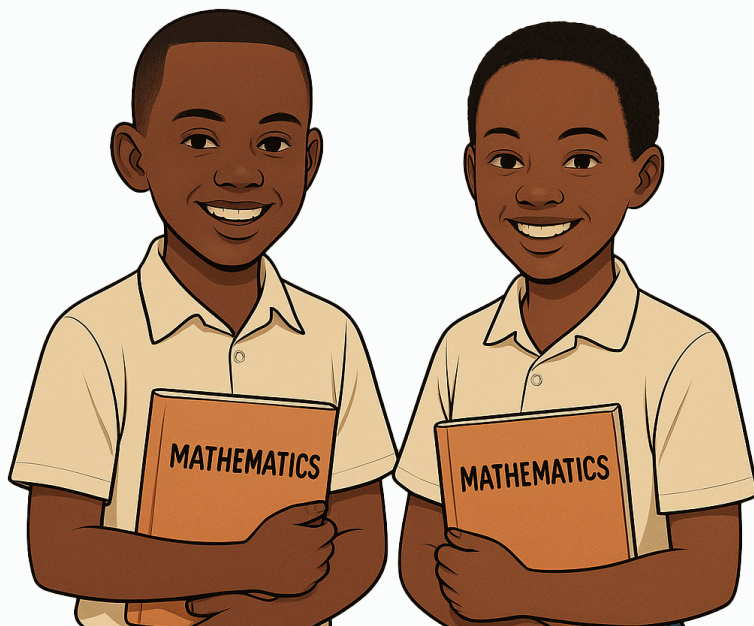


PUPIL'S BOOK

MATHEMATICS

Pupil's Book

Primary 5



Version Edited in 2025

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FOREWORD

Dear Pupil,

The Rwanda Basic Education Board (REB) is honoured to present the Mathematics book for Primary Five. This Textbook is designed to support the effective implementation of the Competence-Based Curriculum (CBC); ensuring learner-centered teaching practices that align with Rwanda's vision of empowering learners to reach their full potential, actively participate in society, and pursue meaningful careers.

As Rwanda advances toward building a knowledge-based economy, education plays a pivotal role. Teachers are central to this transformation, guiding learners to connect classroom knowledge with real-life experiences, thus fostering individual growth and national development.

This student book serves as a comprehensive resource for teaching Mathematics at Primary Five. The subject is an integral part of the CBC, aimed at equipping learners with essential knowledge, skills, values, and attitudes necessary for becoming responsible and competent citizens. Through this subject, learners are encouraged to develop problem-solving abilities and contribute positively to their communities.

In pursuit of quality education, the Government of Rwanda emphasizes the importance of using teaching and learning materials that are aligned with the curriculum.

Although this guide provides answers to all pupil activities, teachers are encouraged to independently work through the exercises to better evaluate and understand learners' responses.

I sincerely thank everyone who contributed to the refinement of this textbook. Special recognition goes to the REB team who coordinated the entire process. We thank all partners, teachers, and individuals who were involved in the process from start to the end. Every feedback was highly valued and helped to improve learning outcomes of our students.



Dr. MBARUSHIMANA Nelson

Director General, REB

ACKNOWLEDGEMENT

I would like to extend my sincere appreciation to everyone who contributed to the refinement of the Mathematics students book for Primary Five. This achievement would not have been possible without the active involvement of various stakeholders.

Special thanks go to Rwanda Basic Education Board (REB) staff; subject experts; university lecturers and teachers for their valuable work in the writing of this textbook.

I am also grateful to all our education development partners whose support was instrumental throughout the process.



MURUNGI Joan

Head of Curriculum, Teaching and Learning Resources Department

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UNIT 1

READING, WRITING, COMPARING AND CALCULATING WHOLE NUMBERS UP TO 1,000,000

Key unit competence: You will be able to read, write, compare and make calculations on whole numbers up to 1,000,000.

Learning objectives:

By the end of this unit, you should be able to:

Knowledge and understanding:

- Name all place values up to 1,000,000 in spoken and written form.
- Identify the place values in written numerals.
- Read written numbers correctly in English.
- Explain the concept and process of addition of 3 numbers of 6 digits with or without carrying.
- Explain the concept and process of subtraction of 2 numbers of 6 digits or more with or without borrowing.
- Explain the concept and process of multiplication of a 3-digit number by a 3-digit number.
- Explain the concept and process of division of numbers.
- Explain the process of solving word problems.



Skills

- Compare numbers of any size using place values with a general understanding.
- Write the number in words correctly.
- Apply comparison of numbers in daily life.
- Carry out addition, subtraction of 3 or more whole numbers.
- Apply the knowledge of addition, subtraction, multiplication and division in solving word problems in daily situations.

Attitudes and values:

- Appreciate the importance of reading and writing numbers correctly.
- Show confidence when using numbers.
- Appreciate the importance of addition, subtraction, multiplication and division in daily life.

1.0 Introduction

This unit equips pupils with the foundational mathematical competence to confidently engage with whole numbers up to 1,000,000. Understanding large numbers is a critical skill that supports problem-solving, logical thinking, and the ability to interpret numerical information in real-life situations.

You will develop the ability to read, write, compare, and order whole numbers using both digits and words. In addition, you will perform the four basic operations, addition, subtraction, multiplication, and division, accurately and efficiently with numbers up to one million. Emphasis will be placed on place value understanding, estimation, and mental math strategies.

This unit fosters numeracy, accuracy, logical reasoning, and problem-solving abilities essential for lifelong learning and daily decision-making.



Introductory Activity

Look at the classroom or outside the classroom to find items that illustrate the role of numbers in our daily lives. Proceed it in the following ways:

Look for something that represents:

- The number of days in a week (7).
- A note of Rwandan francs. It could be a 500Frw note, a one thousand note; or a 5000Frw note;
- Find a clock or watch displaying a specific time;
- A poster that includes data on the population and health-related recommendations.
- Page numbers that are multiple of 10 in a book.
- A bus departure time in one transportation company.

Summarize your findings and be ready to use them to explain the importance of numbers in real life.

1.1 Place Value of digits for whole numbers up to 1,000,000



Activity 1.1

There are numbers greater than 100,000. Numbers from 100,001 to 999,999 have 6 digits but the number 1,000,000 has 7 digits. To write the place value of each digit for example in the number 235,176, we use a table of place values as follows:

Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
2	3	5	1	7	6

The place value of each digit is given here below:

2 – Hundred thousands

3 – Ten thousands

5 – Thousands

1 – Hundreds

7 – Tens

6 – Ones

Therefore, observe these numbers:

(a) 100 000 (b) 473 625 (c) 999 999

Write the place value of each digit.

Say the place value of each digit.

What is the next number after 999 999? What is the place value of digit 1 in your answer?

Discuss your answer.



Summary

Numbers are organized by place values, where each digit's position determines its value. For numbers up to 7 digits, consider the example of 654,321.

Digit	Digit Position	Place Value	Value
6	1 st	Hundred Thousands (H-Th)	600,000 = (6 × 100,000)
5	2 nd	Ten Thousands (T-Th)	50,000 = (5 × 10,000)
4	3 rd	Thousands (Th)	4,000 = (4 × 1,000)
3	4 th	Hundreds (H)	300 = (3 × 100)
2	5 th	Tens (T)	20 = (2 × 10)
1	6 th	Ones (O)	1 = (1 × 1)

For the number 1,000,000, the digit 1 has the place value of millions.



Application Activity 1.1

1) Write the place value of the colored digit. Discuss and present your answer.

(a) 19**8**,065 (b) **7**46,138 / (c) 6**4**0,404 (d) 245,**6**89

2) Use a place value table to identify the digit in the place value of hundred thousands for the following number.

(a) 964 815 (b) 321 456.

3) a) Write the place value of the digit 6 in the number 4567891.

b) What is the place value of the digits 5 and 4 in the number 859174?

1.2 Reading and writing numbers up to 1,000,000 in figures



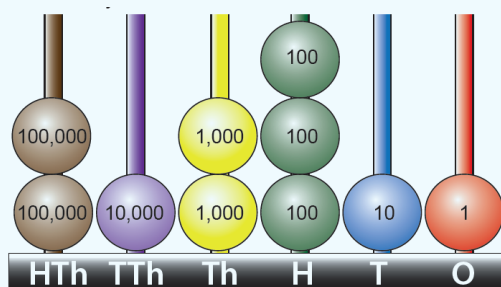
Activity 1.2

1) Take a large abacus with 6 bars. Name the 6th bar as hundred thousands.

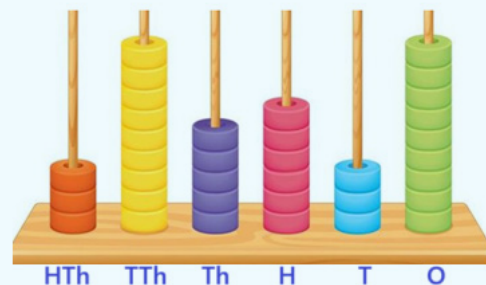
Use beads to form a number with 6 digits,

Write the number in figures in your notebook and read it.

Refer to the example of the number made on the following abacus.



212,311



395,639

The number formed is 212,311: Two hundred twelve thousand, three hundred eleven.

395,639: Three hundred ninety-five thousand, six hundred thirty-nine.

Form other 6 different numbers, write them in figures and read them.

2) Look at the following numbers written on cut-outs of papers:

100,001; 280,465; 405,230; 729,111 and 999,999

- Take each paper, observe the number and represent it on the abacus.
- Say the number of digits for the number.
- Read each number aloud. Write the numbers in your note book.



Summary

When reading a number,

- Start from the right and separate the digits into threes (triples) using commas or spaces.
- **Split the number into groups of 3 digits** (from the right):
Millions, Thousands, and Units.
- Split each triple in **hundreds, tens and ones.**
- Then, read each group separately.

Example 1:

654,321 → 654 (thousand) | 321 (units).

- **Read each group separately, then add the place value:**

654,321:

Six hundred fifty-four thousand, three hundred twenty-one."

Example 2:

Read: 1,000,000:

Solution: 1,000,000 = 1 Million, 0 thousand, 0 units

The number is read: One million.

3) The other method consists of breaking the number into thousands, hundreds, tens and ones.

Example: Read the number 976,946.

Step 1: Break it down into: thousands, hundreds, tens and ones:

976,000 = Nine hundred seventy-six thousand

900 = Nine hundred

40 = Forty

6 = Six

Step 2: Combine them: Nine hundred seventy-six thousand nine hundred forty-six. Try to represent some numbers between 100,001 and 1,000,000 on the abacus and read them



Application Activity 1.2

1. Read each of the following numbers.
(a) 671,379 (b) 286,748 (c) 910,842 (d) 263,450
2. A factory packed 447,313 textbooks in cartons for sale. Write this number in words. Explain the steps you used to arrive at your answer.
3. On Umuganda day, P6 pupils planted 527,174 trees. Write this number in words. Explain steps to your answer.

1.3 Forming and reading numbers using number cards

You can form a number using different digits. In real life, numbers help us to compare the number of objects, and we can identify a big number of objects and a small number of objects.



Practical Activity 1.3

Using different number cards below; pick different digits to form different numbers. The number cards contain the following digits; 1, 3, 5, and 2.



Procedures:

Step 1: Take the number cards with different colours containing the following digits: 1, 3, 5, 2 and then, place them on the table for easy manipulation.



Step 2: Form different numbers and read them. A number card is used once.

Step 3: Identify the number card to start with in order to form the biggest number composed by the digits: 1, 3, 5, and 2. Write and read the obtained number.

Step 3: Identify the number to start with to form the smallest number composed by the digits: 1, 3, 5, and 2. Write and read the number formed.

Step 4: In pair or groups, take the number cards and manipulate the cards to form and read numbers and present the findings in the classroom.



Summary

- To know the **largest number**, arrange the digits in descending order (from highest to lowest).
- This ensures the number starts with the largest possible digit, maximizing its value.
- To form the **smallest number**: Arrange the digits in ascending order (from lowest to highest).

This ensures the number starts with the smallest possible digit, minimizing its value.

If there's a 0, ensure it is not the first digit (as leading zeros are not allowed in standard numbers). In such cases, place the next smallest digit first, followed by 0

Examples

- 1) Arrange these number cards for the given digits 2, 3, 5, and 4 so that they form the biggest number. Write and read the formed number correctly.

Answer:

The biggest number: 

The biggest number is “five thousand four hundred thirty-two”.

The smallest number: 

The smallest number is “two thousand three hundred forty-five”.

- 2) Play a game. Use 10 bottle tops with numbers 0- 9 written on them. Turn them upside down. Turn over any 6 bottle tops and rearrange them to make a number. Read together with your colleague.

Example:

I form a number: 

I read the formed number: One hundred forty-five thousand twenty-eight.



Application Activity 1.3

Use the given number cards to form a smallest possible 6-digits number from the given digits.



Step 1: Take the number cards written on the following digits: 3; 5; 7; 9; 0; 6. Then place them on the table for easy manipulation.

Step 2: Form the smallest number and the biggest number. Then, tell the number to your colleague. Can you write correctly the number obtained in words?

1.4 Reading and writing numbers in figures and in words up to 1,000,000



Activity 1.4

Write these numbers in figures.

- Six hundred thirty-three thousand, four hundred five.
- One hundred thousand, eleven.
- Nine hundred seven thousand, one hundred seven.

Summary

When writing a six-digit number in words,

- Break it down by place value and write as you read.
- Use hyphens for compound numbers between tens and ones from twenty-one and ninety-nine, but not with “thousand” or “hundred.”

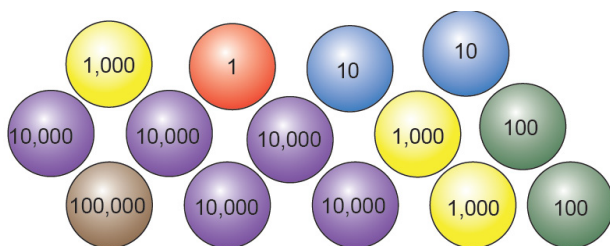
Example:

1) Use the place value counters to break down the number:

One hundred fifty-three thousand, two hundred twenty-one.

Solution:

Select appropriate counters to form the number.



$$100,000 + 50,000 + 3,000 + 200 + 20 + 1 = \mathbf{153,221}$$

2) Write "eight hundred twenty-five thousand, two hundred thirty-four" in figures:

Solution

Thousands: 825 000

Hundreds: 200

Tens : 30

Ones : 4

825 234

Eight hundred twenty-five thousand, two hundred thirty-four: **825234**.

Note: we can sometimes use “and” between the hundreds and the tens/ones in numbers greater than one hundred.

Examples:

101: One hundred **and** one. 215: Two hundred **and** fifteen. 432: Four hundred **and** thirty-two.

However, it would be advisable not to use this word and as it can be confused with the separator of a whole number and a decimal point(.) for a decimal number.



Application Activity 1.4

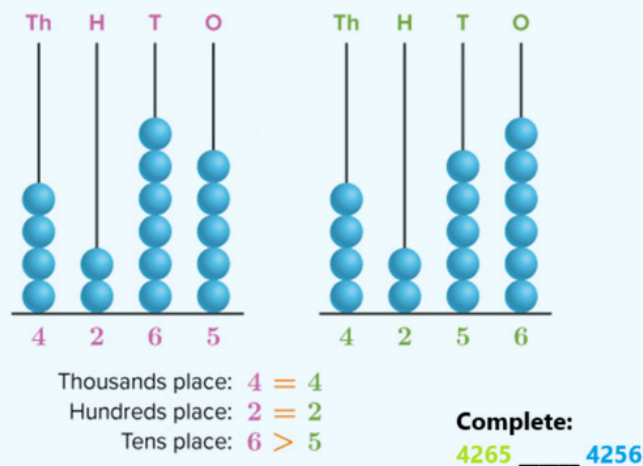
- 1) Write each of the following numbers in figures.
 - a) Seven hundred six thousand, five hundred eighteen.
 - b) One hundred three thousand, six hundred four.
 - c) Twenty-nine thousand, eighty-seven.
- 4) I had three hundred ninety-eight thousand, seven hundred sixty-six Rwandan Francs. Write the amount I had in figures. Is this money sufficient to buy a motorbike?

1.5 Comparing Numbers using $<$, $>$ or $=$



Activity 1.5

- 1) Represent numbers on the abacus as illustrated on the figure below:



Then, use **is greater than** or **is less than** to compare 4,265 and 4,256.

2. Write the following numbers on small paper cutouts.

6 , 2 , 4 , 8 , 9.

Arrange the numbers to form:

- The largest number.
- The smallest number.
- i) Use $>$ to compare the numbers you have formed.
- ii) Use $<$ to compare the numbers you have formed.

Repeat the activity above with other digits. Explain your answers.

Do you sometimes compare money?

Discuss other quantities that can be compared.



Summary

To compare whole numbers:

- Check digits of numbers at the same place values.
- Start at the left and compare the digits in the greatest place value position.
- The greater number has a greater digit at the greatest place value.
- We use: $<$ for “less than”, $>$ for “greater than” and $=$ for “equal to”.

Example:

Use $<$, $=$ or $>$ to compare numbers:

356 481 — 353 406

Solution

Step 1: Write the two numbers on place value chart.

Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
3	5	6	4	8	1
3	5	3	4	0	6

Step 2: Compare the digits from the left towards the right.

The hundred thousands digits are the same. So are the ten thousands digits.

Thousands digits are 6 and 3, but $6 > 3$.

Therefore, $356\,481 > 353\,406$.



Application Activity 1.5

Use $<$, $=$ or $>$ to compare the following pair of numbers.

1) (a) $440\,040$ ____ $460\,040$ (b) $657\,000$ ____ $675\,000$

(c) $649\,362$ ____ $639\,462$ (d) $831\,647$ ____ $861\,347$

2) Anne is a coffee farmer. Anne made 550,000 Frw from sales of her coffee over four months. Musabe is a business person, He made 630,000 Frw from sales in his shop over four months. Who made more money? Explain your answer.

3) Use $<$, $=$ or $>$ to compare the following. Discuss your answers.

(a) $531\,926$ ____ $13\,926$

(b) 145700 ____ 1457×100

(c) $210\,034$ ____ $201\,034$

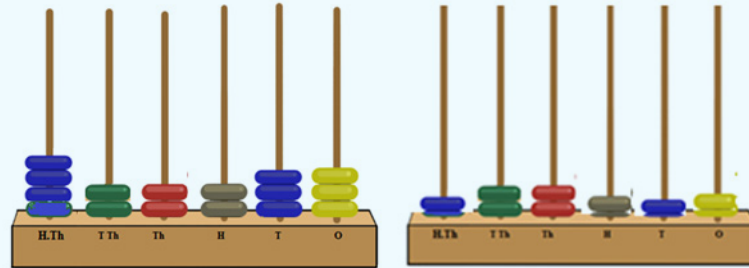
(d) $245\,689$ ____ $245\,689$

1.6 Add whole numbers using a local abacus



Activity 1.6

To add the number 422,233 and the number 122,111, Patrick used a local abacus. He placed the beads representing the first number 422,233 on one abacus. Then, he placed beads for the second number on a second abacus.



a) Explain how the number of beads for each bar is selected.

b) Select the beads representing the second number.

c) Add them on the first abacus respecting the place values.

What number of beads do you get at each place value bar? Now Record, the new number represented on the abacus.

d) Use the answer obtained in (c) to find: $422,233 + 122,111 =$



Summary

To add six-digit numbers, consider the number representing different place values: units, tens, hundreds, thousands, ten-thousands, and hundred-thousands.

- Set the first six-digit number on the abacus by moving the beads to reflect each digit in the correct position.
- Once the initial number is displayed, add the second six-digit number starting from the rightmost bar (units), moving left across the bars.
- As each digit is added, compute the total for each place value.

If the sum of the digits exceeds 9 for any bar, the result is cleared to zero, and a carry is transferred to the next bar on the left.

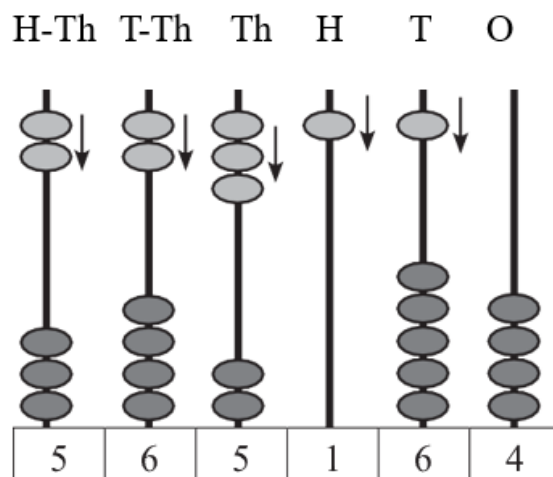
This process continues until all digits have been added.

The beads left on the abacus at the end of the operation visually represent the total of the two numbers.

Example:

Use an abacus to illustrate: $342,054 + 223,110 =$

On the following abacus, the beads for the second number are being placed:



The number to obtain is counted from the different place values shown above.

Therefore, $342,054 + 223,110 = 565,164$.



Application Activity 1.6

A school used 230, 224 litres of water in the first term. In the second term, the school used 411,221 litres of water. Use the abacus to illustrate the total number of litters the school used for two terms.

1.7 Addition of whole numbers of 6 digits without carrying



Activity 1.7

- Use an abacus or objects of different colours to add the following numbers:
 $100,204 + 551,480 + 226,102 =$
- Add those numbers without using the abacus. What did you observe while adding numbers?



Summary

- When adding big numbers, align them according to place values.
- Add ones, add tens, add hundreds, etc.

Example: Workout: $272,142 + 203,512 =$

HTh	TTh	Th	H	T	O	Add ones: $2 + 2 = 4$, write 4 ones
2	7	2	1	4	2	Add tens: $4 + 1 = 5$, write 5 tens
+ 2	0	3	5	1	2	Add hundreds: $1 + 5 = 6$, write 6 hundreds
4	7	5	6	5	4	Add thousands: $2 + 3 = 5$, write 5 thousands Add tens thousands: $7 + 0 = 7$, write 7 ten thousands Add hundred thousands $2 + 2 = 4$, write 4 hundred thousands.

Therefore, $272,142 + 203,512 = 475,654$

Or simply:

$$\begin{array}{r} 272,142 \\ + 203,512 \\ \hline 475,654 \end{array}$$


Application Activity 1.7

1) Work out:

(a) $430\,526 + 323\,250 =$

(b) $252\,143 + 235\,322 + 400\,312 =$

2) A businessman invested 442,300 Frw in the first year of business. The second year he invested 442,100 Frw. The third year he invested 115,600 Frw. How much money did he invest in total? Explain your answer.

3) At a peace campaign rally, there were 8,430 women. Men were 5,660 and children were 7,200. How many people attended the rally? Discuss the steps involved in calculating your answer.

1.8 Addition of 3 or more whole numbers of 6 digits with carrying



Activity 1.8

a) Use the abacus to add: $128,539 + 300,856 + 15,210$.

b) Add the numbers without using the abacus. Explain how you do it.



Summary

To add numbers,

- Start by writing the two six-digit numbers one above the other, aligning them by their place values (units, tens, hundreds, thousands, ten-thousands, hundred-thousands).

- Begin adding from the rightmost column (the units) and move leftward. If the sum of a column exceeds 9, write down the rightmost digit of the sum in that column and carry over the leftmost digit to the next column. Continue this process for all columns until all digits have been added. The final sum, composed of all the digits from the columns and any carried values, gives the total of the two six-digit numbers.

Example: Add: $472598 + 284706 + 163075 =$

	2	1	1	1	1	
	4	7	2	5	9	8
+	2	8	4	7	0	6
	1	6	3	0	7	5
	9	2	0	3	7	9

Ones: $8 + 6 + 5 = 19$ ones, write 9 in ones carry 1 ten to tens

Tens: $9 + 0 + 7 + 1 = 17$ tens, write 7 tens write 7 in tens carry 1 to hundreds

Hundreds: $5 + 7 + 0 + 1 = 13$ hundreds, write 3 in hundreds, carry 1 to thousands

Thousands: $2 + 4 + 3 + 1 = 10$ thousands, write 0 in thousands, carry 1 to ten thousands.

Ten thousands: $7 + 8 + 6 + 1 = 22$ ten thousands, write 2 in ten thousands, carry 2 to hundred thousands.

Hundred thousands: $4 + 2 + 1 + 2 = 9$ hundred thousands.



Application Activity 1.8

1) Work out the following. Present your answers.

(a) $39\,845 + 105\,523 + 351\,214 =$

(b) $193\,584 + 258\,907 + 391\,358 =$

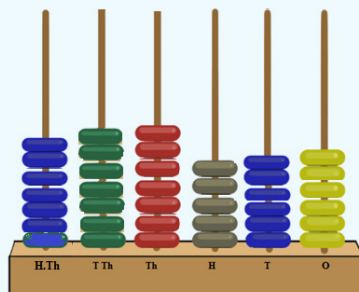
2) A poultry farmer sold 252 797 chickens in one year. The next year he sold 391 358 chickens. The third year he sold 193 583 chickens. How many chickens did he sell in 3 years?

1.9 Subtraction of 2 numbers of 6 digits without borrowing



Activity 1.9

a) Use an abacus to subtract: $777,566 - 352,150 =$



b) Subtract without using the abacus:

i) $808,210 - 205,210 =$

(ii) $394,930 - 192,620 =$



Summary

To subtract vertically,

- Write the two six-digit numbers one below the other, ensuring proper alignment by place values.
- Start to subtract from ones, then tens, hundreds, etc.

Example:

$$\begin{array}{r} 258\ 950 \\ -\ 50\ 300 \\ \hline 208\ 650 \end{array}$$

Steps: Subtract

Ones: $0 - 0 = 0$, write 0

Tens: $5 - 0 = 5$, write 5

Hundreds: $9 - 3 = 6$, write 6

Thousands: $8 - 0 = 8$, write 8

Ten thousands: $5 - 5 = 0$, write 0

Hundred thousands: $2 - 0 = 2$, write 2.



Application Activity 1.9

Find the answer:

(a) $888,980 - 56,360 =$

(b) $393,588 - 372,475 =$

1.10 Subtraction of 2 numbers of 6 digits with borrowing



Activity 1.10

Subtract:

(a) $852\,757 - 193\,583 =$ (b) $710\,534 - 40\,203 =$

What do you notice when subtracting?

Explain the borrowing from one place value to the next to carry out subtraction



Summary

To subtract numbers with borrowing, start by writing the first number (minuend) and the second number (subtrahend) one above the other, ensuring they are aligned by place values (ones, tens, hundreds, etc.).

Begin the subtraction process from the rightmost column (the units) and check if the digit in the first number is smaller than the corresponding digit in the second number. If it is, you need to borrow from the next left column: decrease that left digit in the minuend by 1 and add 10 to the current digit you're working with.

After adjusting for any borrowing, perform the subtraction for each column. Continue this process from right to left until all columns have been processed.

Example 1: Subtract $874,450 - 583,750 =$

$$\begin{array}{r} 7 \overline{) 874\,450} \\ - 583\,750 \\ \hline 290\,700 \end{array}$$

Steps: Subtract Ones: $0 - 0 = 0$, write 0

Tens: $5 - 5 = 0$, write 0

Hundreds: $4 - 7$ not possible. Borrow 10 from 4 thousands to get 14;
 $14 - 7 = 7$, write 7.

Thousands: $3 - 3 = 0$, write 0.

Ten thousands: $7 - 8$, not possible borrow 10 from 8 to get 17;
 $17 - 8 = 9$, write 9.

Hundred thousands: $7 - 5 = 2$, write 2.

Example 2: $1,000,000 - 482,000 = 518,000$

$$\begin{array}{r} 1\,000\,000 \\ - 482\,000 \\ \hline 518\,000 \end{array}$$



Application Activity 1.10

(a) $855,157 - 398,480 =$

(b) $875,864 - 557,993 =$

(c) A company packed 763 389 packets of milk on Monday. 539 897 packets were sold at the same day. How many packets of milk remained to be sold on Tuesday?

1.11 Quick multiplication of a 3-digit number by 5



Activity 1.11

Given that $5 = 10 \div 2$, How can you multiply 883×5 and get a quick answer?



Summary

To multiply a number by 5, multiply it by 10 and divide the result by 2.

Example: $629 \times 5 = (629 \times 10) \div 2 = (6290 \div 2) = 3145$



Application Activity 1.10

Workout:

(a) $130 \times 5 =$

(b) $889 \times 5 =$

(c) $216 \times 5 =$

1.12 Quick multiplication of a 3-digit number by 9 or by 11



Activity 1.12

Given that $9 = 10 - 1$ and $11 = 10 + 1$,

How can you multiply very quickly 726×9 and 726×11 ?



Summary

- To multiply a number by 9, you can use the trick of multiplying it by 10 and then subtracting the original number.
- To multiply a number by 11, you can use the trick of multiplying it by 10 and then adding the original number.

Example:

a) $827 \times 9 = 8270 - 827 = 7,443$

b) $618 \times 11 = 6,180 + 618 = 6,798$



Application Activity 1.10

Workout: a) $449 \times 9 =$

b) $325 \times 11 =$

1.13 Quick multiplication of a 3-digit number by 50 or by 25



Activity 1.12

Given that $50 = 100 \div 2$ and $25 = 50 \div 2$,

How can you multiply very quickly 726×50 and 726×25 ?



Summary

- To multiply a number by 50, you can simply take half of the number and then multiply it by 100 (or multiply the number by 100 and divide by 2).
- To multiply a number by 25, you can take a quarter of the number and then multiply it by 100 (or multiply the number by 100 and divide by 4).

Example:

a) $827 \times 50 = 82700 \div 2 = 41,350$

b) $436 \times 25 = 43600 \div 4 = 10,900$



Application Activity 1.13

Workout: a) $567 \times 25 =$

b) $629 \times 50 =$

1.14 Quick multiplication of a 3-digit number by 19 or by 49



Activity 1.12

Given that $19 = 20 - 1$ and $20 = 2 \times 10$ and $49 = 50 - 1$,

How can you multiply very quickly 352×19 and 430×49 ?



Summary

Number $\times 19 =$ Number $\times (20 - 1) = (\text{Number} \times 20) - \text{Number}$

Number $\times 49 =$ Number $\times (50 - 1) = (\text{Number} \times 50) - \text{Number}$

Example: 352×19

Step 1

Multiply 352 by 2.

$$352 \times 2 = 704$$

Step 2

Multiply the result by 10.

$$704 \times 10 = 7,040$$

Step 3

Subtract the original number from your result.

$$7\,040 - 352 = 6,688.$$

$$\text{Thus, } 352 \times 19 = 6,688$$

**Application Activity 1.14**

Workout: a) $704 \times 19 =$

b) $430 \times 49 =$

1.15 Quick multiplication of a 3-digit number by 90 or by 99**Activity 1.15**

Given that $90 = 100 - 10$ and $99 = 100 - 1$,

How can you multiply very quickly 352×19 and 430×49 ?

**Summary**

Number $\times 90 = \text{Number} \times (100 - 10) = (\text{Number} \times 100) - (\text{Number} \times 10)$

Number $\times 99 = \text{Number} \times (100 - 1) = (\text{Number} \times 100) - \text{Number}$

Example: Find $726 \times 99 =$

Step 1: Multiply the number by 100: $726 \times 100 = 72,600$

Step 2: Subtract the original 3-digit number from your result.

$$72,600 - 726 = 71,874,$$

$$\text{Thus, } 726 \times 99 = 71,874.$$

**Application Activity 1.15**

1) Workout: a) $525 \times 99 =$

b) $449 \times 90 =$

2) A school farm collected 99 eggs daily. Each egg was sold at 110 Frw. Calculate the money the school got from sale of eggs daily.

3) There are 49 pupils in the P5 class. During a mathematics lesson, each pupil brought 125 counters. How many counters were brought altogether?

1.16 Multiplying whole numbers by a 3 digits number



Activity 1.16

1) workout

a) $833 \times 4 =$ b) $833 \times 20 =$ c) $833 \times 100 =$ d) $833 \times 124 =$

2) a) One bottle of juice costs 350 Frw. Find the cost of 112 bottles of juice.

b) Do people need to multiply numbers in real life? Provide examples.



Summary

When multiplying a whole number by a three-digit number, follow these steps:

- **Multiply:** Start by multiplying the whole number by each digit of the three-digit number, beginning with the rightmost digit (the ones place).
- **Align:** Write each product below the line, aligning them according to their place value:
 - For the ones place, write the result directly below.
 - For the tens place, shift the result one place to the left.
 - For the hundreds place, shift the result two places to the left.

Add: Sum all the aligned products to get the final answer

Example:

Multiply $365 \times 241 =$

365		
x 241		
365	(365 x1)	Multiply by ones. $365 \times 1 = 365$
1460	(365 x40)	Multiply by tens. $365 \times 4 = 1\ 460$
730	(365 x 200)	Multiply by hundreds. $365 \times 2 = 730$
87965		

Therefore, $365 \times 241 = 87\ 965$



Application Activity 1.16

1) Work out:

(a) $833 \times 410 =$ (b) $581 \times 611 =$ (c) $648 \times 212 =$

2) Work out and discuss your answers:

a) $349 \times 247 =$ (b) $943 \times 333 =$ (c) $317 \times 149 =$ (d) $623 \times 261 =$

3) There are 258 hotels in a certain country. In one holiday season, each hotel received 415 visitors. How many visitors were there during that season? Explain your answer.

1.17 Division without a remainder of a 3-digit number by a 2-digit number



Activity 1.17

- a) A teacher had 120 books. The books are to be equally shared to 12 pupils. How can you find the number of books each pupil got?

What operation do you use to find the answer? Does division mean sharing objects equally to a given number of individuals?

- b) Now, explain how to divide 120 by 24 and complete; $120 \div 24 = \underline{\hspace{2cm}}$



Summary

Division means sharing equally or making a number of groups where all groups have the same number of members or objects.

When dividing a big number, use the long division method.

Example: Work out: $875 \div 25 =$

$$\begin{array}{r} 35 \\ 25 \overline{) 875} \\ \underline{- 75} \\ 125 \\ \underline{- 125} \\ 0 \end{array}$$

Steps:

Let us try to divide from the digit on the left side: 8;

$8 \div 25$ is not possible.

We try $87 \div 25$. This gives: $87 \div 25 = 3$ rem 12. Record 3.

Drop 5 to be with 12. This gives 125.

$125 \div 25 = 5$ rem 0.

Thus, $875 \div 25 = 35$



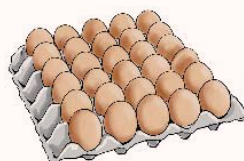
Application Activity 1.17

1) Divide the following:

(a) $792 \div 18 = \underline{\hspace{2cm}}$ (b) $768 \div 32 = \underline{\hspace{2cm}}$ (c) $391 \div 23 = \underline{\hspace{2cm}}$

2) Complete: a) $858 \div 22 = \underline{\hspace{2cm}}$ (b) $405 \div 15 = \underline{\hspace{2cm}}$

3) Diane has 180 eggs. She packs them in trays of 30 eggs each. How many trays did she fill?



1.18 Division by a 2-digit number with a Remainder



Activity 1.18

A headteacher has 398 markers to be shared equally to 30 classes. If the classes must have the same number of markers. How many markers does each class get? How many markers will remain? Why do they remain?

Now, complete; $398 \div 30 =$ _____ Remainder _____



Summary

The remainder is the number of objects that remains because they cannot be shared equally to those who were sharing.

Example: Work out:

1) $830 \div 23 =$

$$\begin{array}{r} 36 \\ 23 \overline{) 830} \\ \underline{-69} \\ 140 \\ \underline{-138} \\ 002 \end{array}$$

Steps:

Let us try to divide from the digit on the left side: 8;

$8 \div 23$ is not possible.

We try $83 \div 23$. This gives: $83 \div 23 = 3$ rem 14. Record 3.

Drop 0 to be with 14. This gives 140.

$140 \div 23 = 5$ rem 2.

Thus, $850 \div 23 = 36$ Remainder 2

2) $600 \div 24 = ?$

$$\begin{array}{r} 25 \\ 24 \overline{) 600} \\ \underline{-48} \\ 120 \\ \underline{-120} \\ 0 \end{array}$$

Therefore, $600 \div 24 = 25$.



Application Activity 1.18

1) Divide the following:

(a) $798 \div 18 =$ _____

(b) $775 \div 34 =$ _____

(c) $398 \div 28 =$ _____

2) Complete: $869 \div 21 =$ _____ Remainder _____

3) A mother has 124 notebooks. She wants to share them equally to 13 children. How many notebooks will each child get? How many notebooks will remain because they cannot be shared?

1.19 Solve real life problems involving addition, subtraction, multiplication or division



Activity 1.19

Tell your colleague how you can solve the following problem:

Joly got an envelope of 22,500 Frw on her birthday party and her twin brother, Emmy, got 26,550 Frw. When the twins reached home, their mother suggests them to share equally the total of their money with their younger sister Ella.

- What is the total of money shared by the 3 children?
- How much money did each child get?
- Do you think that Ella should get the same amount of money as Joly and Emmy?



Summary

To solve a word problem,

- Read carefully to identify what is being asked.
- Highlight key numbers and operations (e.g., total, difference, product, share equally).
- Identify the Operation Needed.
- Write the Math Expression.
- Perform the Calculation.
- Verify the Answer.

Example of Solved problem:

The number of adults in a certain district was 361,958. The number of children was 613,989 for the same district. Given that there is a total of 250,325 people that were not educated, How many people were educated in the District?

Solution:

The total population in that district is made up with the addition of adults and the children.

Hence, Total number of people = $(361,958 + 613,989) = 975,947$

To get the educated people, we subtract the number of those who were not educated; 250325.

Therefore, educated people are = $(361,958 + 613,989) - 250,325$

$= 975,947 - 250,325 = 725,622$ people were educated.



Application Activity 1.19

Read the word problem and find the answer:

- 1) A school bought 32 boxes of textbooks. Each box had 144 test books. On the same day that school was donated 24 cartons of textbooks; each carton had 288 books. How many textbooks did that school get on that day? Explain your answer.
- 2) Umutoni is a poultry farmer in a sector. On Monday her chickens laid 3600 eggs, on Tuesday 4200 eggs were laid and on Wednesday 4750 eggs were laid and on Thursday 4375 eggs were laid.
 - a) How many eggs were laid in four days?
 - b) If an egg costs 85 Frw, calculate how much money Umutoni got in four days.



End unit assessment

Assessing knowledge and understanding:

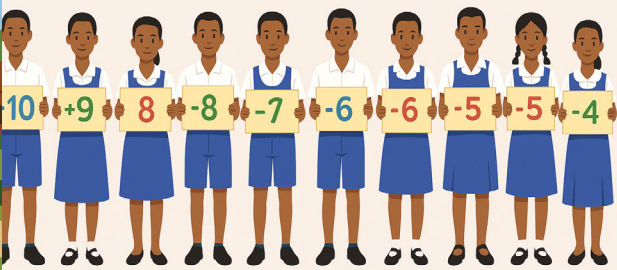
1. Write the following number in words: **382,640**.
2. Write the following number in figures: **Nine hundred seventy-seven thousand six hundred thirty-one**.
3. Use $<$, $=$, or $>$ to compare the numbers: **677,931** _____ **977,631**.

Assessing skills

4. Calculate the total of **524,321 Frw** and **374,329 Frw**.
5. A distributor delivered **99 cartons** of books to each school. How many books were distributed to **265 schools**?
6. Kalisa was sent to the shop with **10,000 Frw** to buy notebooks, assuming each notebook costs **500 Frw**.
 - a) How many notebooks should Kalisa buy?
 - b) At the shop, Kalisa found that each notebook costs 400 Frw. He bought the same number of notebooks as instructed but kept the remaining money. How much did he retain?

Assessing attitudes and Values:

- c) Do you think Kalisa acted responsibly by keeping the extra money without informing his parents? Explain your reasoning.
 - If you were in Kalisa's position, what would you do, and why?



UNIT 2

INTEGERS

Key unit Competence: You will be able to solve problems related to addition and subtraction of negative and positive integers.

Learning objectives:

By the end of this unit, you should be able to:

Knowledge and understanding:

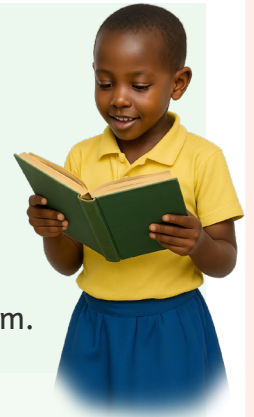
- Describe the concept of addition and subtraction of integers.
- Apply addition and subtraction of integers in solving real words problem.

Skills:

- Add and subtract integers using or without using a number line or counters.
- Calculate the additive inverse of any integer.
- Solve mathematical problems involving addition and subtraction of integers.

Attitudes and values:

- Appreciate the application of negative numbers in practical contexts.
- Appreciate the relationship between positives and negatives in terms of debits.



2.0 Introduction

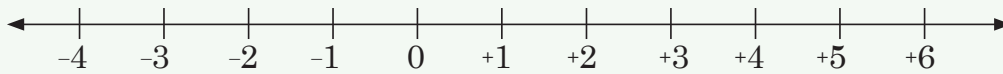
This unit aims to empower learners with the ability to confidently and accurately add and subtract integers in both academic and real-life contexts. Learners will develop critical thinking and numeracy skills as they explore patterns, relationships, and applications of integers.

Learners will connect integer operations to everyday situations such as calculating temperatures increase or decrease, paying loan to the bank and game scores.



Introductory Activity

- 1) On the soil, use a tape measure and draw a number line from -4 to $+6$.



Follow these steps.

- (i) Stand at -3 . Move 4 steps to the right. Where do you stop?

Can you accept that your position is the same as the sum $(-3) + (+4)$?

- (ii) Stand at -3 . Move 4 steps on the left. Where do you stop?

Can you accept that your position is the same as the sum $(-3) + (-4)$?

- 2) Now, think about the following situation: You have a debt of 30Frw, and your friend gives 40Frw. As you are a kind person, you immediately pay the debt. How much money do you keep?

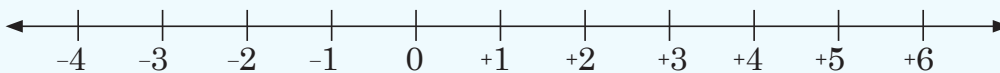
Is it possible to add negative or positive integers? Is this used in real life?

2.1 Addition of integers using a number line



Activity 2.1

On a paper, draw the number line.



Consider that moving right means increasing value (positive), while moving left means decreasing value (negative).

- a) Stand at $+4$. Move 3 steps on the left. Where do you stop?

What numbers does this represent?

Then, complete: $(+4) + (-3) = \underline{\hspace{2cm}}$

- b) Explain how you can use a number line to get $(+3) + (-4)$.



Summary

When adding integers using a number line,

- The first integer is where you start on the number line.
- Adding a positive means moving right.
- Adding a negative means moving left.

The sum of two negative integers will always be more negative.

Adding integers is applicable when adding temperature changes, financial situations (gains/losses), or vertical movements (above/below sea level).

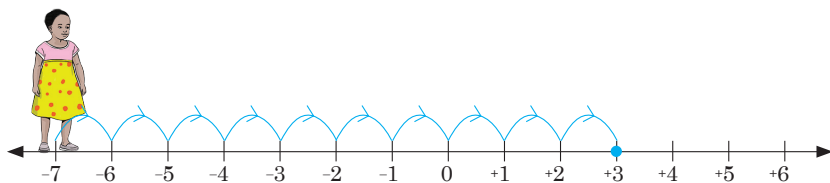
Example:

On a number line, work out $(-7) + (+10)$

Solution

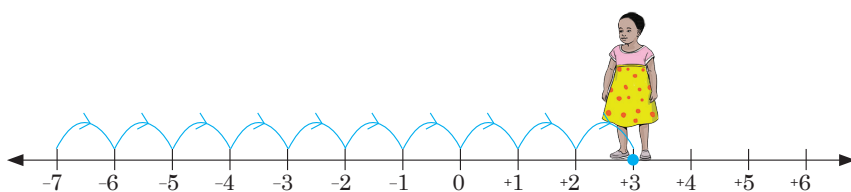
Stand at -7 . Move 10 steps to the right. Where do you stop?

Therefore, $(-7) + (+10) = +3$.



(Start at -7)

You stop at $+3$



(Stops at $+3$)



Application Activity 2.1

1) Using a number line, work out:

a. $(-1) + (+3)$

b. $(-9) + (+4)$

c. $(-10) + (+5)$

d. $(-2) + (-3)$

e. $(+3) + (-4)$

f. $(+10) + (-7)$

g. $(-15) + (-12)$

2) Instead of using a number line, add $(+5) + (-4)$ using counters. Explain how to do it.

You will consider the red side of the counter as positive (+) and the blue side of the counter as negative (-).

Positive



Negative

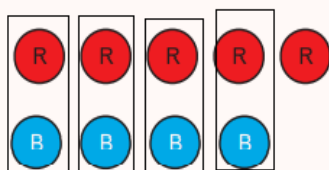


i) To add using counters, match the counters with red faces on top with those with blue faces on top.

Then, count the remained number of counters by noting their color and their corresponding number.

ii) When adding integers using counters you get the same answer as when adding using number line.

Now let us use counters to add $(+5) + (-4)$.



Match (pair) the counters with red faces on top with those with blue faces on top.
Complete $(+5) + (-4) =$

2.2 Addition of integers by calculation



Activity 2.2

1) Work out the following using a number line or counters

(a) $(-3) + (+4) = \underline{\quad}$ (b) $(+4) + (-3) = \underline{\quad}$ (c) $(-3) + (-4) = \underline{\quad}$

Discuss how you can find the answer without using a number line or counters.

2) Try your addition without using a number line.

(a) $(-3) + (+4) = \underline{\quad}$ (b) $(+3) + (-4) = \underline{\quad}$ (c) $(-3) + (-4) = \underline{\quad}$

3) From your working in number 1 and 2, which method was easier?

Discuss your steps in each case.



Summary

- When adding numbers with the same sign, the answer takes that sign.
- When adding numbers with different signs, the answer takes the sign of the number with the larger magnitude.

Example:

a) $(-3) + (-4) = -3 - 4 = -7$

b) $(+3) + (+4) = +3 + 4 = +7$

c) $(-3) + (+4) = +4 - 3 = +1$

d) $(+3) + (-4) = +3 - 4 = -1$



Application Activity 2.2

Work out the following:

1. $(-8) + (+5)$

2. $(-6) + (+2)$

3. $(-20) + (+16)$

4. $(-2) + (+10)$

5. $(-3) + (-3)$

6. $(-12) + (+6)$

7. $(-11) + (+10)$

8. $(+4) + (+4)$

9. $(-12) + (+1)$

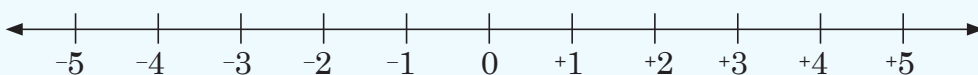
10. On Tuesday, the temperature at noon was $+20$ degrees. The temperature increased by 4°C in the next two hours. What was the final temperature?

2.3 Subtraction of integers using a number line



Activity 2.3

Make a number line from -5 to $+5$ on the field.



a) Start at -1 move 3 steps backwards (to the left), where do you stop?

Complete the expression of your movement: $(-1) - (+3) = \text{---}$

- b) Start at -1 , move 3 steps backward of backward. What does Backward of backward mean? Where do you stop?

Write the expression of the movement: $(-1) - (-3) = \text{---}$



Summary

When subtracting integers using a number line:

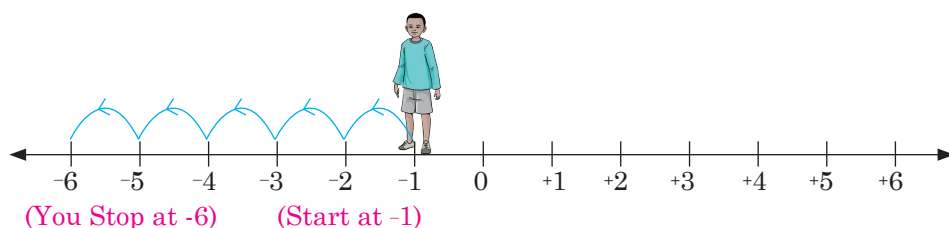
- Identify the First Number: This is your starting point.
- Move Left for Subtraction: Since you are subtracting, you will move to the left on the number line. The distance to move left will be equal to the second number.
- To subtract a positive integer: Move left.
- To subtract a negative integer: Move right.

Example:

Using a number line, work out $(-1) - (+5)$.

Solution

Stand at -1 . Move 5 steps to the left. Where do you stop?



Therefore $(-1) - (+5) = -6$



Application Activity 2.3

1) Work out the following using a number line.

a. $(+8) - (+3)$

b. $(-6) - (+2)$

c. $(-8) - (+3)$

d. $(+7) - (+9)$

e. $(+7) - (+3)$

f. $(+9) - (+9)$

g. In the morning, the temperature was -18°C . Within the next hour, the temperature dropped another 5°C . What was the temperature after the 2 hours?

2) Workout this activity using **counters**:

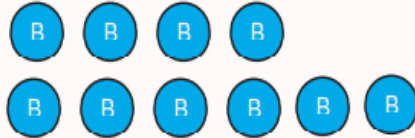
Proceed with the following steps: $(-4) - (-6) = ?$

Step 1: Consider the red side of the counter as positive (+) and the blue side of the counter as negative (-).

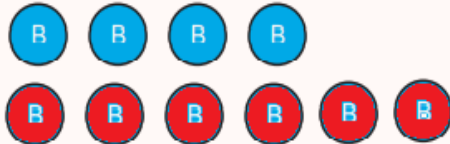
Step 2: For the first -4 , take four counters and place them in a row with blue faces facing up as illustrated below:



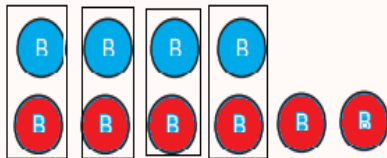
Step 3: Now take other six counters and place them in a second row with blue faces facing up.



Step 4: Now, keep the counters of the first row as they are and place the counters of the second row after inverting sides in which the blue faces on top become the red faces on top. (**Note:** If the two faces of the blue counter are blue, in this case, replace the second row by the red counters).



Step 5: Match red-faced counters with blue-faced counters. Count remaining unmatched counters along with their colours.



Question: How many counters are unmatched? What color are they?

Hence Deduce $(-4) - (-6) = \underline{\hspace{2cm}}$

2.4 Subtraction of integers using calculation



Activity 2.4

Given the expression $(-1) - (-3) = \underline{\hspace{2cm}}$

a) Find the answer using a number line

b) Consider that the opposite of the left side is the right side, we conclude that $-(-1) = +1$.

Use this rule to work out $(-1) - (-3)$ without using a number line.

Do you find the same answer as in (a)?



Summary

- To subtract an integer, you add its opposite.
- If integers to subtract have the same sign, add their values and keep the common sign.
- If the integers have different signs, subtract the smaller value from the larger value and take the sign of the integer with the larger value.

Example:

- a) $(-6) - (+3) = -6 - 3 = -9$
- b) $(+6) - (-3) = 6 + 3 = 9$
- c) Convert to addition: $7 + (-3) = 7 + (-3)$. Since the signs are different, subtract: $7 - 3 = +4$.
- d) $4 - (-2) = 4 + 2 = 6$.



Application Activity 2.4

Work out the following:

- | | | |
|--------------------|-------------------|-------------------|
| 1. $(+12) - (+10)$ | 2. $(-15) - (+5)$ | 3. $(-7) - (-3)$ |
| 4. $(-8) - (+2)$ | 5. $(+16) - (-3)$ | 6. $(-11) - (+6)$ |
| 7. $(+7) - (+13)$ | 8. $(-10) - (-8)$ | 9. $(+9) - (-4)$ |

2.5 Additive inverses (opposite) of numbers



Activity 2.5

Add the following integers.

1. $(-4) + (+4) =$ 2. $(-5) + (+5) =$ 3. $(-6) + (+6) =$

What do you notice when you add?

What is the integer to be added to -7 to give the same result as the one obtained here above?

What is the one to be added to $+8$?



Summary

- The **additive inverse** of a number is the value that, when added to the original number, gives a sum of zero. For example, the additive inverse of 5 is -5 because: $5 + (-5) = 0$
- Additive inverse of a number is also called the opposite of that number.

Example:

Find the additive inverse of: (i) -9 (ii) $+6$

Solution

Additive inverse of $(+a)$ is $(-a)$, so that $(+a) + (-a) = 0$. So we have:

- (i) additive inverse of -9 is $+9$.
- (ii) additive inverse of $+6$ is -6 .



Application Activity 2.5

Write the additive inverse of the following.

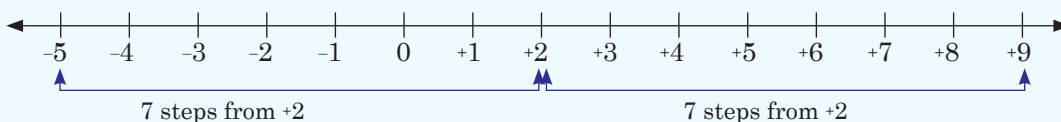
- | | | |
|----------|----------|----------|
| a) -3 | b) -10 | c) -11 |
| d) -14 | e) -15 | f) $+4$ |

2.6 Solving problems involving addition and subtraction of integers



Activity 2.6

Draw a number line to show integers.



Then read carefully and find me. “I am the integer; I am greater than 2 and I am 7 steps from $+2$ ”. Which integer am I?



Summary

When solving a word problem involving integers, here's a step-by-step approach:

1. Read the Problem Carefully: Understand what the problem is asking. Highlight or note important numbers and keywords (e.g., *increase*, *decrease*, *below zero*, *profit*, *loss*).
2. Identify the Integers Involved: Determine which numbers are positive or negative.

Positive integers represent gains, increases, above zero, etc. Negative integers represent losses, decreases, debts, below zero, etc.

3. Determine the operation needed.

Look for key words:

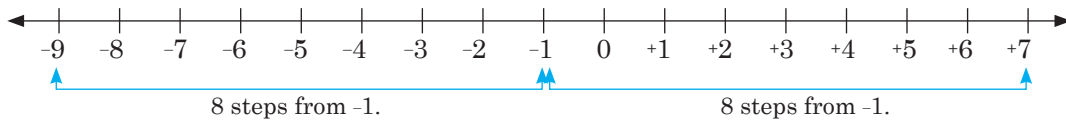
- Addition (+): *Combined*, *total*, *sum*, *gained*, *rose*
 - Subtraction (-): *Difference*, *decreased by*, *lost*, *withdrew*, *fell*, *dropped*.
4. Write the Mathematical Expression: Translate the words into numbers and symbols.
 5. Solve the Problem: Perform the calculation correctly, following integer rules.
 6. Write the answer.

Examples:

- 1) I am 8 steps from -1 . I am greater than $+6$. Where am I?

Solution

Draw a number line.



I am either at -9 or $+7$. But I am greater than $+6$. We know $-9 < +6$ and $+7 > +6$. Therefore, I am $+7$.

- 2) One day, the temperature in Kitabi was $+15$ degrees in the morning. In the evening its temperature is $+6$ degrees. Did the temperature in Kitabi increase or reduce on that day? What was the difference of the temperature in Kitabi?

Solution:

The temperature in Kitabi **decreased** from the morning to the evening.

Morning Temperature: $+15$ degrees

Evening Temperature: $+6$ degrees

Change in Temperature:

Difference = Morning Temperature - Evening Temperature

Difference = $15 - 6 = 9$ degrees

Conclusion: The temperature in Kitabi **reduced by 9 degrees** from the morning to the evening.

- 3) The highest temperature recorded on earth is 56°C .

The lowest temperature recorded on earth is -89°C . Find the difference between the temperatures.

Solution:

High: $+56$ degrees

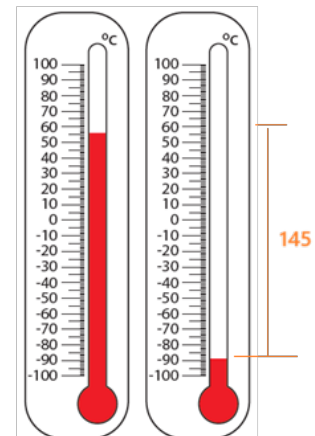
Low: -89 degrees

Difference = ?

Difference = high temperature - low temperature

= $+56$ degrees - $(-89$ degrees) = $+56$ degrees + 89 degrees

= 145 degrees.



The difference is 145°C .



Application Activity 2.6

- 1) I am a positive number. Exactly 11 steps from 0. What am I?
- 2) Mary is 17 steps from $+7$. She is next to -9 . Where is she?
- 3) I am a negative number. Exactly 9 steps from -1 . Where am I?
- 4) A positive number is 8 steps away from -3 . It is greater than 4. What is the number?
- 5) A number is 15 steps from $+10$. The number is less than -4 . What is the number?



End unit assessment

Assessing knowledge and understanding

1. Multiple Choice Questions

- a) What is the result of $(+9) - (+4)$?
- A) 13
B) -5
C) +5
- b) The **additive inverse** of +7 is:
- A) +7
B) -7
C) +14
- c) The expression $(-10) + (+4)$ is the same as:
- A) Adding 4
B) Subtracting 6
C) Adding 14
- d) Which of the following statements is **true**?
- A) The sum of two positive integers is always positive.
B) The sum of a negative and positive integer is always positive.
C) The sum of two negative integers is positive.
D) The additive inverse of a number is always positive.

Assessing skills

2. Solve the following problems:

- a) Calculate: $(-7) + (+10) = \underline{\hspace{2cm}}$
- b) Solve: $(+6) - (-4) = \underline{\hspace{2cm}}$
- c) Find the result of: $(-8) + (-5) = \underline{\hspace{2cm}}$
- d) What is: $(+12) - (+15) = \underline{\hspace{2cm}}$
- e) If the temperature was $+10^{\circ}\text{C}$ and dropped by 7°C , the final temperature is $\underline{\hspace{2cm}}$.

3. Word Problems

- a) A mountain climber starts at +300 meters above sea level. He then descends 450 meters. What is his final elevation?
- b) Jackline is 10 steps away from +3 on a number line. She is standing on a negative number less than -6. Where is she standing? Justify your answer.

Assessing attitudes and Values

4. Do you think addition and subtraction of integers are useful in **managing money**? Give an example to support your answer.



Prime factorization of 40 = $2 \times 2 \times 2 \times 5$
 = $2^3 \times 5$

UNIT 3

PRIME FACTORIZATION AND DIVISIBILITY TESTS

Key Unit competence: You will be able to prime factorize, exploit the rule of divisibility tests for numbers less than 10, find the Lowest Common Multiple (LCM) and the Greatest Common Factor (GCF) of whole numbers.

Learning activities:

By the end of this unit, you should be able to:

Knowledge and understanding:

- Define prime numbers.
- Explain LCM and GCF.
- Write the rule of divisibility tests of numbers.
- Explain the concept of indices (powers) as shorthand for repeated factors



Skills:

- Factor numbers using prime factors.
- Calculate the LCM and GCF of numbers.
- Solve problems involving LCM in daily life situations.
- Calculate and show the rule of divisibility tests.

Attitudes and values:

- Appreciate the importance of LCM in daily life situations.
- Be confident and accurate when carrying out different calculations.

3.0 Introduction

This unit is designed to develop learners' critical thinking, logical reasoning, and problem-solving abilities as they explore the concepts of prime numbers, factors, and divisibility rules. Learners will investigate how numbers are built from prime factors and how divisibility tests can help in simplifying mathematical tasks, such as identifying common factors, simplifying fractions, or solving word problems.

In everyday life, divisibility tests help with quick mental math like splitting bills evenly or checking if a number is divisible for scheduling tasks. Even inventory management relies

on factorization to group products efficiently. These mathematical tools simplify complex problems by breaking them into manageable parts, proving essential in technology, finance, and logistics.



Introductory Activity

In one country, lower primary lesson lasts 30 minutes, and upper primary lesson lasts 40 minutes. The bells for lower and upper primary ring at the same time in the morning at 8:40 AM.

1) a) Discuss and complete the list to show the time at which the bell rings:

i) The bell for lower primary rings after: 30, 60, 90, ...

ii) The bell for Upper primary rings after: 40, 80, 120, ...

2) How many minutes will pass before the two bells ring together again?

3) What time will it be when the two bells ring together again?

Is there another way of finding the minutes that pass before the two bells ring together?

3.1 Prime factorization of numbers and its uniqueness



Activity 3.1

Consider the following number: 60

Find the factors for 60.

Which ones are prime numbers?

Write 60 as a product of its prime factors.



Summary

A prime number is a number that has only two different factors. That is 1 and itself.

Some examples of prime numbers are 2, 3, 5, 7, 11 and 13.

Prime factorization means to write a number using its prime factors.

Examples:

Prime factorize: a) 40 b) 30

Solution

Prime factorize 40				Prime factorize 30	
Prime Factors	Number			Prime Factors	Number
2	40			2	30
2	20			3	15
2	10			5	5
5	5				1
1	1				
$40 = 2 \times 2 \times 2 \times 5$				$30 = 2 \times 3 \times 5$	



Application Activity 3.1

Prime factorize the following numbers. Explain your answer.

a) 40 b) 120 c) 170 d) 80 e) 200 f) 320 g) 540 h) 670.

3.2 Prime factorizing using indices as shorthand for repeated factors



Activity 3.2

Given that $3 \times 3 \times 3 \times 3 = 3^4$, 4 is an index or power of base 3, use indices (or powers) to show repeated prime factors when prime factorizing the following numbers:

- Prime factorize 120. Then, use powers to show repeated prime factors.
- Prime factorize 280. Then, use powers to show repeated prime factors

Explain what you have noticed.



Summary

We can express numbers as products of prime factors.

We can use powers or indices on repeated prime factors.

The number 2 to the power of 3 means $2 \times 2 \times 2 = 2^3$. We can prime factorize a number by writing the product using indices or powers.

Example: Use powers to prime factorize:

- a) 68 b) 16 c) 60

Solution:

(a) 68			(b) 16		
Prime Factors	Number	$68 = 2 \times 2 \times 17 = 2^2 \times 17$ (This is because is such that 2 is repeated two times)	Prime Factors	Number	$16 = 2 \times 2 \times 2 \times 2 = 2^4$ (This is because 2 is repeated 4 times). We read this as 2 to the power of 4 or 2 raised to 4
2	68		2	16	
2	34		2	8	
17	17		2	4	
	1		2	2	

c) Prime factorize 60. Show the prime factors using indices.

Prime Factors	Number	$60 = 2 \times 2 \times 3 \times 5$ $60 = 2^2 \times 3 \times 5$ 2^2 is 2 to the power 2 or 2×2 (2 two times)
2	60	
2	30	
3	15	
5	5	
	1	

Therefore $60 = 2^2 \times 3 \times 5$ has been written using factors in powers/indices.



Application Activity 3.2

Prime factorize the following. Express prime factors in index form and explain.

(a) 27

(b) 75

(c) 36

(d) 76

(e) 98

(f) 48

3.3 Calculation of the Lowest Common Multiple (LCM)



Activity 3.3

- 1) What is the multiple of a number?
- 2) Find the multiples of each of the following numbers: **3, 9 and 12.**
- 3) Indicate the common multiples of these numbers: **3, 9 and 12.**
- 4) Chose the smallest common multiple (LCM) of them.
- 5) If Sam waters plants every 4 days and Ada every 6 days, at which first day will they both water plants together?



Summary

To find LCM of numbers:

- Find the multiples of each of the numbers.
- Indicate the common multiples of these numbers
- Select the smallest common multiple. This will be the LCM of those numbers.

We can also find the LCM of numbers using prime factorization. Example Find the LCM of 12 and 18.

Step 1: Prime Factorize Each Number

$$12 = 2 \times 2 \times 3 = 2^2 \times 3^1$$

$$18 = 2 \times 3 \times 3 = 2^1 \times 3^2$$

Step 2: List all prime factors with the highest exponents: For each prime number, take the **highest exponent** present in any factorization. Example: 2^2 (from 12) and 3^2 (from 18).

Step 3: Multiply the selected factors: Multiply the highest powers of all primes to get the LCM.

$$\text{LCM}(12; 18) = 2^2 \times 3^2 = 4 \times 9 = 36.$$

Examples:

1) Find the LCM of 2, 4 and 8.

Solution:

The multiples of 2 include: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28

The multiples of 4 include: 4, 8, 12, 16, 20, 24, 28

The multiples of 8 include: 8, 16, 24

Common multiples: (8, 16, 24).

The smallest common multiple (LCM) is: 8.

2) What is the Lowest Common Multiple of 6, 12 and 18?

Solution

Prime Factors	Numbers			
2	6	12	18	Steps
2	3	6	9	Divide 6, 12 and 18 by prime numbers. Start with the smallest till all have been reduced to 1.
3	3	3	9	
3	1	1	3	Multiply the prime numbers $2 \times 2 \times 3 \times 3 = 36$
	1	1	1	The product is the LCM

Therefore, Lowest Common Multiple of 6, 12 and 18 is 36.

(a) What is the Lowest Common Multiple of 4, 6 and 8?

Solution

In index form:

$$4 = 2 \times 2 = 2^2$$

$$6 = 2 \times 3$$

$$8 = 2 \times 2 \times 2 = 2^3. \quad 2^3 \text{ has the greatest exponent } 3.$$

The LCM of 4, 6, 8 is $2^3 \times 3 = 8 \times 3 = 24$.



Application Activity 3.3

- Find the LCM of the following numbers using multiples
(a) 2, 5 and 10 (b) 5, 6 and 9 (c) 2, 6 and 8
- Find the LCM of the numbers below. Explain the steps to your answer.
(a) 4, 5 and 12 (b) 4, 6 and 9 (c) 6, 15 and 10
- Three traffic lights flash after **30 seconds, 45 seconds, and 60 seconds** respectively. When will they flash at the same time?

3.4 Calculation of Greatest Common Factors (GCF)



Activity 3.4

- Find the factors for each number: 18, 12, and 24.
Find the common factors. Which common factor is the greatest among them?
- Prime factorize each of the following numbers: 18, 12 and 24.
 - Identify the Common Prime Factors
 - Take the Smallest Exponent for Each Common Prime factor
 - Multiply the Selected Primes Together. What number do you find?What can you say about the answers found in the two different methods?



Summary

To find the GCF of numbers;

- Prime factorize each of the numbers,
- Identify the Common Prime Factors,
- Take the Smallest Exponent for Each Common Prime factor.
- Multiply the Selected Primes Together

Example

- Find the GCF of 24 and 36

$$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3^1$$

$$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$$

Common primes are: 2 and 3

Take the **Smallest Exponent** for Each Common Prime

For 2: 24 has 2^3 , 36 has 2^2 . We take 2^2 (smaller exponent)

For 3: 24 has 3^1 , 36 has 3^2 . We take 3^1 (smaller exponent)

Multiply the Selected Primes Together: $GCF = 2^2 \times 3^1 = 4 \times 3 = 12$

2) Find the Greatest Common Factor (GCF) of 28, 42 and 56.

Solution

Method 1				Method 2
Start dividing by the smallest prime number that divides all the numbers.				Express 28, 42 and 56 in indices forms: $28 = 2 \times 2 \times 7 = 2^2 \times 7$ $42 = 2 \times 3 \times 7$ $56 = 2 \times 2 \times 2 \times 7 = 2^3 \times 7$ Observation on common factors in indices: $2, 2^2, 2^3$ and 7 are common. 3 is not common. So, GCF is $2 \times 7 = 14$. We use common factors with lowest indices.
Prime factors	Numbers			
2	28	42	56	
7	14	21	28	
	2	3	4	
Therefore, GCF is $2 \times 7 = 14$.				



Application Activity 3.4

- 1) Calculate the GCF of the following. Discuss your steps.
(a) 14, 20 and 36 (b) 24, 36 and 40 (c) 72, 84 and 108 (d) 84, 140 and 224
- 2) **Simplifying Fractions:** Mukama gave $\frac{12}{30}$ cows to his daughter. Simplify the fraction to deal with small numbers.
- 3) **Dividing Items into Equal Groups:** A teacher has **24 pencils** and **36 notebooks**. She wants to create identical prize packs for students with no leftovers. How many packs will the teacher make?
- 4) **Tiling or Flooring Layouts:** A homeowner wants to tile a **24m × 60m** bathroom floor with square tiles (no cuts). What's the largest tile size possible?

3.5 Divisibility test for 2



Activity 3.5

Divide the following numbers by 2.

(a) 3 241 (b) 573 428 (c) 361 800 (d) 520 042

- Which numbers are divisible by 2? Check their last digits. What do you notice?
- Which numbers are not divisible by 2? Check their last digits. What do you notice?
- What can you say about the last digit of the numbers divisible by 2? Present your findings.



Summary

A number is divisible by 2 if the last digit is an even number or zero. The last digits are: 0, 2, 4, 6, or 8.

Example

1. Is 49 140 divisible by 2?

Solution

The last digit in 49 140 is 0.

Therefore, the number 49 140 is divisible by 2.

2. Test if the following are divisible by 2.

(a) 90 712

(b) 90 721

Solution

a) The last digit 2 in 90 712 is an even number. Therefore, the number 90 712 is divisible by 2.

b) The last digit 1 is an odd even number. Therefore, 90 721 is not divisible by 2.



Application Activity 3.5

1. Which of the following numbers are divisible by 2?

(a) 4 480

(b) 6 429

(c) 5 258

(d) 21 224

(e) 49 242

(f) 15 504

(g) 470 881

(h) 636 027

(i) 36 085

2. Test and write numbers divisible by 2. Discuss how you found your answers.

(a) 52 100

(b) 148 516

(c) 462 946

(d) 90 712

(e) 54 213

(f) 41 768

(g) 87 742

(h) 49 112

3.6 Divisibility test for 3



Activity 3.5

Divide the following numbers by 3.

(a) 39

(b) 214

(c) 171

(d) 8 811

Find the sum of the digits of each number above. Divide the sum for each number by 3. What do you discover? Present your findings.



Summary

A number is divisible by 3 if the sum of its digits is a multiple of 3.

Example

- (a) Test if 1 824 is divisible by 3?
(b) Test if 23 416 is divisible by 3.

Solution

- (a) Add the digits for the number 1 824.

$$1 + 8 + 2 + 4 = 15.$$

Now, $15 \div 3 = 5$. So, 15 is divisible by 3.

Therefore, 1 824 is divisible by 3.

- (b) Add the digits for 23 416. We have: $2 + 3 + 4 + 1 + 6 = 16$.

Now $16 \div 3 = 5$ with remainder of 1.

So, 16 is not divisible by 3.

Therefore, 23 416 is not divisible by 3.



Application Activity 3.6

Test and give the numbers that are divisible by 3. Explain the steps to your answers.

- | | | | | |
|------------|------------|-------------|-------------|------------|
| (a) 1 836 | (b) 5 613 | (c) 9 786 | (d) 6 123 | (e) 56004 |
| (f) 23 112 | (g) 62 172 | (h) 456 312 | (i) 214 701 | (k) 306171 |

3.7 Divisibility test for 4



Activity 3.7

- Divide the following numbers by 4.

(a) 2 472

(b) 2 814

(c) 17 936

Which of them are divisible by 4?

- Test whether the last 2 digits of each number is divisible by 4 or not. What do you notice? Present your findings.



Summary

A number is divisible by 4 if the number of its last 2 digits is divisible by 4.

Example

- a) Is 456 312 divisible by 4?
b) Is 106 526 divisible by 4?

Solution

- a) The last 2 digits of 456 312 form 12. Now, $12 \div 4 = 3$. So, 12 is divisible by 4.
Therefore, 456 312 is divisible by 4.

- b) The last 2 digits of 106 526 form 26. Now, $26 \div 4 = 6$ with remainder of 2. So, 26 is not divisible by 4.



Application Activity 3.7

Test for numbers divisible by 4. Discuss your steps.

- | | | | |
|-------------|-------------|-------------|-------------|
| (a) 3 148 | (b) 98 541 | (c) 83 710 | (d) 182 510 |
| (e) 426 940 | (f) 201 084 | (g) 390 712 | (h) 873 008 |

3.8 Divisibility test for 5



Activity 3.8

Divide the following numbers by 5

- | | | | |
|------------|----------|------------|------------|
| (a) 99 000 | b) 27 43 | (c) 47 861 | (d) 78 390 |
|------------|----------|------------|------------|

Which numbers are divisible by 5? Check their last digit.

Which numbers are not divisible by 5? Check their last digit. What do you notice about the last digit of numbers divisible by 5?



Summary

A number is divisible by 5 if its last digit is 0 or 5.

Example

Which of the following numbers is divisible by 5?

- | | | |
|------------|-------------|-------------|
| (a) 56 480 | (b) 225 445 | (c) 741 024 |
|------------|-------------|-------------|

Solution

(a) 56 480 has the last digit 0. Therefore, 56 480 is divisible by 5.

(b) 225 445 has the last digit 5. Therefore, 225 445 is divisible by 5.

(c) 741 024 has the last digit 4. Therefore, 741 024 is not divisible by 5.



Application Activity 3.8

Test for numbers divisible by 5. Explain your steps.

- | | | | | |
|-------------|-------------|-------------|-------------|-------------|
| (a) 736 920 | (b) 878 945 | (c) 572 315 | (d) 640 635 | (e) 670 670 |
| (f) 654 285 | (g) 563 759 | (h) 410 458 | (i) 369 000 | (j) 225 445 |

3.9 Divisibility test for 6



Activity 3.9

Look at the numbers below: 2760 and 990

Divide the numbers by 2.

Divide the numbers by 3 again. Divide the same numbers by 6.

What do you notice about the numbers divisible by 6?



Summary

A number is divisible by 6 if it is also divisible by 2 and 3.

Example

Which of the numbers below is divisible by 6? Explain your steps.

(a) 2 700 (b) 458 716

Solution

The last digit for 2 700 is 0. So, 2 700 is divisible by 2.

- $2 + 7 + 0 + 0 = 9$. The sum of the digits of 2 700 is 9. So, 9 is divisible by 3. Therefore, 2 700 is divisible by 3.
- Finally, 2 700 is divisible by 6.

(b) The last digit of 458 716 is 6. Now, 6 is an even number. Thus, 458 716 divisible by 2.

- $4 + 5 + 8 + 7 + 1 + 6 = 31$. The sum of the digits of 458 716 is 31.
Now, $31 \div 3 = 10$ remainder 1, or 31 is not divisible by 3. Thus, 458 716 is not divisible by 3.
- Finally, 458 716 is not divisible by 6.



Application Activity 3.9

Test to find numbers divisible by 6.

(a) 458 710 (b) 51 200 (c) 216 (d) 144 (e) 928
(f) 93 621 (g) 3 759 (h) 48 780 (i) 56 800 (i) 278

3.10 Divisibility test for 8



Activity 3.10

Divide the numbers below by 8.

(a) 5 328 (b) 17 428 (c) 93 640

Which one are divisible by 8? Now form a number from the last three digits of each number. Divide the obtained number by 8. What do you notice? Explain your observations.



Summary

A number is divisible by 8 if the last three digits form a number divisible by 8, or are 000.

Example

Investigate for the numbers that are divisible by 8

(a) 404 320

(b) 200 072

(c) 323 638

Solution

Check if the number formed by the last 3 digits is divisible by 8.

- a) From 404 320, the last digits form 320. Now $320 \div 8 = 40$. Since 320 is divisible by 8, thus 404 320 is divisible by 8.
- b) From 202 072, the last 3 digits form 072. Now $072 \div 8 = 9$. Since 072 is divisible by 8, thus 202 072 is divisible by 8.
- c) From 323 638, the last 3 digits form 638. Now $638 \div 8 = 79$ with remainder of 6, is not divisible by 8. Thus, 323 638 is not divisible by 8.



Application Activity 3.10

Test and write the numbers that are divisible by 8. Explain your steps.

(a) 679 168

(b) 217 800

(c) 436 756

(d) 842 056 (e) 300 400

(f) 374 912

(g) 276 480

(h) 248 263

(i) 322 642 (j) 543 120

3.11 Divisibility Test for 9



Activity 3.11

- Divide these numbers by 9.
(a) 8 109 (b) 2 916 (c) 20 007 (d) 108 450
- Which ones are divisible by 9?
- Add the digits of each number given above. Then, divide the sum of the digits by 9. Are they all divisible by 9? What do you notice about numbers divisible by 9? Present your findings.



Summary

A number is divisible by 9 if the sum of its digits forms a number divisible by 9.

Example

Which of the following numbers is divisible by 9?

(a) 64 737

(b) 607 131

(c) 128 000

Solution

Step 1: Add the digits of the numbers.

(a) 64 737: $6 + 4 + 7 + 3 + 7 = 27$

(b) 607 131: $6 + 0 + 7 + 1 + 3 + 1 = 18$

(c) 128 000: $1 + 2 + 8 + 0 + 0 + 0 = 11$

Step 2: Divide the sum by 9. State which numbers are divisible by 9.

(a) $27 \div 9 = 3$. Therefore 64 737 is divisible by 9.

(b) $18 \div 9 = 2$. Therefore 607 131 is divisible by 9.

(c) $11 \div 9 = 1$ with remainder of 2.

Therefore 128 000 is not divisible by 9.



Application Activity 3.11

Test and write the numbers divisible by 9. Discuss your answer.

(a) 713 610

(b) 819 234

(c) 999 045

(d) 515 230

(e) 304 133

(f) 530 280

3.12 Divisibility test for 10



Activity 3.12

Divide the following numbers by 10.

(a) 8 730

(b) 6 940

(c) 5 285

(d) 94 000

(e) 20 184

Which numbers are divisible by 10? Which numbers are not divisible by 10?

Check the numbers that are not divisible by 10 again? What are their last digits?
Discuss your observations.



Summary

A number is divisible by 10 if it ends with 0.

Example 3.12

Which of the following numbers are divisible by 10?

(a) 49 140 (b) 199 000 (c) 447 861 (d) 872 930

Solution

The numbers with a last digit of 0 are:

(a) 49 140 (b) 199 000 and (d) 872 930

Therefore 49 140, 199 000, 872 930 are divisible by 10.

(c) 447 861 is not divisible by 10. It ends with 1.



Application Activity 3.12

1. Test which numbers are divisible by 10. Discuss and present your findings.

- (a) 874 930 (b) 582 140 (c) 529 900 (d) 81 420 (e) 793 00
(f) 405 330 (g) 555 355 (h) 725 660 (i) 554 740 (j) 457834

2. List five numbers less than 70 that are divisible by 10.



End unit assessment

Assessing Knowledge

1. Define the following terms:

- a) Prime number
- b) Prime factorisation
- c) Lowest Common Multiple (LCM)
- d) Greatest Common Factor (GCF)

2. List the first five prime numbers.

3. State the divisibility rule for:

- a) 4
- b) 6
- c) 9
- d) 10

Assessing Skills

4. a) Express 240 as a product of its prime factors.

b) Write 180 using indices (powers) for repeated factors.

5. LCM and GCF:

- a) Find the LCM of 6, 9 and 12.
- b) Find the GCF of 48, 40 and 72.

6. Divisibility Tests:

Check if the following numbers are divisible by the given numbers.

a) Name the numbers divisible by 2. Explain your answers.

- (i) 467 181 (ii) 891 484 (iii) 541 656

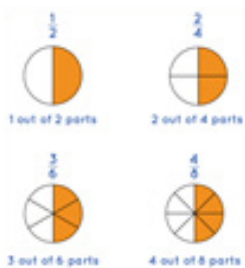
b) Which of the following numbers are divisible by 6? Discuss your steps.

- (i) 891 480 (ii) 556 680 (iii) 497 185

7. A traffic light changes every 30 seconds, 45 seconds, and 60 seconds for three different roads. After how many minutes will all three traffic lights change at the same time?
8. A teacher has 36 pencils, 54 erasers, and 72 rulers. She wants to distribute them equally among students with no leftovers. What is the greatest number of students she can give equal supplies to?

Assessing attitudes and Values

9. Read and answer
 - a) Why is learning about prime factorization and divisibility important in daily life?
 - b) Give an example of how teamwork helped you understand a concept better in this unit.



UNIT 4

EQUIVALENT FRACTIONS AND OPERATIONS

Key unit competence: You will be able to calculate, add and subtract equivalent fractions and apply this knowledge to solve related problems



Learning activities:

By the end of this unit, you should be able to:

Knowledge and understanding:

- Explain the concept of equivalent fractions.
- Explain how to find equivalent fractions.
- Give examples of equivalent fractions.
- Explain how to add or subtract fractions with different denominators using equivalent fractions and the Lowest Common Multiple (LCM).
- Describe the process of solving simple problems involving fractions.

Skills:

- Shade equivalent fractions.
- Find the fractions that are equivalent to a given fraction.
- Use the LCM of denominators to add or subtract fractions.
- Solve word problems involving the addition and subtraction of fractions.

Attitudes and values:

- Be confident and accurate when finding equivalent fractions.
- Develop personal confidence in the use of fractions in real life.
- Appreciate the importance of accuracy when working out equivalent fractions, adding and subtracting fractions.

4.0 Introduction

Equivalent fractions are different fractions that represent the same value or proportion, even though they may look different. They are obtained by multiplying or dividing both the numerator (top number) and denominator (bottom number) by the same non-zero integer.

In this unit, you will understand, compare, and perform operations with equivalent fractions in both mathematical and real-life contexts. Through active participation in practical tasks such as using visual models, measuring ingredients, and sharing items equally, you will explore the meaning of equivalent fractions and how they are used in daily life.



Introductory Activity

Consider the sugar cane divided in 8 equal parts.



a) show or shade one quarter of that sugar cane.

b) Show or shade one eighth of that sugar cane,

Then complete the following:

i) The quarter of the sugar cane is equal to ____ eighths of the whole.

Therefore, $\frac{1}{4} = \frac{\text{---}}{8}$

ii) When the two parts are put together, they make ____ eighths of the whole.

Therefore, $\frac{1}{4} + \frac{1}{8} = \frac{\text{---}}{8}$

4.1 Exploring and comparing fractions using models



Activity 4.1

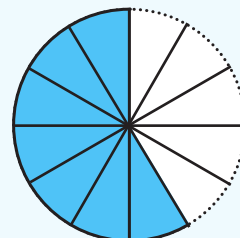
Take circular set divided into 12 equal parts. Take out 7 parts from it.

(a) What fraction does it represent?

(b) What fraction does the remained part represent?

(c) Which part is bigger than the other?

(d) Now, use > or < to compare the two fractions obtained in (a) and (b).



Summary

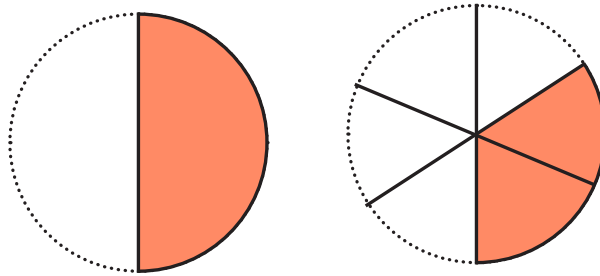
- After using visual models (like fraction strips, circles, or area models), look at the lengths or shaded areas.
- If the model of one fraction is longer or covers more area than another, that fraction is **greater**.
- If it is shorter or covers less area, that fraction is **smaller**.
- A fraction with larger denominator means the whole is divided into more pieces, making each piece smaller.

Examples:

- 1) Take out two circular sheets of the same size, one divided into two parts and another six parts.

Take out fractions $\frac{1}{2}$ and $\frac{2}{6}$ from the circular sheets respectively and compare them by placing one on another.

Which one is bigger? Which one is smaller? Try to use the comparison signs to compare fractions they represent.

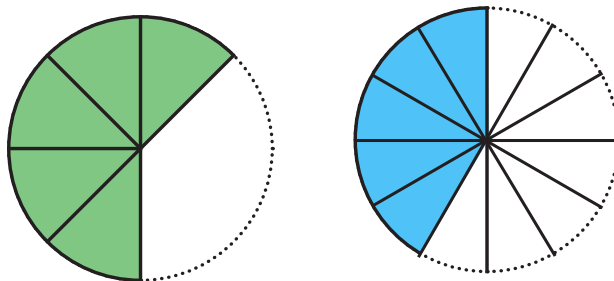


The shaded part of $\frac{1}{2}$ piece is bigger than the shaded part of $\frac{2}{6}$, therefore, $\frac{1}{2} > \frac{2}{6}$.

- 2) Take out other two circular sheets, one divided into eight parts and another twelve parts.

Take out fractions $\frac{5}{8}$ and $\frac{5}{12}$ from the circular sheets respectively and compare them by placing one on another.

Which one is bigger? Which one is smaller?



The shaded part of $\frac{5}{8}$ is bigger than the shaded part of $\frac{5}{12}$. Therefore $\frac{5}{8} > \frac{5}{12}$.



Application Activity 4.1

- 1) Use rectangular model or the circle set fractions to represent $\frac{1}{4}$ and $\frac{3}{8}$.

Then, compare the 2 fractions.

- 2) Observe the following shaded regions in model A and Model B of fractions made from a same whole object. What fraction is shaded in A, what fraction is shaded in B? Then, compare the two fractions. Which one is greater?



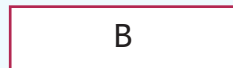
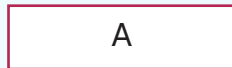
4.2 Meaning of equivalent fractions



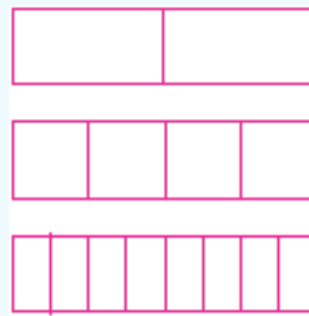
Activity 4.1

Cut three strips of paper as shown below.

They are in the same sizes



- i) Fold paper A into two equal parts.
 - ii) Fold paper B into four equal parts.
 - iii) Fold paper C into eight equal parts.
- Open them and draw lines along the folds.



iv) Shade $\frac{1}{2}$ in A.

v) Shade $\frac{2}{4}$ in B.

vi) Shade $\frac{4}{8}$ in C.

vii) Compare the sizes of shaded parts in A, B and C.

viii) What do you notice? Present your findings.

From the previous activity, we find that $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$



Summary

- When we shade the equal parts of the same whole, the different fractions they represent are called equivalent fractions.
- **Equivalent fractions** are fractions that represent the same value or proportion of a whole, even though they may have different numerators and denominators.

Examples:

1) Shade $\frac{1}{3}$ and $\frac{2}{6}$ on paper cutouts of the same size. Compare $\frac{1}{3}$ and $\frac{2}{6}$.

Solution



The shaded part of $\frac{1}{3}$ is equal to the shaded part of $\frac{2}{6}$.

2) Shade $\frac{3}{4}$, $\frac{6}{8}$ and $\frac{12}{16}$ on same size of paper cutouts. Compare the size of shaded parts.

What do you notice?

Solution



The shaded parts are equal sizes.

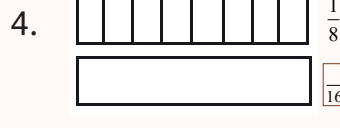
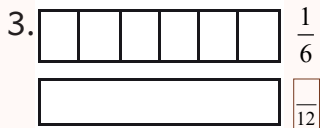
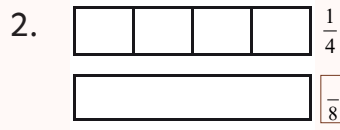
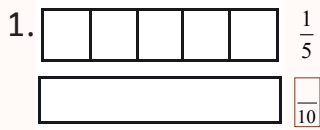
• Thus, $\frac{3}{4}$, $\frac{6}{8}$ and $\frac{12}{16}$ are equivalent fractions.

$$\frac{3}{4} = \frac{6}{8} = \frac{12}{16}$$

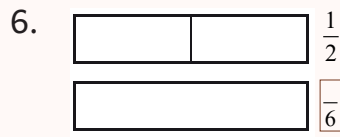
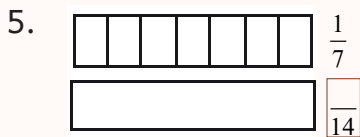


Application Activity 4.2

Shade and write the fraction equivalent to the fraction below:



Shade the equivalent fractions. Explain your steps.

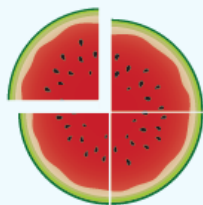
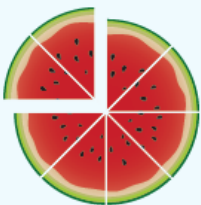


4.3 Finding equivalent fractions

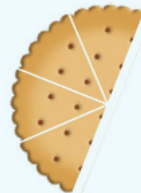


Activity 4.3

1) Observe and Complete:



$$\frac{2}{8} = \frac{?}{4}$$



$$\frac{1}{2} = \frac{?}{4}$$

2) Multiply $\frac{2}{3}$ by $\frac{3}{3}$ as $\frac{2}{3} \times \frac{3}{3} = \frac{2 \times 3}{3 \times 3} =$

What fraction do you get?

Use the same size paper cutouts to shade.



Compare $\frac{2}{3}$ and $\frac{6}{9}$. Are they equivalent or not?

Repeat the same steps to find equivalent fractions of: (a) $\frac{4}{5}$ (b) $\frac{5}{6}$

Where do we use equivalent fractions in daily life?



Summary

To find the equivalent fraction of a given fraction,

- Multiply numerator and denominator by a same whole number.
- A whole number to use include 2, 3, 4, 5, etc. If you multiply by 1, you get the same fraction.

Example:

- 1) Write 3 equivalent fractions of $\frac{3}{7}$.
- 2) Find two equivalent fractions for $\frac{4}{11}$
- 3) Find the equivalent fractions of $\frac{1}{2}; \frac{1}{5}; \frac{3}{10}$ with the denominator 20.

Solution

- 1) Three equivalent fractions of $\frac{3}{7}$. We can get them by multiplying by whole numbers 2, 3 and 4. We have:

The first fraction: $\frac{3}{7} \times \frac{2}{2} = \frac{6}{14}$. The second fraction: $\frac{3}{7} \times \frac{3}{3} = \frac{9}{21}$, The third fraction: $\frac{3}{7} \times \frac{4}{4} = \frac{12}{28}$.

- 2) Two equivalent fractions for $\frac{4}{11}$:

The first fraction: $\frac{4}{11} \times \frac{2}{2} = \frac{8}{22}$. The second fraction: $\frac{4}{11} \times \frac{3}{3} = \frac{12}{33}$.

- 3) Equivalent fractions of $\frac{1}{2}; \frac{1}{5}; \frac{3}{10}$ with the denominator 20

- To find equivalent fraction of $\frac{1}{2}$ with the denominator 20, we find the whole number that was multiplied by the denominator 2 to get the denominator 20. This is $20 \div 2 = 10$.

So, the equivalent fraction of $\frac{1}{2}$ with the denominator 20 is $\frac{1}{2} \times \frac{10}{10} = \frac{10}{20}$

- To find equivalent fraction of $\frac{1}{5}$ with the denominator 20, we find the whole number that was multiplied by the denominator 5 to get the denominator 20. This is $20 \div 5 = 4$.

So, the equivalent fraction of $\frac{1}{5}$ with the denominator 20 is $\frac{1}{5} \times \frac{4}{4} = \frac{4}{20}$.

- To find equivalent fraction of $\frac{3}{10}$ with the denominator 20, we find the whole number that was multiplied by the denominator 10 to get the denominator 20. This is $20 \div 10 = 2$.

So, the equivalent fraction of $\frac{3}{10}$ with the denominator 20 is $\frac{3}{10} \times \frac{2}{2} = \frac{6}{20}$.



Application Activity 4.3

I) Find the equivalent fractions of the given fraction. Then fill in the missing blanks and explain your answer.

1. $\frac{1}{4} = \frac{\square}{52}$

2. $\frac{2}{3} = \frac{\square}{24}$

3. $\frac{3}{10} = \frac{\square}{50}$

4. $\frac{5}{6} = \frac{\square}{54}$

5. $\frac{\square}{7} = \frac{16}{28}$

6. $\frac{\square}{9} = \frac{27}{81}$

II) Find two equivalent fractions for the following fractions

1. $\frac{7}{9}$

2. $\frac{8}{12}$

3. $\frac{3}{5}$

4. $\frac{6}{7}$

III) Find three equivalent fractions for each of the following.

1. $\frac{1}{5}$

2. $\frac{8}{15}$

3. $\frac{5}{9}$

4. $\frac{5}{12}$

5. $\frac{5}{6}$

IV) Change the fractions below so that their denominators are 60.

Present or explain your findings.

(a) $\frac{2}{3}$

(b) $\frac{3}{4}$

(c) $\frac{4}{5}$

(d) $\frac{20}{600}$

(e) $\frac{10}{120}$

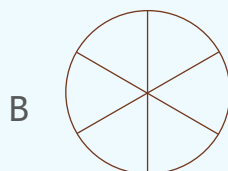
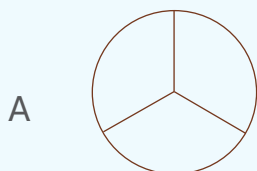
(f) $\frac{7}{15}$

4.4 Shading the equivalent fractions

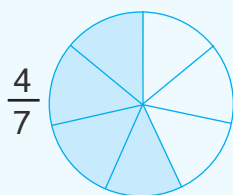


Activity 4.4

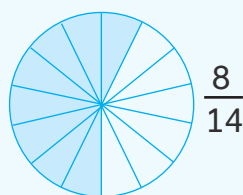
- a) Draw and shade $\frac{2}{3}$ in A. Shade the equivalent fraction B where the whole is divided in 6 equal parts.



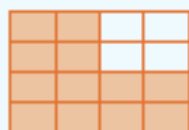
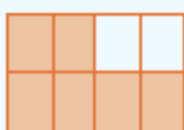
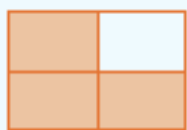
- b) Do the same for $\frac{4}{7}$ in the model where the whole is divided into 14 equal parts.



Equivalent fraction



- c) Given that the whole is divided into equal parts, write the shaded fractions. Are the obtained fractions equivalent? Explain your answer.

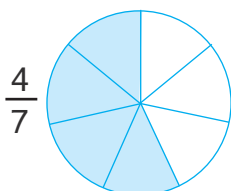


Summary

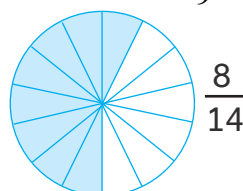
Equivalent fractions are fractions that represent the same value or proportion of a whole, even though they may have different numerators and denominators.

Examples

- 1) Using models show equivalent fractions of $\frac{2}{9}$.



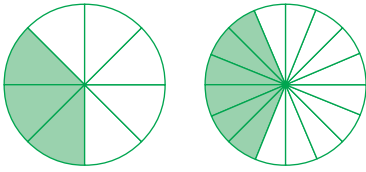
Equivalent fraction



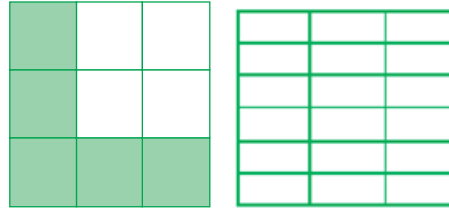
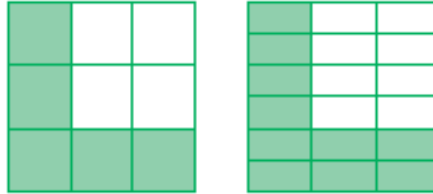
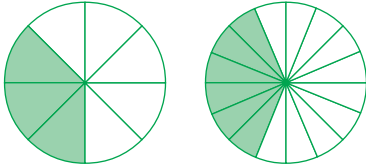
The fraction $\frac{2}{9}$ is equivalent to $\frac{4}{18}$ because they represent two equal parts of a whole

- 2) Write the shaded fraction in A and shade its equivalent fraction in B.

A



B

**Solution:**

In A, the shaded fraction is $\frac{3}{8}$ (the whole is divided in 8 equal parts and 3 are shaded). When the whole is divided into 16, the region equals to the shaded one has 6 parts. This

means that we shade 6 out of 16. They are represented by the fraction $\frac{6}{16}$. Therefore, $\frac{3}{8}$

is equivalent to $\frac{6}{16}$.

$$\frac{3}{8} = \frac{6}{16}.$$

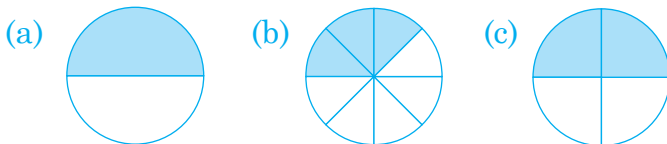
In B, the shaded fraction is $\frac{5}{9}$ (the whole is divided in 9 equal parts and 5 are shaded). When the whole is divided into 18, the region equals to the shaded one has 10 parts. This means that we shade 10 out of 18.

They are represented by the fraction $\frac{10}{18}$. Therefore, $\frac{5}{9}$ is equivalent to $\frac{10}{18}$.

$$\frac{5}{9} = \frac{10}{18}.$$

3) (i) Which of the shaded parts show equivalent fractions?

(ii) Write the equivalent fractions.

**Solution**

(i) The shaded parts in (a) and (c) are equal.

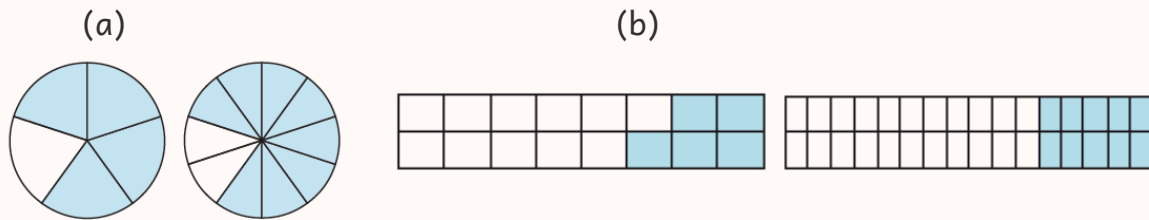
(ii) The fraction in (a) is $\frac{1}{2}$. The fraction in (c) is $\frac{2}{4}$.

The equivalent fractions shown are $\frac{1}{2}$ and $\frac{2}{4}$. It means $\frac{1}{2} = \frac{2}{4}$.



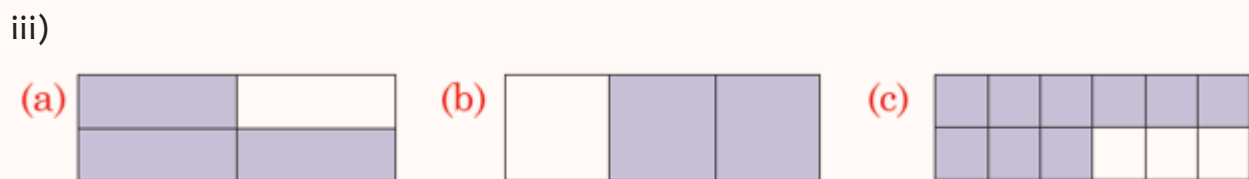
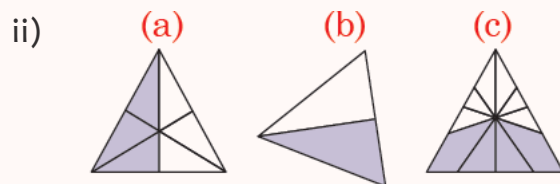
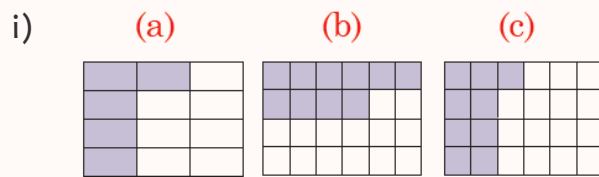
Application Activity 4.3

1) Write the equivalent fractions for the models. Discuss your answer



2) From b) and c) Which shaded part shows equivalent fraction of the fraction illustrated in (a)?

Write the equivalent fractions in each case. Discuss your answer.



4.5 Addition of fractions with different denominators using models or circle set fractions



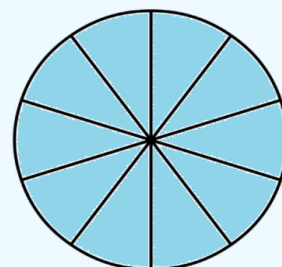
Practical Activity 4.5

In this activity, you will use plain papers, pencil, pair of scissors, plastic circle and set fractions.

Using plastic circle and set fractions let us add: $\frac{1}{10} + \frac{3}{5}$

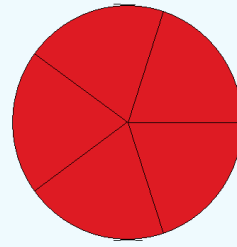
Step 1: Circle 1: Take a circular object and split it into 10 equal parts.

Circle 1:

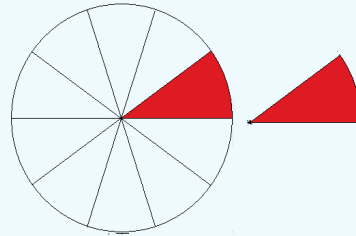


Step 2: Circle 2: Take a similar circular object and split it into 5 equal parts.

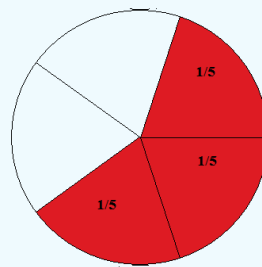
Circle 2:



Step 3: Remove 1 piece from **Circle 1**.
What fraction is it?



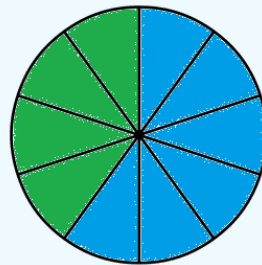
Step 4: Remove 3 pieces from **Circle 2**.
What fraction is it?



Is it possible to add the pieces from **Circle 1** and pieces from **Circle 2** together?

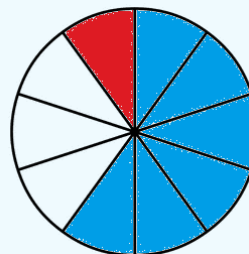
Question: How?

Step 5: In order to add sectors of our circles, let them have the same sizes by splitting pieces of **Circle 2** into 10 equal pieces.



What does the part “3 pieces out of 5” become? What fraction does it represent?

Step 6: Add one split from circle 1 to the part represented by the part of circle 2 obtained on step 5.



What fraction do you get?



Summary

When adding fractions of different denominators, let the whole (circle) be split into equal number of pieces that can be easily added by considering the denominator.

$$\frac{1}{10} + \frac{3}{5} = \frac{1}{10} + \frac{6}{10} = \frac{7}{10}$$

This way of using a whole divided into the same number of equal parts is the same as reducing the fractions to be added to the same denominator.

$$\frac{1}{10} + \frac{3}{5} = \frac{1}{10} + \frac{2 \times 3}{2 \times 5} = \frac{1}{10} + \frac{6}{10} = \frac{7}{10}.$$



Application Activity 4.5

Use circle set fractions or other cut-outs for fractions to practically add $\frac{1}{4} + \frac{3}{8}$.

4.6 Addition of fractions with different denominators using equivalent fractions

4.6.1 When a denominator for one fraction is a multiple of the denominator of the other fraction



Activity 4.6.1

Observe the fractions $\frac{2}{3}$ and $\frac{1}{6}$. Do they have the same denominator? Can you find the equivalent fraction of $\frac{2}{3}$ that has 6 as the denominator?

Find the equivalent fraction of $\frac{2}{3}$ with the denominator 6.

Now, use your results to complete the addition of $\frac{2}{3} + \frac{1}{6}$ as follows: $\frac{2}{3} + \frac{1}{6} = \frac{\dots}{6} + \frac{1}{6} = \frac{\dots+1}{6} = \frac{5}{6}$

Discuss your findings.



Summary

To add fractions with different denominators,

- Identify the different denominators.
- Check if denominators are multiples of each other. Then use the largest denominator as a common denominator.

- (iii) Convert the other fraction to an equivalent fraction having the selected common denominator.
- (iv) Add the numerators of these new fractions.
- (v) Write the sum over the common denominator.
- (vi) Simplify the resulting fraction if possible.

Examples:

Add: $\frac{1}{3} + \frac{4}{9} =$

Solution

Denominators 3 and 9 are different. We change $\frac{1}{3}$ into an equivalent fraction with 9 as its new denominator.

• $\frac{1}{3} = \frac{1}{3} \times \frac{3}{3} = \frac{3}{9}$

Thus, $\frac{1}{3} + \frac{4}{9} = \frac{3}{9} + \frac{4}{9} = \frac{3+4}{9} = \frac{7}{9}$



Application Activity 4.6.1

I) Fill in the missing numbers when adding fractions:

1. $\frac{1}{10} + \frac{3}{5} = \frac{1}{10} + \frac{\square}{10} = \frac{1+\square}{10} =$

2. $\frac{3}{8} + \frac{1}{2} = \frac{3}{8} + \frac{\square}{8} = \frac{3+\square}{8} =$

3. $\frac{2}{3} + \frac{1}{9} = \frac{\square}{9} + \frac{1}{9} = \frac{\square+1}{9} =$

4. $\frac{1}{5} + \frac{3}{10} = \frac{\square}{10} + \frac{3}{10} = \frac{\square+3}{10} =$

II) Use equivalent fractions to add the following

1. $\frac{1}{2} + \frac{1}{4} =$

2. $\frac{1}{4} + \frac{1}{8} =$

3. $\frac{1}{4} + \frac{1}{12} =$

4. $\frac{1}{2} + \frac{1}{8} =$

5. $\frac{1}{5} + \frac{1}{10} =$

6. $\frac{1}{3} + \frac{1}{9} =$

7. $\frac{1}{2} + \frac{5}{12} =$

8. $\frac{2}{3} + \frac{1}{6} =$

9. $\frac{3}{5} + \frac{1}{10} =$

10. $\frac{2}{9} + \frac{2}{3} =$

11. $\frac{2}{7} + \frac{3}{14} =$

12. $\frac{5}{12} + \frac{3}{6} =$

4.6.2 When denominators of the fractions are totally different



Activity 4.6.2

1) Find equivalent fractions with the denominator 6 for:

(i) $\frac{1}{3}$

(ii) $\frac{1}{2}$

What is the easiest way to do that?

Now add $\frac{1}{3} + \frac{1}{2} = \frac{\dots}{6} + \frac{\dots}{6} = \frac{\dots}{6}$

2) Using equivalent fractions, add the following fractions:

a) $\frac{1}{2} + \frac{1}{5}$

b) $\frac{1}{3} + \frac{1}{4}$

What answers did you get?



Summary

To add fractions with different denominators,

Find a common denominator, then **rewrite each fraction** with that common denominator, and finally **add the numerators**.

Step-by-step:

- Find the least common denominator (LCD) of the fractions.
- Convert each fraction to an equivalent fraction with the LCD as the denominator.
- Add the numerators of these new fractions.
- Write the sum over the common denominator.

Simplify the resulting fraction if possible.

Example:

1) Use the Least Common Multiple (LCM) of denominators of fractions as the common denominator to add $\frac{1}{3} + \frac{2}{5}$. Explain the steps you followed.

Solution:

Numbers 3 and 5 are different denominators. Their common denominator is their LCM. This is

$$3 \times 5 = 15.$$

Equivalent fractions are:

$$\frac{1}{3} = \frac{1}{3} \times \frac{5}{5} = \frac{5}{15}$$

$$\frac{2}{5} = \frac{2}{5} \times \frac{3}{3} = \frac{6}{15}$$

Thus, $\frac{1}{3} + \frac{2}{5} = \frac{5}{15} + \frac{6}{15} = \frac{5+6}{15} = \frac{11}{15}$

2) Use the LCM to add $\frac{1}{3} + \frac{1}{15}$. Explain your steps

Solution

Find LCM of 3 and 15

3	3	15
5	1	5
	1	1

LCM is $3 \times 5 = 15$

We can now use the LCM to add

Method 1:

$$\frac{1}{3} + \frac{1}{15} = \frac{(15 \div 3) \times 1 + (15 \div 15) \times 1}{15} = \frac{5 + 1}{15} = \frac{6}{15}$$

Method 2:

The common denominator is the LCM. The equivalent fraction of $\frac{1}{3}$ is $\frac{1}{3} \times \frac{5}{5} = \frac{5}{15}$.

$$\frac{1}{3} = \frac{1}{3} \times \frac{5}{5} = \frac{5}{15}$$

Therefore, $\frac{1}{3} + \frac{1}{15} = \frac{5}{15} + \frac{1}{15} = \frac{5+1}{15} = \frac{6}{15}$

3) Use the LCM to add $\frac{2}{5} + \frac{2}{6} =$

Solution

Their denominators 5 and 6 are different, so we have to use the LCM of 5 and 6 as a common denominator.

2	6	5	5
3	3	1	
1			

LCM is $2 \times 3 \times 5 = 30$

Equivalent fractions: $\frac{2}{5} = \frac{2}{5} \times \frac{6}{6} = \frac{12}{30}$ and $\frac{2}{6} = \frac{2}{6} \times \frac{5}{5} = \frac{10}{30}$

Therefore, $\frac{2}{5} + \frac{2}{6} = \frac{12}{30} + \frac{10}{30} = \frac{12+10}{30} = \frac{22}{30} = \frac{11}{15}$

Or as we have the common denominator as 30, we can proceed as follows:

$$\frac{2}{5} + \frac{2}{6} = \frac{2(30 \div 5) + 2(30 \div 6)}{30} = \frac{12 + 10}{30} = \frac{22}{30} = \frac{11}{15}$$



Application Activity 4.6.2

I) Work out the addition of fractions:

1. $\frac{1}{2} + \frac{3}{5} =$

2. $\frac{6}{7} + \frac{2}{3} =$

3. $\frac{1}{3} + \frac{6}{7} =$

4. $\frac{1}{2} + \frac{1}{3} =$

5. $\frac{1}{2} + \frac{1}{5} =$

6. $\frac{3}{4} + \frac{2}{5} =$

II) Using the LCM, work out:

1. $\frac{1}{8} + \frac{1}{2} =$

2. $\frac{1}{6} + \frac{1}{12} =$

3. $\frac{1}{4} + \frac{1}{8} =$

4. $\frac{1}{4} + \frac{1}{12} =$

5. $\frac{1}{7} + \frac{1}{14} =$

6. $\frac{1}{2} + \frac{1}{4} =$

III) Use the LCM to work out the following:

1. $\frac{1}{6} + \frac{2}{7} =$

2. $\frac{1}{3} + \frac{5}{12} =$

3. $\frac{2}{3} + \frac{3}{10} =$

4. $\frac{3}{5} + \frac{2}{7} =$

5. $\frac{1}{6} + \frac{2}{3} =$

6. $\frac{2}{5} + \frac{1}{2} =$

7. $\frac{4}{9} + \frac{3}{7} =$

8. $\frac{4}{12} + \frac{1}{9} =$

9. $\frac{2}{6} + \frac{2}{9} =$

10. $\frac{2}{8} + \frac{3}{6} =$

11. $\frac{3}{7} + \frac{1}{4} =$

12. $\frac{2}{7} + \frac{1}{6} =$

4.7 Addition of mixed numbers and fractions with different denominators



Activity 4.7

1) Given that $1 = \frac{5}{5}$ and $2 = \frac{10}{5}$, work out the following:

a) $2 + \frac{1}{5} =$

b) $3 + \frac{3}{8} =$

2) Write the following as improper fractions:

a) $2\frac{1}{5} = \underline{\hspace{2cm}}$

b) $3\frac{3}{8} = \underline{\hspace{2cm}}$

c) $3\frac{1}{4} = \underline{\hspace{2cm}}$

d) $4\frac{1}{9} = \underline{\hspace{2cm}}$

3) Now, Add (a) $2\frac{1}{5} + 3\frac{1}{4} =$

(b) $3\frac{3}{8} + 4\frac{1}{9} =$



Summary

To add mixed fractions, follow these steps:

- Convert each mixed fraction to an improper fraction.
- Find a common denominator for the improper fractions.
- Rewrite each improper fraction with the common denominator.
- Add the numerators.
- Express the sum as an improper fraction or mixed number.
- Simplify, if possible, and convert back to a mixed number if needed

Note: you can also add whole numbers first and add fractions separately.

Example:

Add: Add: (a) $2\frac{1}{6} + 3\frac{1}{6} =$ (b) $5\frac{3}{4} + 2\frac{2}{3} =$

Explain your steps. Explain your steps.

Solution:

$$\text{a) } 2\frac{1}{6} + 3\frac{1}{6} = \frac{(2 \times 6) + 1}{6} + \frac{(3 \times 6) + 1}{6} = \frac{13}{6} + \frac{19}{6} = \frac{32}{6} = 5\frac{2}{6}$$

$$\text{Or } 2\frac{1}{6} + 3\frac{1}{6} = (2 + 3) + \left(\frac{1}{6} + \frac{1}{6}\right) = 5 + \frac{1+1}{6} = 5 + \frac{2}{6} = 5\frac{2}{6}$$

The two methods provide the same answer.

$$\text{b) } 5\frac{3}{4} + 2\frac{2}{3} = \frac{23}{4} + \frac{8}{3} = \frac{(3 \times 23)}{12} + \frac{(8 \times 4)}{12} = 8\frac{5}{12}$$

Or Add whole numbers and add fractions using LCM:

$$\begin{aligned} 5\frac{3}{4} + 2\frac{2}{3} &= (5 + 2) + \left(\frac{3}{4} + \frac{2}{3}\right) = 7\frac{9+8}{12} \\ &= 7\frac{17}{12} \quad \left(\frac{17}{12} = \frac{12}{12} + \frac{5}{12} = 1\frac{5}{12}\right) \\ &= 7 + 1\frac{5}{12} = 8\frac{5}{12} \end{aligned}$$



Application Activity 4.6.2

Using the LCM to work out the following:

1. $3\frac{3}{8} + 1\frac{1}{4} =$

2. $4\frac{1}{5} + 3\frac{1}{2} =$

3. $2\frac{2}{9} + 1\frac{3}{4} =$

4. $1\frac{3}{5} + 2\frac{1}{2} =$

5. $2\frac{1}{6} + 5\frac{1}{3} =$

6. $1\frac{3}{8} + 3\frac{3}{4} =$

7. $3\frac{5}{7} + 4\frac{1}{3} =$

8. $3\frac{9}{10} + 4\frac{4}{5} =$

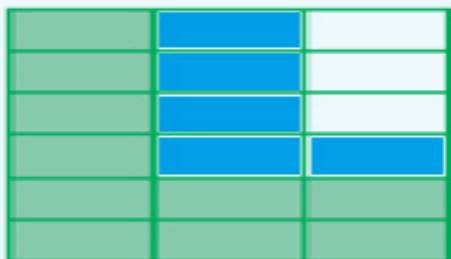
9. $4\frac{3}{7} + 2\frac{3}{4} =$

4.8 Subtraction of fractions with different denominators using models or circle set fractions



Activity 4.8

a) Observe the following picture:



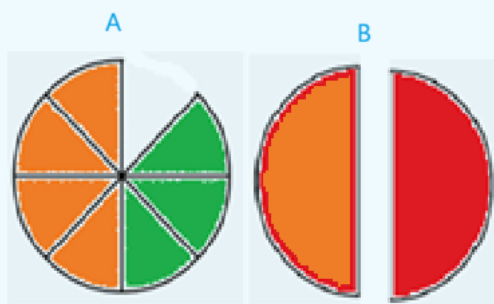
What is the fraction of all shaded parts?

What is the fraction of the parts shaded in blue?

If you remove the part shaded in blue, what fraction do you remain with?

b) Practical activity:

Observe the circle set fractions A and B.



How many equal parts were divided from the whole in A?

How many parts were divided in the whole of B?

Write the fraction of the remained parts in A after removing 1 part.

Write the fraction representing the remained part in B after removing one part.

Question: Is it possible to cut the piece equal to one split piece of **Circle B** from **Circle A**?

How can you do it? What is the remaining part? Now Complete: $\frac{7}{8} - \frac{1}{2} =$



Summary

Subtracting fractions involves removing parts of a whole.

Example:

1) Observe the shaded region



- a) Write the fraction of the shaded region.
- b) Write the fraction of the region shaded in green colour.

c) Now, complete: $\frac{15}{24} - \frac{4}{24} =$

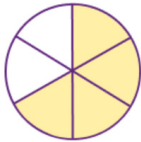
Solution:

a) The shaded region is $\frac{15}{24}$.

b) The fraction of the region shaded in green colour is $\frac{4}{24}$

c) Now, complete: $\frac{15}{24} - \frac{4}{24} = \frac{11}{24}$

2) Observe the following figure:

	Shade $\frac{4}{6} - \frac{1}{6}$ and complete the answer $\frac{4}{6} - \frac{1}{6} =$
Solution The total of shaded parts makes $\frac{4}{6}$ 	$\frac{1}{6}$ taken away is shaded with blue colour. Therefore, the remaining part is $\frac{4}{6} - \frac{1}{6} = \frac{3}{6}$



Application Activity 4.8

Refer to the example and complete:



$$\frac{3}{4} - \frac{2}{4} = \frac{1}{4}$$



$$\frac{4}{6} - \frac{1}{6} =$$

4.9 Subtraction of fractions with different denominators using equivalent fractions



Activity 4.9

1) Find the equivalent fraction of $\frac{2}{3}$ with the denominator 6.

Find the equivalent fraction $\frac{1}{2}$ with the denominator 8.

Find the equivalent fraction of $\frac{5}{6}$ with the denominator 12.

2) Use your finding to work out the following:

(a) $\frac{2}{3} - \frac{1}{6} = \frac{\square}{6} - \frac{1}{6} = \frac{\square - 1}{6} =$

(b) $\frac{7}{8} - \frac{1}{2} = \frac{7}{8} - \frac{\square}{8} = \frac{7 - \square}{8} =$

(c) $\frac{5}{6} - \frac{5}{12} = \frac{\square}{12} - \frac{5}{12} = \frac{\square - 5}{12} =$

(d) $\frac{3}{4} - \frac{1}{2} = \frac{3}{4} - \frac{\square}{4} = \frac{3 - \square}{4} =$



Summary

When subtracting fractions using equivalent fractions, the rule is:

1. Find a common denominator (usually by finding the least common multiple, LCM).
2. Convert both fractions to equivalent fractions with this common denominator.
3. Subtract the numerators while keeping the common denominator.

Simplify the resulting fraction if possible.

Examples

1) Workout: $\frac{7}{8} - \frac{3}{4}$

Solution

The denominator 8 is the multiple of 4. We take 8 as a common denominator.

The equivalent fraction of $\frac{3}{4}$ is $\frac{3}{4} \times \frac{2}{2} = \frac{6}{8}$

Therefore, $\frac{7}{8} - \frac{3}{4} = \frac{7}{8} - \frac{6}{8} = \frac{7-6}{8} = \frac{1}{8}$

2) Work out $\frac{7}{8} - \frac{2}{3}$

Solution

We see that the two denominators 8 and 3 are different. The common denominator for 8 and 3 is their LCM which is $8 \times 3 = 24$.

We find their equivalent fractions having 24 as a denominator:

$$\bullet \quad \frac{7}{8} = \frac{7}{8} \times \frac{3}{3} = \frac{21}{24} \qquad \bullet \quad \frac{2}{3} = \frac{2}{3} \times \frac{8}{8} = \frac{16}{24}$$

Therefore, $\frac{7}{8} - \frac{2}{3} = \frac{21}{24} - \frac{16}{24} = \frac{21-16}{24} = \frac{5}{24}$

3) Work out using the LCM $\frac{2}{3} - \frac{1}{5}$

Solution:

Find LCM of 3 and 5

3	3	5
5	1	5
	1	1

LCM is $3 \times 5 = 15$

Method 1:

The common denominator is 15

$$\frac{2}{3} = \frac{2}{3} \times \frac{5}{5} = \frac{10}{15}$$

$$\frac{1}{5} = \frac{1}{5} \times \frac{3}{3} = \frac{3}{15}$$

$$\frac{2}{3} - \frac{1}{5} = \frac{10}{15} - \frac{3}{15} = \frac{10-3}{15} = \frac{7}{15}$$

Method 2

$$\begin{aligned} \frac{2}{3} - \frac{1}{5} &= \frac{(15 \div 3) \times 2 - (15 \div 5) \times 1}{15} \\ &= \frac{10 - 3}{15} \\ &= \frac{7}{15} \end{aligned}$$



Application Activity 4.9

I) Work out the following using equivalent fractions. Discuss your answer

1. $\frac{7}{8} - \frac{1}{2} = \frac{7}{8} - \frac{\square}{8} = \frac{7-\square}{8}$

2. $\frac{4}{5} - \frac{1}{10} = \frac{\square}{10} - \frac{1}{10} = \frac{\square-1}{10} =$

3. $\frac{5}{9} - \frac{1}{3} = \frac{5}{9} - \frac{\square}{9} =$

4. $\frac{1}{3} - \frac{1}{6} = \frac{\square}{6} - \frac{1}{6} =$

II) Use equivalent fractions to solve. Discuss your answer

1. $\frac{1}{3} - \frac{1}{6} =$

2. $\frac{1}{7} - \frac{1}{14} =$

3. $\frac{1}{9} - \frac{1}{18} =$

4. $\frac{1}{2} - \frac{1}{12} =$

5. $\frac{1}{6} - \frac{1}{18} =$

III) Using equivalent fractions, work out the following

1. $\frac{9}{10} - \frac{3}{6} =$

2. $\frac{5}{6} - \frac{7}{9} =$

3. $\frac{7}{8} - \frac{1}{3} =$

4. $\frac{8}{9} - \frac{1}{5} =$

5. $\frac{2}{3} - \frac{2}{4} =$

6. $\frac{6}{7} - \frac{2}{5} =$

IV) Use the LCM to work out:

1. $\frac{1}{3} - \frac{1}{7} =$

2. $\frac{1}{3} - \frac{1}{9} =$

3. $\frac{1}{5} - \frac{1}{10} =$

4. $\frac{1}{2} - \frac{1}{9} =$

5. $\frac{1}{4} - \frac{1}{12} =$

6. $\frac{1}{5} - \frac{1}{6} =$

7. $\frac{1}{2} - \frac{1}{12} =$

8. $\frac{6}{8} - \frac{1}{3} =$

9. $\frac{2}{6} - \frac{2}{9} =$

10. $\frac{7}{9} - \frac{7}{12} =$

11. $\frac{2}{3} - \frac{2}{10} =$

12. $\frac{4}{5} - \frac{3}{4} =$

4.10 Subtraction of mixed numbers and fractions with different denominators



Activity 4.10

1) Write a) $16\frac{1}{2}$ b) $20\frac{9}{10}$ c) $6\frac{5}{8}$ d) $9\frac{1}{9}$ as improper fractions.

2) Use answers from (1) to work out: a) $23 - 20\frac{9}{10}$ b) $15 - 9\frac{1}{9}$

What answers did you get? Discuss your steps.



Summary

When subtracting mixed numbers, there are three methods that give the same answer:

1. Subtract the whole numbers, Subtract the fractions and then, Combine results

Example: $5\frac{3}{4} - 2\frac{1}{4} = \underline{\hspace{2cm}}$

We can take $5 - 2 = 3$ and $\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$, then combine $3 + \frac{2}{4} = 3\frac{2}{4} = 3\frac{1}{2}$

2. Borrowing (When Fraction Part is Smaller in the First Number)

Example: $4\frac{1}{4} - 2\frac{3}{4} = \underline{\hspace{2cm}}$

Here, $\frac{1}{4} < \frac{3}{4}$; we borrow 1 from 4 and add it to $\frac{1}{4}$: $1 + \frac{1}{4} = \frac{4}{4} + \frac{1}{4} = \frac{5}{4}$. Then, we take $3 - 2 = 1$

to be combined with $\frac{5}{4} - \frac{3}{4} = \frac{2}{4}$. Therefore, $4\frac{1}{4} - 2\frac{3}{4} = 1\frac{2}{4} = 1\frac{1}{2}$

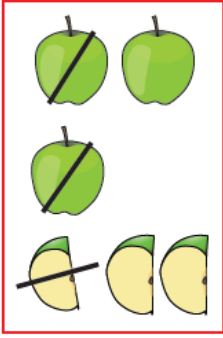
3. Converting to Improper Fractions (Alternative Method)

Example: $3\frac{2}{5} - 1\frac{4}{5} = \underline{\hspace{2cm}}$

This requires converting everything in improper fraction and subtract.

$$3\frac{2}{5} - 1\frac{4}{5} = \frac{17}{5} - \frac{9}{5} = \frac{17-9}{5} = \frac{12}{5} = 2\frac{2}{5}$$

More examples: 1) Work out: $3\frac{3}{4} - 2\frac{1}{4} = \underline{\hspace{2cm}}$

Pictures	Calculations
	• $3 - 2 = 1$ • $\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$ • $3\frac{3}{4} - 2\frac{1}{4} = ?$ • $\frac{3}{4} - \frac{1}{4}$ • The answer is $1\frac{2}{4}$

2) Work out using the LCM.

a) $2 - \frac{1}{3}$

b) $3\frac{3}{4} - 1\frac{1}{4}$

Solution

a) $2 - \frac{1}{3} = 1 + 1 - \frac{1}{3}$ Borrow 1 from 2 and change it to three thirds

$$= 1 + (1 - \frac{1}{3}) = 1 + (\frac{3}{3} - \frac{1}{3}) = 1\frac{2}{3}$$

b) $3\frac{3}{4} - 1\frac{1}{4}$

We can first subtract whole numbers. Then subtract the fractions.

$$3\frac{3}{4} - 1\frac{1}{4} = (3 - 1) + (\frac{3}{4} - \frac{1}{4}) = 2\frac{2}{4} = 2\frac{1}{2}$$

We can also convert mixed numbers in improper fractions. The two methods give the same answer.

3) Work out $5\frac{1}{4} - 1\frac{5}{6}$

Solution

$5\frac{1}{4} - 1\frac{5}{6} = \frac{(5 \times 4) + 1}{4} - \frac{(1 \times 6) + 5}{6} = \frac{21}{4} - \frac{11}{6}$ let us use the LCM of 4 and 6 as the common denominator. This is 12. Therefore,

$$5\frac{1}{4} - 1\frac{5}{6} = \frac{(5 \times 4) + 1}{4} - \frac{(1 \times 6) + 5}{6} = \frac{21}{4} - \frac{11}{6} = \frac{63 - 22}{12} = \frac{41}{12} = 3\frac{5}{12}$$



Application Activity 4.10

Use the LCM to work out:

1. $8\frac{5}{6} - 3\frac{2}{6}$

2. $6\frac{3}{5} - 3\frac{1}{5}$

3. $4\frac{7}{8} - 1\frac{3}{8}$

4. $5\frac{5}{9} - 2\frac{2}{9}$

5. $6\frac{2}{3} - 3\frac{5}{6}$

5. $7\frac{3}{4} - 3\frac{2}{3}$

6. $2\frac{1}{4} - 1\frac{1}{4}$

7. $4\frac{7}{9} - 3\frac{2}{3}$

8. $3\frac{7}{11} - 1\frac{1}{2}$

9. $4\frac{1}{6} - 1\frac{6}{9}$

4.11 Solving Word problems involving addition of fractions



Activity 4.11

Mum wanted to prepare a good meal for lunch. She then bought $\frac{1}{4}$ kilogram of meat of beef and $\frac{1}{2}$ kilogram of meat of pork. To find the weight of both beef and pork, Kamana decided to add the two fractions. Help Kamana to find the total mass of meat the Mum bought.

Solved example:

During a prayer meeting in the church, $\frac{1}{3}$ of the people were women. $\frac{3}{5}$ of the people were children. The rest were men. Calculate the fraction representing women and children.

Solution

Fraction of women is $\frac{1}{3}$. Fraction of children is $\frac{3}{5}$.

Total is $\frac{1}{3} + \frac{3}{5} =$

The common denominator is the LCM of 3 and 5:

$$\begin{array}{r|l} 3 & 3 \\ 1 & 1 \end{array} \quad \begin{array}{r|l} 5 & 5 \\ 1 & 1 \end{array}$$

LCM of 3 and 5 is $3 \times 5 = 15$.

The fraction representing women and children is:

$$\frac{1}{3} + \frac{3}{5} = \frac{(1 \times 5) + (3 \times 3)}{15} = \frac{5 + 9}{15} = \frac{14}{15}$$



Application Activity 4.11

1. Carene was celebrating her birthday. Her mother bought her a cake. Carene shared the cake with her mother. Carene ate $\frac{4}{9}$ of the cake. Her mother took $\frac{1}{3}$ of the same cake. The remaining part of the cake was eaten by her father. What is the total fraction of the cake eaten by Carene and her mother? Explain why we should share what we have.
2. In the morning, a cook wanted to make some tea. He mixed $\frac{1}{4}$ of liter of milk and $\frac{1}{8}$ liter of water. He then boiled them. Find the quantity of liquid the cook boiled. Discuss your steps.

4.12 Solving Word problems involving subtraction of fractions



Activity 4.12

Use the LCM to solve the following: A farmer has $\frac{7}{8}$ ha of land. A half of ha from the land is planted with crops. The rest is for the farm. How much land is used for the farm?



Summary

- Common words and phrases that indicate subtraction in word problems: Minus, Difference, Remaining, Left, take away, Subtract, Fewer than, Reduced by and Decrease by.
- Read the problem carefully to understand the problem, identify the given data and the requested information.
- Find the operation to be used and then calculate.

Solved example

A mother had $\frac{5}{9}$ liters of milk for her baby. During the day, the baby drank some of the milk.

The milk that remained was $\frac{1}{3}$ litres. How much milk did the baby drink during the day?

Solution

Amount of milk initially is $\frac{5}{9}$ litres.

Remaining milk is $\frac{1}{3}$ litres.

Requested: Milk drank = ?

Operation: we use the subtraction.

$$\text{Milk drank} = \frac{5}{9} - \frac{1}{3} = \frac{5 - 1 \times (9 \div 3)}{9} = \frac{5}{9} - \frac{1 \times (9 \div 3)}{9} = \frac{5}{9} - \frac{3}{9} = \frac{5 - 3}{9} = \frac{2}{9}$$

The milk drank is $\frac{2}{9}$ litres.



Application Activity 4.12

1. During an activity on measuring length, Jane and Michael had different sticks. Jane had a stick that is $\frac{7}{8}$ m long. Michael had a stick that is $\frac{5}{6}$ m long. By how many metres is Jane's stick longer than Michael's stick?
2. A teacher had $\frac{3}{4}$ meters of a thread. The teacher used $\frac{1}{2}$ a meter from the thread to mend a cloth. What length of thread remained? Explain your answer.



4.13 End unit assessment

Assessing knowledge:

Select the correct answer for each question:

1. Which of the following represents an equivalent fraction to $\frac{1}{2}$?
a) $\frac{2}{3}$ b) $\frac{2}{4}$ c) $\frac{3}{4}$ d) $\frac{3}{6}$
2. I shaded $\frac{1}{4}$ of a circle. Which of the following shows an equivalent fraction on another circle?
a) $\frac{2}{4}$ b) $\frac{2}{8}$ c) $\frac{4}{8}$ d) $\frac{1}{3}$
3. To add $\frac{3}{8}$ and $\frac{1}{4}$, which common denominator should we use?
a) 4 b) 8 c) 3 d) 9
4. What is the equivalent fraction of $\frac{4}{5}$ with denominator 15?
a) $\frac{12}{15}$ b) $\frac{20}{15}$ c) $\frac{16}{15}$ d) $\frac{15}{20}$
5. Which is the proper way to find an equivalent fraction?
a) Add numerator and denominator
b) Multiply numerator and denominator by the same number

- c) Subtract numerator from denominator
- d) Divide numerator by denominator

Assessing Skills

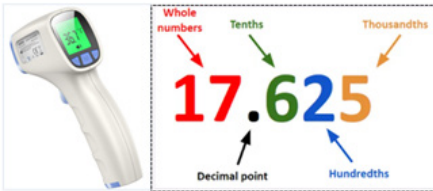
6. Write the equivalent fraction of $\frac{1}{3}$ with denominator 9.
7. Add the fractions: $\frac{1}{4} + \frac{1}{2}$. Show your working steps.
8. Find a common denominator for $\frac{3}{8}$ and $\frac{1}{4}$, then add the two fractions
9. A cake is divided into 8 equal slices. If Ruth eats 3 slices, what fraction of the cake did she eat?
10. Subtract $\frac{3}{8}$ from $\frac{5}{8}$. Write your answer in simplest form.
11. Lily cut a cake into 12 equal slices. She ate 3 slices, and her sister ate 4 slices. What fraction of the cake was eaten?
12. A glass of juice is filled to $\frac{3}{4}$. If you pour out $\frac{1}{4}$, what fraction of the juice remains?
13. Two pupils measured the distance from their school to their homes. The distance from Martha's home to the school was $\frac{5}{8}$ km. The distance from Uwineza's home to the school was $\frac{1}{4}$ km. Every morning, each pupil walks to the school.
Who walks longer distance every morning? And by how many kilometres?

Assessing attitudes and values:

14. Explain why equivalent fractions are useful in everyday life.
15. How can understanding fractions help us make good decisions when sharing with friends or family?

UNIT 5

MULTIPLICATION AND DIVISION OF DECIMAL NUMBERS



Key Unit competence: You will be able to compare, multiply and divide decimal numbers up to 3 decimal places and apply this knowledge to solve related problems.

Learning activities:

By the end of this unit, you should be able to:

Knowledge and understanding

- Explain the concept of decimal numbers using place values up to 3 decimal places.
- Identify the place values in written decimals.
- Explain how to multiply and divide decimal numbers.

Skills:

- Read and write decimal numbers correctly in figures and in words.
- Convert fractions to decimals fractions and vice versa.
- Compare 2 or more decimal numbers using $>$, $<$ or $=$
- Carry out multiplication or division of decimal numbers.
- Apply decimal concepts in solving daily life mathematics problems.

Attitudes and values:

- Show confidence when using decimal numbers.
- Appreciate the importance of decimal numbers in real life.



5.0 Introduction

Decimal numbers are numbers that have a whole part and a fractional part, separated by a decimal point. Decimal numbers are used in everyday life for precise measurements and calculations. In banks that use dollars, decimals represent cents to ensure accurate exchanges. Scientists and engineers rely on decimals to record precise length measurements such as 2.75 meters in construction. Doctors use decimals to precise the temperature such as 37.5°C in medical readings. Cooks use decimals for ingredient quantities like 1.5 teaspoons of sugar, to maintain consistency. Additionally, decimals appear in fuel consumption rates; for example 15.6 km per liters, ensuring efficient vehicle performance.

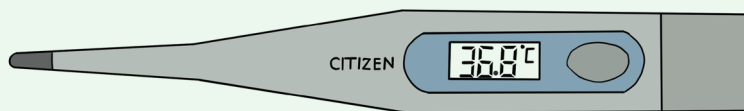
In this unit, you are introduced to decimal place values up to thousandths and you will practice writing decimal numbers in words and reading them while in figures.

Conversion of tenths, hundredths and thousandths into decimals, recognition of place value and division are all covered. Fractions that can be written with denominators of 10, 100 or 1,000 are also converted into decimals.



Introductory Activity

Observe the following thermometer. The nurse was using it to measure the **temperature**, which helps determine if a patient has a **fever**, an elevated body temperature, often indicating illness.



- What is the temperature does it indicate?
Can you read it?
- How much temperature is missing to get 40 degrees?
- Provide other instruments that indicate the type of numbers you have.
- Try to multiply $36.8 \times 10 = \underline{\hspace{2cm}}$

5.1 Illustration of the concept of decimal fractions through examples

A tenth



Activity 5.1.1

- (a) use a knife to cut an orange into ten equal parts.



A whole orange



Ten pieces of orange



$$\text{one tenth} = \frac{1}{10}$$

- (b) Name each part you have cut. Count them using your own words.
- (c) Try to write $\frac{4}{10}$ in words.
- (d) Explain your observations to the class.

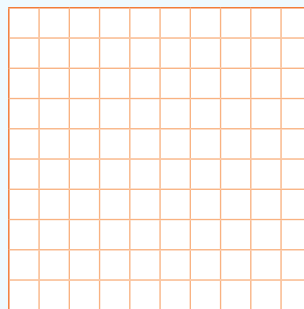
A hundredth



Activity 5.1.2

Materials: a pair of scissors, manila paper, a ruler, a pencil

- Draw a square of 10 cm for side. Do it on manila paper or a gridded sheet of paper.
- Draw smaller squares each measuring 1 cm inside the bigger square.
- Count the number of small squares.
- Shade four of the small squares.
- What fraction have you shaded?
- Write this fraction in words.
- Present and explain your work.



A thousandth

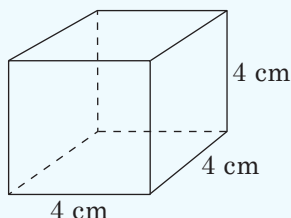


Activity 5.1.3

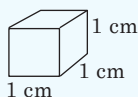
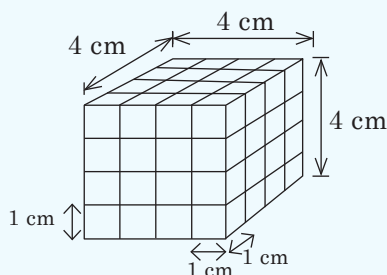
Materials: a pair of scissors, manila paper, a ruler, a pencil

Do the following:

- Draw a cube measuring 4 cm.



- In it, draw 1 cubic centimetre.



- Count the number of 1 centimeter cubes you have drawn. Let us make a 4 centimeters cube using wet clay. We can cut a 1 centimeter cube using a sharp knife.
- Repeat the process above for a cube with 10 cm of sides. What decimal number is

$\frac{705}{1000}$? Write it in words. Discuss your findings.



Summary

- A **decimal fraction** is a fraction whose denominator is a power of 10 (10, 100, 1000, etc.).
- A decimal fraction can be converted into a **decimal number**: A number that contains a decimal point to represent whole numbers and fractions of ten.

Example

$\frac{1}{10}$ = one tenth = 0.1. The decimal number is read as zero point one.

$\frac{1}{100}$ = one hundredth = 0.01. The decimal number is read as zero point zero one.

$\frac{1}{1000}$ = one thousandth = 0.001. The decimal number is read as zero point zero, zero, one.

Example:

(a) Read and write the decimals in words.

(i) 0.02 (ii) 0.3 (iii) 0.005 (iv) 0.85 (v) 0.850

(b) Write the following in figures.

(i) Three hundredths (ii) Seven tenths (iii) Nine thousandths (iv) Four point nine

Solution

(a) The decimals in words:

(i) 0.02 is read as zero point zero two. It is written as two hundredths.

(ii) 0.3 is read as zero point three. It is written as three tenths.

(iii) 0.005 is read as zero point zero zero five. It is written as five thousandths.

(iv) 0.85 is read as zero point eight five. It is written as eighty-five hundredths.

(v) 0.850 is read as zero point eight five zero. It is written as eight hundred fifty thousandths.

(b) (i) 0.03 (ii) 0.7 (iii) 0.009 (iv) 4.9 .



Application Activity 5.1

Work out the following:

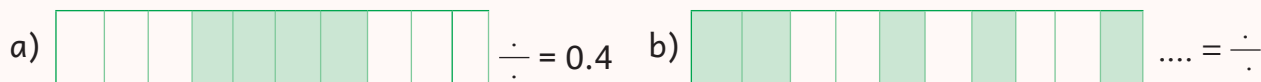
1) Fill in the table below.

Written in figures	Read as	Written in words
0.1	zero point one	one tenth
0.8	----	-----
0.01	----	one hundredth
----	----	Four and twenty-three thousandths.

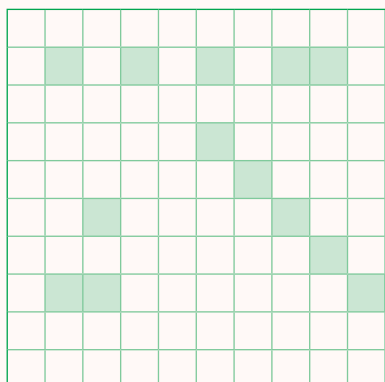
2) From the diagram below, we can write the fraction of the shaded part and decimal number as follow:



Then, complete



3) Study the figure below. Explain your observation.



- a) Write the decimal fraction for the shaded part.
b) Write the decimal number for the part that is not shaded.

4. Read and write the following decimal numbers in words.

- (a) 0.256 (b) 436.2 (d) 196.261 (e) 0.75 (f) 0.4

5. Read and write the following decimal numbers in figures.

- (a) Zero point two three five (b) Six hundredths
(c) Eight hundred seven thousandths.

5.2 Place value of decimals



Activity 5.2

Fill in the missing numbers in the following table. Then, say the place value for each digit.

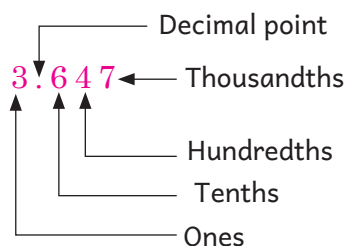
Decimal number	Place value for decimals				
	Ones	Decimal point	Tenths	Hundredths	Thousandths
0.23	0	.	2	3	
	0	.	0	0	4
0.456					
9.25					
	0	.	0	0	5

Discuss your results.

Summary

Tenths, hundredths and thousandths are examples of place values for decimals.

Example: The place values of the digits in 3.647 are:



Application Activity 5.2

1. Fill in the missing numbers in the following table.

Decimal number in figures	Place value							
	Thousands	Hundreds	Tens	Ones	Decimal point	Tenths	Hundredths	Thousandths
0.25				0	.	2	5	
				0	.	4	5	8
5.236								
84.5								
		6	2	5	.	0	1	
467.2								
4321.036								

2. What is the place value of the digit 4 in the following numbers?

(a) 356.4 (b) 236.254

(c) 0.004 (d) 0.245

3. Discuss and write the name of the place value of the digit 2 in the following numbers.

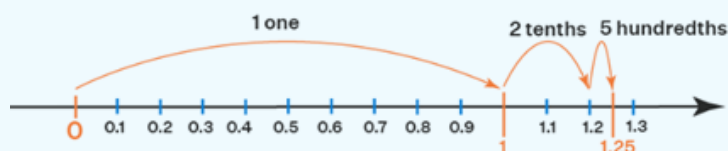
a) 56.235 b) 43.325 (c) 0.002.

5.3 Comparing and ordering decimal numbers



Activity 5.3

When we locate two decimal numbers on a number line, we can compare them by observing their position. For example, $0.4 < 1.25$ because 0.4 comes before 1.25 on the number line. On a number line, a number on the left is less than a number on the right side.



Therefore, represent the following numbers on a number line and compare them using $>$, $<$ or $=$ symbols.

(a) 0.3 ____ 4 (b) 0.07 ____ 0.09 (c) 0.001 ____ 0.009

(d) 0.01 ____ 0.010 (e) 0.2 ____ 0.02 ____ 0.002

Explain your answers.



Summary

We use these symbols $<$ (less than), $>$ (greater than), $=$ (equals) to compare decimals. Tenths are greater than hundredths and thousandths.

Thousandths are less than hundredths and tenths, hundredths are less than tens but greater than thousandths.

To compare decimal numbers,

- Align the Decimal Points and Write the numbers vertically, ensuring the decimal points are directly in line.

Add trailing zeros if needed to make each number have the same number of decimal places.

Example: Compare 3.25 and 3.4 $\rightarrow$ Rewrite as 3.25 and 3.40.

- Compare Whole Number Parts (Left of the Decimal) $\begin{array}{r} 3.25 \\ 3.40 \end{array}$
as $4 > 2$, we have $3.4 > 3.25$.

If one number has a larger whole number part, it is the greater number.

Example: $5.7 > 4.98$ (since $5 > 4$).

- If Whole Numbers Are Equal, Compare Tenths Place

Look at the first digit after the decimal (tenths). The larger digit means the number is greater.

Example: $2.3 > 2.15$ ($3 > 1$ in the tenths place).

- If Tenths Are Equal, Move to Hundredths, Then Thousandths, etc.

Continue comparing each decimal place from left to right until one digit is larger.

Example: $0.047 < 0.05$ (0.040 vs. $0.050 \rightarrow 4 < 5$ in hundredths).

If all digits are identical, the numbers are equal

Example: $6.00 = 6.0 = 6$

More examples:

Use $>$, $<$ or $=$ to compare the following:

(a) 0.2 ____ 0.4

(b) 0.05 ____ 0.08

(c) 0.009 ____ 0.004

(d) 0.009 ____ 0.04 ____ 0.1

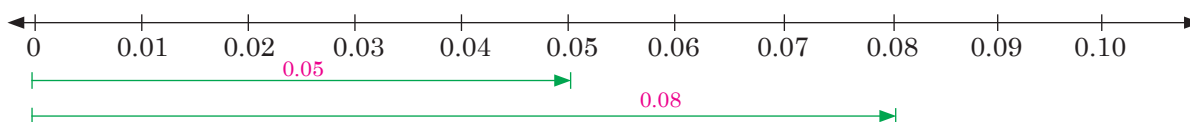
Solution

(a) Draw similar strips below. Shade 0.2 and 0.4.



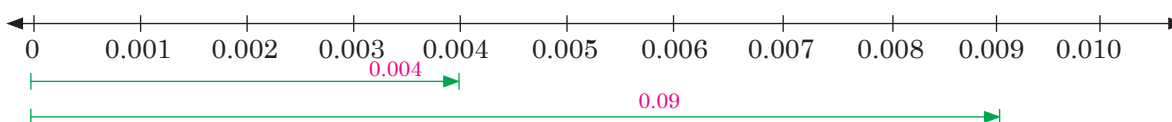
From the diagram, 0.4 is greater than 0.2. We can say 0.2 is less than 0.4. Thus, $0.2 < 0.4$.

b) Draw a number line and represent the numbers on it.



0.05 is less than 0.08. Thus, $0.05 < 0.08$

(c) Draw a number line and represent the numbers 0.009 and 0.004 on it.



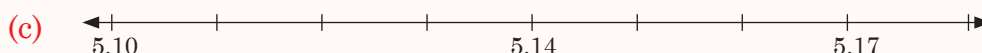
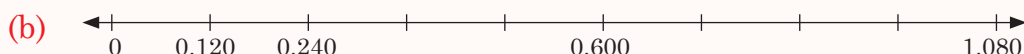
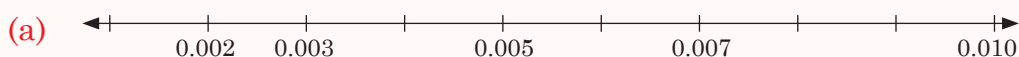
0.009 is greater than 0.004. We write $0.009 > 0.004$

(d) If we draw number lines, tenths are greater than hundredths. Hundredths are greater than thousandths. Thus, $0.009 < 0.04 < 0.1$. Draw this on a paper and discuss your answer.



Application Activity 5.3

1) Copy and complete the number lines below.



2) Use $>$, $<$ and $=$ to fill the blanks correctly.

a) 0.005 ____ 0.007 b) 3.40 ____ 3.040 c) 0.77 ____ 0.770 d) 0.825 ____ 0.826

3) Arrange the following from the smallest to the largest. You can use > or <

a) 0.01; 0.05; 0.02; 0.04.

b) 0.452; 0.252; 0.436.

c) 0.5; 0.4; 0.6; 0.8.

4) A farmer recorded the amount of milk from her farm as follows:

Days of the week	Monday	Tuesday	Wednesday	Thursday
Amount of milk	0.25hℓ	0.23hℓ	0.34hℓ	0.30hℓ

(a) In which day had the farmer recorded the highest amount of milk?

(b) Arrange the recorded amount of milk from the largest to the smallest. Justify your answer.

5.4 Conversion of fractions to decimals

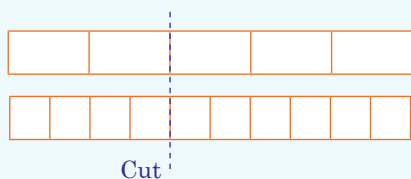


Activity 5.4

We can change a fraction into a decimal. For example: $\frac{3}{10} = 0.3$

Do the activity below:

- Get two strips of manila paper that are the same size.
- Fold one paper into five equal parts. Cut out two of the five parts. What fraction can represent them?
- Then fold the second paper strip into ten equal parts. Cut out four of the ten parts. What fraction can represent them?



- Compare the parts you have cut. Write them as fractions.
- What did you discover?
- Now, write the cut parts as decimals.
- Explain your observations.



Summary

To convert a fraction to decimal,

- Divide the numerator (top number) by the denominator (bottom number).

Example: $\frac{3}{4} = 3 \div 4 = 0.75$.

- For fractions with denominators that are powers of 10 (10, 100, 1000, etc.):
Write the numerator directly as a decimal, using the denominator's place value.

Example: $\frac{7}{10} = 0.7$. $\frac{5}{10} = 0.5$, etc.

- Terminating and repeating decimals:

If the division ends (no remainder), it's a **terminating decimal**

(e.g., $\frac{1}{2} = 0.5$).

If the division repeats indefinitely, it's a **repeating decimal** (e.g., $\frac{1}{3} = 0.3333\dots$).

- Shortcut for Common Fractions:

Memorize conversions like $\frac{1}{4} = 0.25$. $\frac{1}{8} = 0.125$ etc.

Examples:

a) Write $\frac{2}{5}$ as a decimal.

Solution

Method 1: Divide 2 by 5.

$$\begin{array}{r} 0.4 \\ 5 \overline{)20} \\ \underline{-20} \\ 00 \end{array}$$

Method 2:

$\frac{2}{5}$ Multiply the numerator and denominator by a common number.
Make the denominator 10.

$$\frac{2}{5} \times \frac{2}{2} = \frac{4}{10} \text{ This is four tenths. Thus } \frac{4}{10} = 0.4$$

b) Write $\frac{15}{25}$ as a decimal

Method 1: Divide 15 by 25.

$$\begin{array}{r} 0.6 \\ 25 \overline{)150} \\ \underline{-150} \\ 000 \end{array}$$

$$\text{Thus, } \frac{15}{25} = 0.6$$

Method 2: Multiply the numerator and denominator by a common number. Make the denominator 100:

$$\frac{15}{25} \times \frac{4}{4} = \frac{60}{100} . \text{ This is sixty hundredths. Thus } \frac{60}{100} = 0.60$$



Application Activity 5.4

1. Write the following as decimals.

a) $\frac{5}{10}$ b) $\frac{6}{25}$ c) $\frac{8}{1000}$ d) $\frac{21}{25}$ e) $\frac{2}{50}$ f) $\frac{3}{8}$

2. Match the fractions to the decimals. Explain.

	Fractions	Decimals
(a)	$\frac{8}{100}$	0.1
(b)	$\frac{8}{1000}$	0.9
(c)	$\frac{1}{10}$	0.008
(d)	$\frac{9}{10}$	0.08
(e)	$\frac{165}{100}$	0.165
(f)	$\frac{165}{1000}$	1.65

3. John gave an orange to four pupils to share equally. Find the decimal fraction of the orange each got. Use a diagram to present decimal fraction of an orange for each pupil

5.5 Conversion of decimals to fractions



Activity 5.5

We can change a decimal into a fraction. For example, $0.5 = \frac{5}{10} = \frac{1}{2}$.

Now, Change the following into fractions.

(a) 0.8 (b) 0.7 (c) 0.45 (d) 0.658

What steps do you follow?



Summary

To convert a decimal into a fraction, you have to know the decimal place values.

- Write the decimal as a fraction with 1 as the denominator:
- Multiply numerator and denominator by 10 power n (where n is the number of decimal places).

- Simplify the fraction by dividing numerator and denominator by their greatest common divisor (GCD).

For **example:** 0.4 is 4 tenths and $0.4 = \frac{0.4}{1} = \frac{0.4 \times 10}{1 \times 10} = \frac{4}{10} = \frac{2}{5}$.

0.46 is 46 hundredths, $0.46 = \frac{0.46}{1} = \frac{0.46 \times 100}{1 \times 100} = \frac{46}{100}$.

We then simplify the fraction.

Examples:

Convert the decimals into fractions.

(a) 0.475 (b) 0.89

Solution

$$(a) 0.475 = \frac{475}{1000}$$

Remember:

(i) 0.475 is 475 thousandths. So

$$0.475 = \frac{475}{1000}.$$

(ii) Simplify by dividing with a common number.



$$\frac{\overset{19}{\cancel{475}}}{\overset{19}{\cancel{1000}}} = \frac{19}{40}$$

(b)



0.89 is 89 hundredths.

$$\text{So } 0.89 = \frac{89}{100}.$$

Check if it can be simplified. **No.**

$$\text{Therefore, } 0.89 = \frac{89}{100}$$



Application Activity 5.5

1. Change the following decimals into fractions

(a) 0.75 (b) 0.455 (c) 0.625 (d) 0.075 (e) 0.41

2. Which one is greater? Justify your answer.

(a) $\frac{3}{5}$ or 0.007 (b) $\frac{1}{5}$ or 0.75 (c) $\frac{2}{5}$ or 0.25

3. Discuss and arrange the following from the smallest to the largest.

(a) 0.56, $\frac{3}{10}$, 0.09 (b) $\frac{8}{10}$, 0.84, 0.25 (c) 0.44, $\frac{1}{4}$, 0.5

5.6 Multiplication of decimal numbers

We can multiply a decimal number by a whole number. We can also multiply a decimal number by a decimal number. Let us study the following activity.



Activity 5.6

- Cut 2 oranges into halves. Each half is 0.5 of an orange. Put three halves together. What decimal number is three halves?
- Now, multiply the following:
 $0.5 \times 3 =$ (ii) $0.5 \times 6 =$ (iii) $0.5 \times 0.5 =$
What do you notice? Present your findings.



Summary

To multiply a decimal number either by a whole number or a decimal number,

- first, temporarily ignore the decimal points and multiply the numbers as if they were whole numbers.
- Secondly, count the total number of decimal places in the factors.
- Thirdly, place the decimal point in the product so as to include that same number of decimal places.

Examples:

Multiply the following:

(a) $0.8 \times 4 =$ (b) $0.2 \times 0.7 =$ (c) $0.4 \times 0.16 =$

Solution:



Express decimal number in terms of place values
e.g. 0.8 is $\frac{8}{10}$ or 8 tenths. Multiply whole numbers
and denominators separately.

(a) 0.8×4 is $\frac{8}{10} \times \frac{4}{1} = \frac{8 \times 4}{10 \times 1} = \frac{32}{10} = 3.2$

(b) 0.2×0.7 is $\frac{2}{10} \times \frac{7}{10} = \frac{2 \times 7}{10 \times 10} = \frac{14}{100} = 0.14$

(c) 0.4×0.16 is $\frac{4}{10} \times \frac{16}{100} = \frac{4 \times 16}{10 \times 100} = \frac{64}{1000} = 0.064$

Note: 0.2×0.7 . There are 2 decimal places

$$2 \times 7 = 14 = 014$$

Therefore, $0.2 \times 0.7 = 0.14$



Application Activity 5.6

1. Work out the following.

(a) 0.06×7 (b) 7.04×4 (c) 2.66×11

2. Multiply each of the following decimal fractions.

(a) 0.2×0.6 (b) 0.14×0.2 (c) 0.17×0.3

3. Multiply the following. Justify your answers.

(a) 0.089×100 (b) 2.533×100 (c) 1.485×100

4. Multiply the following. Discuss and present your steps.

(a) $0.006 \times 1\,000$ (b) $13.507 \times 1\,000$ (c) $4.005 \times 1\,000$

5. A motorcycle consumes 1 litre of fuel to cover 5.25 km.

Calculate the distance it would cover with 1.5 litres of petrol.

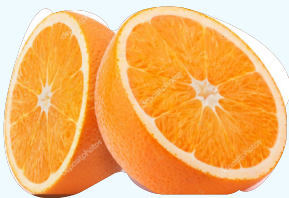
5.7 Division of decimal numbers



Activity 5.7

1. Let us do the activity below.

Cut an orange into two equal parts. Share the half of the orange equally among 4 pupils.



Halves



Halves divided in 4 equal parts

What fraction of orange does each of the four get?

2. Now, discuss the following and present your findings.

i) $0.5 \div 4 =$ ii) $0.5 \div 5 =$ iii) $0.5 \div 0.5 =$ iv) $0.005 \div 0.04 =$



Summary

To divide decimal numbers, we identify the number of decimals in the denominator. If it is

in tenths, we multiply by $\frac{10}{10}$. If it is in hundredths, we multiply by $\frac{100}{100}$.

If the denominator is in thousandths, we multiply by $\frac{1000}{1000}$.

Examples:

1. Work out $5 \div 0.2$

Solution

Write it as a fraction: $5 \div 0.2 = \frac{5}{0.2}$

(How can we make the denominator a whole number? Multiply both the numerator and the denominator by 10)

$$\frac{5}{0.2} = \frac{5 \times 10}{0.2 \times 10} = \frac{50}{2} = 25$$

2. Work out $3.609 \div 0.03$

Solution

Write it as a fraction

$$3.609 \div 0.03 = \frac{3.609}{0.03} \quad (\text{Since the denominator is in hundredths, multiply it by } \frac{100}{100})$$

$$\frac{3.609}{0.03} = \frac{3.609 \times 100}{0.03 \times 100} = \frac{360.9}{3} = 120.3$$



Application Activity 5.7

1. Work out the following.

(a) $0.2 \div 5 =$ (b) $4 \div 0.02 =$ (c) $0.44 \div 1.1 =$
(d) $1.792 \div 0.07 =$ (e) $6.4 \div 1.6 =$ (f) $2.4 \div 0.08 =$

2. Work out the following. Discuss the steps you followed.

a) $12.22 \div 26 =$ (b) $8.648 \div 0.23 =$ (c) $0.13 \div 0.05 =$

5.8 Mixed operations for multiplication and division



Activity 5.8

Work out the following

a) $0.6 \times 0.2 \div 0.04 =$ b) $\frac{0.02 \times 0.6}{0.04} =$



Summary

For mixed operations for multiplication and division,

- we first put it into decimal fractions form,
- then we identify the number of decimals in the denominator.

If it is in tenths, we multiply by $\frac{10}{10}$.

If it is in hundredths, we multiply by $\frac{100}{100}$.

If the denominator is in thousandths, we multiply by $\frac{1000}{1000}$.

Examples:

1. Work out the following.

a) $0.06 \times 0.2 \div 0.04$

b) $\frac{0.1 \times 0.36}{0.09}$

Solution

$$\begin{aligned} \text{a) } 0.06 \times 0.2 \div 0.04 &= \frac{0.02 \times 0.6}{0.04} = \frac{0.02 \times 0.6 \times 100}{0.04 \times 100} \\ &= \frac{6 \times 0.2}{4} = \frac{3 \times 0.2}{2} = \frac{3}{1} \times 0.1 = 0.3 \end{aligned}$$

$$\text{b) } \frac{0.1 \times 0.36}{0.09} = \frac{0.1 \times 36}{9} = \frac{0.1 \times 0.36}{9} = 0.1 \times 4 = 0.4$$



Application Activity 5.8

1. Work out the following.

a) $0.4 \times 0.2 \div 0.8 =$ b) $5 \times 1.6 \div 0.08 =$ c) $29.14 \times 9.2 \div 0.2 =$

d) $0.04 \times 0.2 \div 0.4 =$ e) $0.5 \times 0.2 \div 0.4 =$

2. Work out and explain the steps you followed.

a) $\frac{1.2 \times 1.2}{1.2} =$ b) $\frac{40 \times 0.4}{0.16} =$ c) $\frac{0.3 \times 0.03}{0.009} =$

5.9 Problems involving multiplication and division of decimals



Activity 5.9

Work out the following

- 1) How many pieces of tenths would I cut from 9 oranges? Explain your answer.
- 2) A quarter of an orange is shared equally by 5 pupils. What size of orange does each pupil get? Explain your steps to answer.



Summary

To work out problems involving multiplication and division of decimals, we follow these steps:

For **multiplication**, we use the decimals as whole numbers, then we add the total number of decimal places in the factors and place the decimal in the product accordingly.

For **division**, we make the divisor a whole number by shifting the decimal point, and shift the decimal in the dividend the same number of places.

Examples:

1. A small bottle holds 0.3 litres of milk. Discuss how much milk is held by 12 such small bottles?

Solution

One small bottle holds 0.3litres. To know the quantity of milk held by 12 such small bottles, we multiply 0.3litres to 12 bottles. Which means, $0.3\text{litres} \times 12 = 3.6$ litres.

2. A roll of cloth 540 m long was cut into equal pieces, each 3.6 m. Each piece was enough to make a dress. Calculate the number of dresses made from the roll.

Solution

To get the number of dresses made from the roll, we divide 540m by 3.6m.

$$\text{Number of dresses} = \frac{540\text{m}}{3.6\text{m}} = \frac{540\text{m} \times 10}{3.6\text{m} \times 10} = \frac{5400}{36} = 150 \text{ dresses}$$



Application Activity 5.9

1. 20 pupils were each given 0.5 loaf of bread. How many loaves of bread were given in total? Discuss your answer.
2. A bottle contains 0.3 litres. How many litres would 3 bottles contain?
3. David drives 2.2km a day. How far does he travel in 7 days?
4. I have 8.45kg of coffee. I want it in 0.25kg bags. How many bags do I need?



End unit assessment

Assessing knowledge

1. What is the **place value** of 5 in 264.235?
2. Write 0.236 as a fraction.
3. Identify the **place value** of the digit 2 in 0.253. Explain your steps.
4. Write 52.067 in words.

5. Read and write the following in figures:
- (a) Six hundred sixty-seven thousandths.
 - (b) Seventy-two hundredths.
 - (c) One and one tenth.
6. Which symbol ($>$, $<$, or $=$) correctly compares these decimals?
- (a) **0.081** _____ **0.095**
 - (b) **0.25** _____ **0.205**
 - (c) **0.65** _____ **0.650**

Assessing skills

7. Calculate:
- (a) Multiply **0.23×0.23** .
 - (b) Divide **$3 \div 0.3$** .
 - (c) Work out **$1.44 \div 1.2$** .
 - (d) Solve **$0.2 \times 0.5 \div 0.01$** (explain steps).
8. **Arrange** in ascending order (From smallest to largest): **0.02; 0.85; 0.26**.
9. **Convert** the fraction $\frac{1}{2}$ into a decimal number. Discuss the steps.
10. **Share two oranges equally among twenty people**. What decimal number represents each person's share? Explain.

Assessing attitudes and values

11. Why is it important to understand decimal numbers in real-life when dealing with money or measurements?
12. If you were sharing food or money among a group of children using fraction or decimals, why is it important to be precise and fair?

Number of pupils	Counters
	 5
	 10
	_____
	_____

UNIT 6

PROPORTION

Key unit competence: You will be able to apply direct proportion in a practical context.

Learning activities:

By the end of this unit, you should be able to:

Knowledge and understanding

- Explain the concept of direct proportions.
- Explain how to work out problems involving direct proportions.
- State where to apply direct proportions in a practical context.

Skills:

- Calculate numbers involving direct proportions.
- Solve mathematical problems involving direct proportions.

Attitudes and values:

- Appreciate the importance of using direct proportions in daily life situations.
- Be confident and accurate when working out direct proportions.



6.0 Introduction

Direct proportion is a powerful idea that helps us solve real-life problems involving sharing, scaling, and comparing quantities.

In this unit, you will learn that when one amount increases, another increases at the same rate. When one amount decreases, another decreases at the same rate. That is what we call **direct proportion**.

You will discover that direct proportion is all around us, in cooking, shopping, travel, and even when mixing paint or creating art.



Introductory Activity

On a scale drawing, 1 cm represents 5 km of a road. What length of road is represented by 5cm? What length of road is represented by 10cm?

What length of the drawing that represents 20km of a road?

Do you think that when the length of the drawing increases, the road represented increases also? Explain your answer.

Provide examples of other variables that increases when the other increases.

6.1 Meaning of direct proportion



Activity 6.1

Let each pupil take five books or counters. Record your counters as shown:

- How many counters gotten if two pupils put their counters together. Record the number of counters.
- How many counters gotten if 3 pupils put their counters together. Record the number of counters.
- Carry on up to four pupils of you and fill in the blanks accordingly.

Number of pupils	Counters
	 5
	 10

Is it possible to determine the number of counters for 6 pupils, 8 pupils or any number of pupils? If Yes, how can you get the number?



Summary

Rule for direct proportion:

When one quantity increases, the second quantity increases in a similar way.

When one quantity decreases, the second quantity decreases in a similar way.

Examples:

- (a) On a scale drawing, 1 cm represents 10 km of road. What length of road is represented by 3 cm?

Solution

1 cm represents 10 km

3 cm represent ___?_

It is clear that 3 cm will represent a longer length of road.

3 cm represents $3 \times 10 \text{ km} = 30 \text{ km}$

Length of the drawing	1cm	2cm	3cm	4cm
Actual length of the road	10km	20km	30km	40km

- (b) I take 30 minutes to walk to school. How much time do I need to walk to school and back home?

Solution

The distance to school from home is fixed. I take 30 minutes one way.

To walk to school and back home is two ways. I take $30 \text{ minutes} \times 2 = 60 \text{ minutes} = 1 \text{ hour}$.



Application Activity 6.1

Work out the following:

- 1) Fill in the table below if pupils eat the same quantity of bread.

Number of pupils	1	2	3	4	5	___
Quantity of bread eaten	$\frac{1}{2}$	1	___	___	___	20

- 2) Study the table below. Fill in the missing numbers if pupils have the same number of exercise books. Justify your answers.

Number of pupils	1	___	4	20	___	35
Number of Exercise books	4	8	___	___	100	___

- 3) I have six water tanks. Each tank holds 1,500 litres of water. How many litres of water can my tanks hold? Explain your answer.
- 5) We are twenty pupils. Each of us has 10 notebooks. What is the total number of our notebooks?

6.2 Ratios and direct proportion



Activity 6.2

- Give three books to each volunteer.
- Now, have 2 volunteers put their books together. How many books do they have?
- Have four volunteers put their books together. How many books do they have?

Divide: $\frac{\text{number of books}}{\text{number of volunteers}}$.

This is the book to volunteer ratio.

- Count the number of boys and girls in your group. What ratio is it?
- Explain your findings.
- Discuss situations where ratios are used in daily life.



Summary

In direct proportions, we compare quantities of different items.

For example, 1 boy has 2 books. A boy and books are different items.

In this case, 1:2 is the ratio of boy to books.

In a direct proportion, the **ratio indicates a constant relationship between two quantities**.

Example:

Three children had nine sweets.

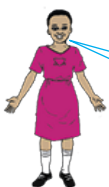
(a) What is the ratio of child to sweets?

(b) How many sweets are needed for 4 children?

Solution

$$\text{a) Ratio} = \frac{\text{number of children}}{\text{number of sweets}} = \frac{3 \text{ children}}{9 \text{ sweets}} = 1 \text{ child to 3 sweets or } 1 : 3$$

Note:



3 children to 9 sweets is a ratio.
It is simplified to 1 : 3 by
dividing by 3.

b) 1 child gets 3 sweets.

4 children get 3×4 sweets = 12 sweets



Application Activity 6.2

Work out the following.

1) Express each of the ratios in its simplest form.

(a) 8:24 (b) 21:42 (c) 9:27

2) The weight of mathematics book for Peter is 420 g. The weight of his dictionary is 560 g. What is the ratio of his mathematics book to his dictionary?

3) Dusabimana has 360 oranges and 120 mangoes. Find the ratio of her mangoes to her oranges. Discuss your answer.

4) In a game park, there are 120 giraffes and 360 antelopes. Find the ratio of giraffe to antelope in its simplest form.

6.3 Constant of proportionality and the application of direct proportion in real life



Activity 6.3

Work out the following:

If James donated 2 apples to 4 children, and then he donated 4 apples to 8 children, what is the number of apples for 1 child? This number is called constant of proportionality.



Summary

The constant of proportionality is a constant number that, when multiplied by one variable, results in the other variable.

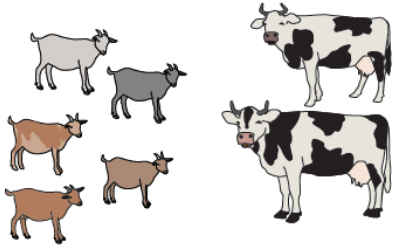
When the quantity **m** increases, the other quantity **n** in the way that **n = k x m**.

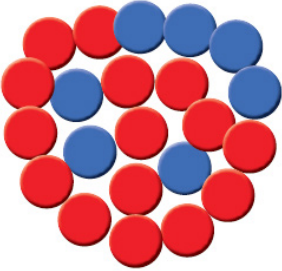
Or **n/m = k**. we say that the number **k** is a constant of proportionality.

When **m = 1**, **n = k x 1 = k**.

Examples:

1) Write the ratios for the following and work out the answers to the questions below:

Picture	Ratio	Question and answer
	Number of goats to cows is 5 to 2 $\frac{5}{2} = 2.5 = K$	If there are 20 goats, how many cows? Answer: The number of goats becomes $20 \div 2.5 = 8$ The constant is 2.5 Therefore, there will be 8 cows

Picture	Ratio	Question and answer
	Number of books to children is 6 to 1 . $K = \frac{6}{1} = 6$	If there are 7 children, how many books? Answer: The number of children becomes $7 = 1 \times 7$. The constant is 6. Therefore, the number of books will be $6 \text{ books} \times 7 = 42 \text{ books}$.
	Number of red to blue is 15 to 7 . $K = \frac{15}{7}$	If there are 14 blue counters, how many red should be there? Answer: The number of blue counters becomes 14. The constant is $\frac{15}{7}$. Therefore, the number of red counters will be $15 \text{ red counters} \times \frac{14}{7} = 30 \text{ red counters}$.

2) There are 30 pupils in a classroom. They each need 4 pieces of papers for exam. How many pieces of papers are needed in total?

Solution

One pupil needs 4 pieces. $k = 4$.

Then, 30 pupils need $4 \times 30 = 120$ pieces of papers.

2) 3 buses carry a total of 75 people. How many people will 4 buses carry?

Solution

2) buses carry a total of 75 people; $k = \frac{75}{3} = 25$

1 bus will carry $75 \div 3 = 25$ people

4 buses will carry $25 \text{ people} \times 4 = 100$ people.



Application Activity 6.3

- It takes 40 minutes to walk to the market. How much time do I need to walk to the market and back home every day for one week?
- Dusabimana travelled from 6:00 a.m to 10:00 a.m on Monday to the field. He was covering 60 km every hour. What was the total distance of his journey? Present your answer.
- 3cm on the map represents 27km.
 - What distance is represented by 6cm?
 - What distance is represented by 10cm?

6.4 Scale drawing and direct proportion



Activity 6.4

Work out the following:

- Measure the real object (e.g., height = 40cm).
- Measure the size of the height of the drawing (e.g., height = 8cm).
- Calculate the **scale factor** and complete:

$$\text{Scale factor} = \frac{\text{real object size}}{\text{drawing size}} = \underline{\hspace{2cm}}$$

- If the height of the drawing is 16cm, what is the height of the real object?



Summary

- Scale drawings links the drawing size to real sizes through **direct proportion**.
- The **scale factor** links the drawing's size and the actual size.
- In the activity 6.4, the drawing is scaled down by a factor of 5. This means every part of the drawing is **directly proportional** to the real object by the factor **1:5** (The real object is divided by 5 or the drawing is multiplied by 5 to get the real size).

Example

Look at the different scaled drawings of two rectangles with different sizes and calculate each scale factor: For drawing 1: real size = 50cm, drawing size = 10cm, for drawing 2: real size = 60cm, drawing size = 12cm

Solution:

For drawing 1: Real size = 50cm, drawing size = 10cm $\rightarrow$ scale = $\frac{50}{10} = 5$

For drawing 2: Real size = 60cm, drawing size = 12cm $\rightarrow$ scale factor = $\frac{60}{12} = 5$

Thus, **all parts** are scaled proportionally when the same scale factor applies.



Application Activity 6.4

- "A model car is 15cm long, and the real car is 6 meters long. Find the scale factor
- If the model is 15cm, and you want to draw a larger model with a scale of 1:20, what would be the real size?
- If a drawing is 6cm and the scale factor is 1:10, what is the actual size of the object?"

6.5 Problems involving ratios



Activity 6.5

A minibus carries 28 passengers. A bus carries 64 passengers. What is the ratio of passengers carried by minibus to bus? Present your answer.



Summary

When solving **ratio problems**, we follow the steps here below:

- **Understand the ratio:** A ratio compares two or more quantities (e.g., 3:2 or 4:5:6). It tells you how many parts of each quantity are involved.
- **Write the ratio as a fraction (if needed).** Example: A ratio of 3:2 can be written as $\frac{3}{2}$ or as parts of a total (e.g., 3 parts to 2 parts = 5 total parts).
- **Set up proportions (for comparison problems):** If given one part and asked to find another, set up a proportion:
$$\frac{\text{part 1}}{\text{part 2}} = \frac{\text{known value}}{\text{unknown value}}$$
- **Use total parts to find specific values (for whole amount problems):** Add all parts in the ratio. Divide the total amount by the total number of parts to find the value of one part. Multiply by the relevant part to find individual quantities.
- **Check units and consistency:** Make sure all values are in the same units. Verify that the ratio remains the same after solving.

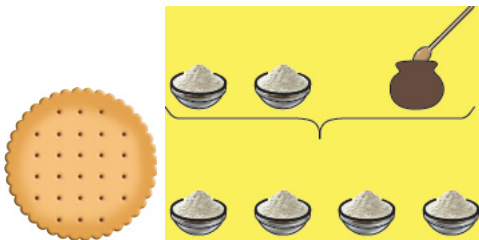
Examples:

- 1) A bag contains red and blue beads in a ratio of 3:5. If there are 40 beads in total, how many of each colour are there?

Solution:

- Total parts = $3 + 5 = 8$
- Value of one part = $\frac{40}{8} = 5$
- Red beads = $3 \times 5 = 15$, Blue beads = $5 \times 5 = 25$

- 2) Biscuits contain flour and butter in a ratio of 2:1.



To make one batch of biscuits, we use 200g of flour and 100g of butter. If we use 400g of flour, how much butter would we need to use?

Solution:

Ratio: 2: 1

200 g of flour : 100g of butter.

To find the number of grams for butter, we divide the number of flour by 2.

If we use 400 g of flour, we use $(400 \div 2)$ of butter = 200 g of butter.



Application Activity 6.5

1. My necklace uses yellow and red colours at a ratio of 1:8. How many red beads do I need for 6 necklaces?
2. A certain farmer has goats and chickens in her farm. The ratio of goats to chickens is 3:5 in her farm. The total number of chickens and goats is 320.
 - a) How many chickens are there in her farm?
 - b) Calculate the number of goats in her farm. Explain your steps.
3. The ratio of mass of maize to rice was 3:5. The total mass of maize and rice was 96 kg. Find the mass of rice and maize. Explain your steps to answer.

6.6 Problems involving direct proportion



Activity 6.6

Solve the problems below:

A family uses 90 litres of water every day from their tank.

- (a) How many days will it take the family to use 2,700 litres of water from the tank?
- (b) How many litres of water from the tank are used in 20 days? Explain your steps.
- (c) Discuss examples where you use direct proportions.



Summary

For problems involving direct proportions,

If two things are directly proportional, they increase or decrease together. Furthermore, directly proportional means there is a constant ratio between the two quantities.

Examples:

- 1) In a peace rally, 3 speakers talk to people in 7 districts. How many speakers are needed for 42 districts? Discuss the importance of peace in our country.

Solution

3 speakers talk to 7 districts.

We can write this as $\frac{3}{7}$ speakers per district.

Then 42 districts require $42 \times \frac{3}{7}$ speakers = (6×3) speakers = 18 speakers

18 speakers can talk about peace in 42 districts.

2) In a certain town, the ratio of adults to children is 4:5. The number of adults is 400.

(a) Calculate the number of children in the town.

(b) From Monday to Friday, 100 adults go for their jobs away from the town. Similarly, 200 children go to their schools away from the town. Find the total number of adults and children in the town on Tuesday at 10:00 a.m.

Solution

(a) Ratio of adult to children = 4:5 = 400:children

Clearly number of children = $5 \times 100 = 500$.

Or $\frac{4}{9} \times \text{total population} = 400$

Total population = $\frac{9}{4} \times 400 = 900$, here adults are 400.

So, children are $900 - 400 = 500$.

(b) We know, there are 400 adults and 500 children. On Tuesday, we have $(400 - 100)$ adults = 300 adults and $(500 - 200)$ children = 300 children. Total number of adults and children are $300 + 300 = 600$.



Application Activity 6.6

1. The weight of eight copies of P5 mathematics book is 480 g.

What is the weight of one copy?



2. A car travels 12 km on one litre of petrol.

How many kilometres will the car travel on three litres?

3. In transport business, three minibuses carry 54 passengers. How many passengers will 8 such minibuses carry? Discuss the importance of transport business.

4. A truck uses 3 litres of petrol to travel 72 kilometres. How much petrol does it use in a journey of 648 kilometres? Explain your answer.



5. A house cleaner uses 8 litres of water every day to clean a house. How many days will 64 litres of water last for his work?

6. A mother feeds her baby with 4 glasses of milk every day. This is in order to keep the baby healthy. How many days will the baby take to drink 132 glasses of milk? Discuss your steps to answer.



7. The weight of 15 boys is 300 kg. The boys have the same weight.

Calculate the weight of one boy? Present your answer.



8. One aircraft carries 100 passengers.

How many passengers are carried by 6 such aircrafts? Explain your answer.



End unit assessment

Assessing Knowledge and Understanding

- 1) Study the following table.

It shows the number of teachers to pupils in a certain country. Fill in the missing numbers. State the ratio used and explain your observation.

Number of teachers	1	2	3	4	5	_____	7
Number of pupils	30	_____	_____	_____	180	150	_____

- 2) In a school, each pupil is given 5 exercise books. How many exercise books will 30 pupils get from the school?
- 3) Abel's farm has 20 mango trees and 100 coffee trees. Find the ratio of mango to coffee trees on his farm.

Assessing skills

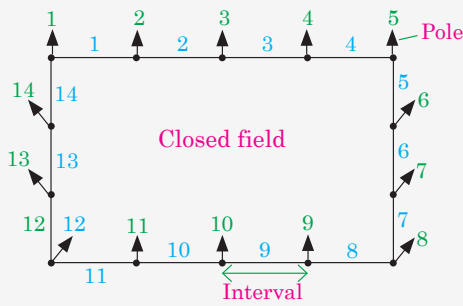
- 4) At breakfast 2 loaves of bread are served to 8 children. How many loaves are needed for 64 children?
- 5) For good health, a pupil should drink 5 glasses of water every day. Calculate how many glasses of water a pupil should drink in 10 days for good health.
- 6) It takes 40 minutes to walk to the market. How much time do I need to walk to the market and back home every day for one week?

Assessing attitudes and Values

- 7) After solving Question 3 (Abel's farm), explain: **Why is planting trees important for communities?**
- 8) In Question 4 (bread loaves), discuss: **Why is breakfast considered the most important meal of the day?**
- 9) Based on Question 5 (water consumption), write a short note on: **How does staying hydrated contribute to a pupil's learning?**

UNIT 7

SOLVING PROBLEMS INVOLVING MEASUREMENTS OF LENGTH, CAPACITY AND MASS



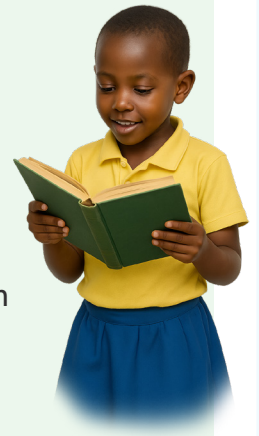
Key Unit competence: You will be able to solve problems involving measurements of length, capacity, mass and calculating number of intervals.

Learning activities:

By the end of this unit, you should be able to:

Knowledge and understanding

- Explain how to convert units of length, capacity and mass measurements.
- Define the relationship between the number of poles and intervals on a closed and open line.
- Explain steps to follow to solve problems involving length, capacity and mass.



Skills:

- Convert units of length, capacity and mass measurement.
- Calculate the number of intervals and poles on an open and closed line.
- Solve problems involving length, capacity and mass measurements.
- Apply the knowledge of length in solving problems involving intervals.

Attitudes and values:

- Appreciate the importance of measurement of length, capacity and mass in daily life situations.

7.0 Introduction

In this unit, you will explore the world through measurement! You will learn how to solve real-life problems using three key types of measurement:

Length: how long, tall, or far something is.

Capacity: how much liquid something can hold.

Mass: how heavy something is.

We use measurement everywhere, in the kitchen, on the road, at the shops, and even in sports.



Introductory Activity

- What do you think happens when equal number of objects are placed at the same distance apart on an open line and a closed line?
- Did you ever try to think about the relationship between the number of objects on any given line and the interval distance?
- Where do you think the concept of interval distance can be applied?

7.1 Revision on problems on length, capacity and mass



Activity 7.1

1. Complete the conversions here below.

(a) Length

km	hm	dam	m	dm	cm	mm
1	0	0	0	0	0	0
				1	0	0

From table;

i) $1 \text{ dm} = 100 \text{ mm}$

ii) $1 \text{ dam} = \underline{\hspace{1cm}} \text{ m}$

iii) $1 \text{ m} = \underline{\hspace{1cm}} \text{ cm}$

(b) Capacity

kl	hl	dal	l	dl	cl	ml
1	0	0	0	0	0	0
			1	0	0	0

From table;

i) $1 \text{ l} = 1\,000 \text{ ml}$

ii) $1 \text{ dal} = \underline{\hspace{1cm}} \text{ l}$

iii) $1 \text{ l} = \underline{\hspace{1cm}} \text{ dl}$

(c) Mass

kg	hg	dag	g	dg	cg	mg
1	0	0	0	0	0	0
			1	0		

From table;

i) $1 \text{ g} = 10 \text{ dg}$

ii) $1 \text{ dag} = \underline{\hspace{1cm}} \text{ g}$

iii) $1 \text{ kg} = \underline{\hspace{1cm}} \text{ mg}$

t	q	—	kg
	1	0	0
		1	0



Summary

When calculating,

- Always **convert units**,
- It is better to use the conversion table to avoid confusion,
- **Underline key values** and units in word problems,
- Write the **unit** in every answer.

Examples:

1) Convert the following into centimetres. Explain your steps.

- (a) 30 mm (b) 60 mm (c) 0.7 km

Solution:

a) 30 mm = **3 cm**

b) 60 mm = **6 cm**

c) 0.7 km = **70,000 cm**

2) (a) Subtract 2m 6dm 4cm from 9m. Give answer in dm.

(b) A person's stride is 90 cm. How many strides can she take in 27dam to her school?

Solution:

a) 2 m 6 dm 4 cm = ? dm

6 dm = 6 dm

4 cm = 0.4 dm

Thus, 2 m 6 dm 4 cm = 26.4 dm

9m = 90 dm

Subtract: 90dm - 26.4 dm = 63.6dm

(b) Change 27dam to cm.

km	hm	dam	m	dm	cm	mm
	2	7	0	0	0	

From the conversion table, 27dam = 27,000 cm

Now, Interval = Stride's length = 90 cm.

Distance = 27 dam or 27,000 cm.

So we calculate the number of strides:

$$\text{Number of strides} = \frac{\text{distance}}{\text{interval}} = \frac{27,000}{90} = \mathbf{300 \text{ strides}}$$



Application Activity 7.1

1. Convert the following measurements into metres.

- (a) 260 cm (b) 4,000 cm (c) 6 km (d) 60 cm

2. Convert the following measurements into kilometres.

- (a) 600 cm (b) 360,000 mm (c) 800 m (d) 14,000 cm

3. Work out the following.

(a) $4.5 \text{ kg} + 13.6 \text{ dag} = \underline{\hspace{2cm}} \text{ kg}$

(b) $4 \text{ hl} - 20 \text{ dal} = \underline{\hspace{2cm}} \text{ litres}$

(c) $2 \text{ dam } 3 \text{ m} \times 5 = \underline{\hspace{2cm}} \text{ hm}$

4. Change the following into litres.

(a) 30 dl (b) 105 dl (c) 1,050 ml (d) 2,500 ml

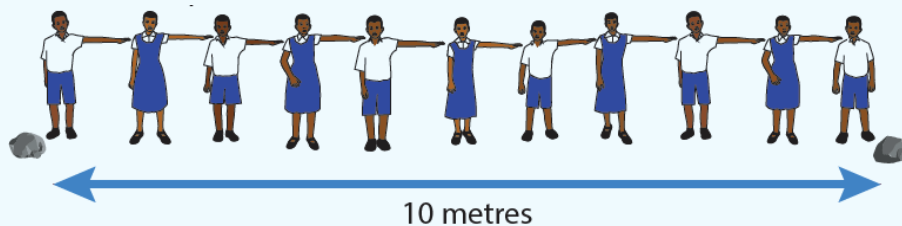
5. Write the following weights in tonnes. (a) 3,450 kg (b) 2,050 kg (c) 170,000 kg.

7.2 Number of intervals between objects on an open line



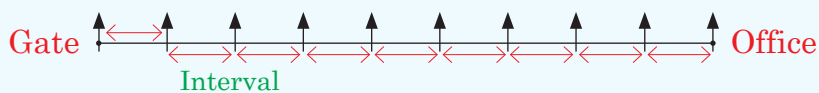
Activity 7.2

- 1) Stand along the line with one arm stretched between you and the next child such that there is 1m between two consecutive pupils. The child at the end keeps their arm down because there is no one next to them to touch.



How many are you? How many intervals are there? Is the number of intervals equal to the number of pupils?

- 2) Make a 1 m stick. Use it to mark fixed distances from one point to another. Fixed distances are called intervals. Make markings by standing along the lines at intervals of 1 m. Look at the figure below:



- (i) How many of you stood along the line?
(ii) How many intervals are there?
(iii) What other ways could you stand along the line

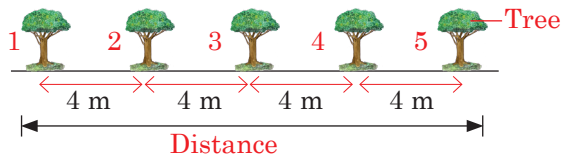


Summary

The **number of intervals** between poles or trees fixed at equal intervals calculated as follow:

Look at an open line below:

1) When no interval space left at the ends:



- 4 m is the interval.
- Distance = $4 \text{ m} \times 4 = 16 \text{ m}$
- Number of intervals = 4 or $(5 - 1)$

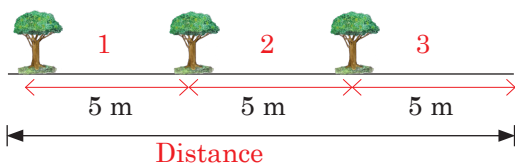
Number of intervals = number of trees (poles) - 1

- Number of trees = 5 or $(4 + 1)$

$$\text{Number of trees} = \frac{\text{distance}}{\text{interval}} + 1$$

$$\text{e.g. } \frac{16\text{m}}{4\text{m}} + 1 = 4 + 1 = 5 \text{ trees}$$

2) When interval space is left at one end:



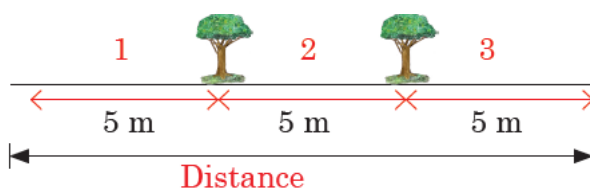
- 5 m is the interval.
- Distance = $5 \text{ m} \times 3 = 15 \text{ m}$
- **Number of intervals = 3**

Number of intervals = number of trees (poles)

- Number of trees = 3
- In this case, **number of trees** = $\frac{\text{distance}}{\text{interval}}$

$$\text{e.g. } \frac{15\text{m}}{5\text{m}} = 3 \text{ trees}$$

3) When interval space is left at both ends:



- 5 m is the interval
- Distance = $5 \text{ m} \times 3 = 15 \text{ m}$
- Number of intervals = 3

Number of intervals = number of trees +1

- Number of trees = 2
- In this case, **number of trees** = $\frac{\text{distance}}{\text{interval}} - 1$.

Examples:

A man's stride is 10 dm long. He walks a distance of 10 dam. How many strides does he take to cover the distance?

Solution

Change 10 dam to dm.

km	hm	dam	m	dm	cm	mm
	1	0	0	0		

From the conversion table, 10 dam = 1,000 dm

Now, Interval = Stride's length = 10 dm. Distance = 1,000 dm or 10 dam.

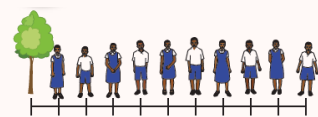
So, we calculate the number of strides: Number of intervals.

$$\text{Number of strides} = \frac{\text{distance}}{\text{interval Length}} = \frac{1000}{10} = \mathbf{100 \text{ strides}}$$

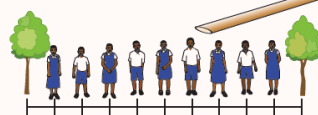


Application Activity 7.2

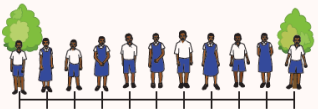
- 1) A road is 2 km long. Trees were planted 2 m apart along one side of the road. An interval of 2 m was left at one end without a tree due to an existing shop. How many trees were planted along the road?
- 2) Match the rule to the correct example and work out the answers. Total length 20m, length of interval 2m.



$$\text{Number of children} = \frac{\text{total distance}}{\text{interval length}} - 1$$



$$\text{Number of children} = \frac{\text{total distance}}{\text{interval length}} + 1$$



$$\text{Number of children} = \frac{\text{total distance}}{\text{interval length}}$$

- 3) A path is 5 dam long. Trees are to be planted at intervals of 5 dm on 117 both sides. How many trees are needed?
- 4) Electric poles are fixed along one side of 16km section of the road. This was to light the road. The poles are placed 10 m apart from each other. How many poles are fixed? Discuss why it is important to light the road?

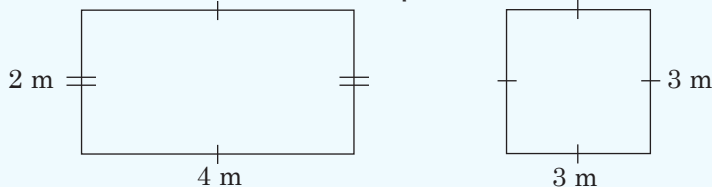
7.3 Finding the number of intervals on a closed line



Activity 7.3

1) Measure one-metre-long stick.

- Use a set square to make a square and a rectangle on the ground using the measured stick. Let their perimeters be 12 m.

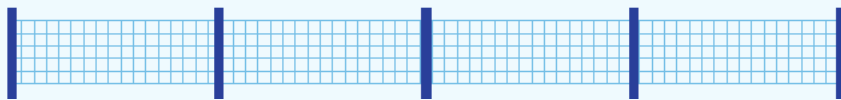


- Starting at one corner, fix small stones at equal intervals of 1 m.

(i) How many small stones have you used?

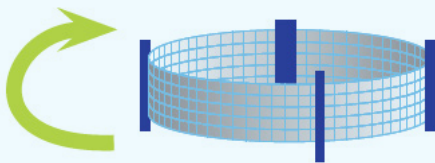
(ii) How many intervals are there?

2) a) Take a thin piece of paper as a strip of 20cm long. Mark lines at 5cm intervals starting on 0cm.



How many lines are there?

b) Curve the strip to make the line a closed line (circle).



How many lines do you need now?

Do you get the same answer as in a) in the linear strip?

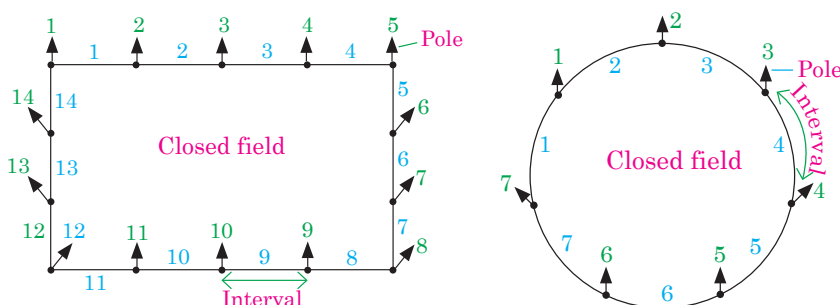
d) Tell the daily life situations where we find equal intervals and poles on closed line.



Summary

When **poles are placed at equal distances on a closed line** (like a circle or rectangle), the **number of intervals is equal to the number of poles**.

Look at these figures:



On every closed line or field;

- Number of intervals = number of poles.
- Number of intervals $\times$ interval length = distance of closed field line.

Example:

The length of a rectangular piece of land is 48 m by 12 m. Poles were fixed at intervals of 2 m to fence it.

- What is the distance round the land?
- How many poles were used to fence the land?

Solution:

$$\begin{aligned}\text{(a) Distance round the land} &= \text{perimeter} = (L + l) \times 2 = (48 \text{ m} + 12 \text{ m}) \times 2 \\ &= 60 \text{ m} \times 2 \\ &= 120 \text{ m}\end{aligned}$$

$$\text{Number of poles} = \frac{\text{Distance round or perimeter}}{\text{Interval length}} = \frac{120\text{m}}{2\text{m}} = \mathbf{60 \text{ poles}}$$



Application Activity 7.3

- In a town, a square plot has sides of 50 m. Poles were fixed to fence it at intervals of 2 m. How many poles were used? Where can you fence and why?
- A circular fishpond has a circumference of 154 m. Poles are to be fixed at intervals of 3.5 m. How many poles are required to fence around the entire pond? Tell the importance of rearing fish.
- A rectangular tank was built to store water. The tank had a length of 125 m and a width of 100 m. It was fenced using 150 poles fixed at equal intervals. Calculate the length of each interval.
- The fence distance round an animal park is 4.2 km. Along the park fence at equal intervals, 210 trees are planted. Calculate the length of each interval in metres. Explain your answer. Why do we fence round the animal park?



End unit assessment

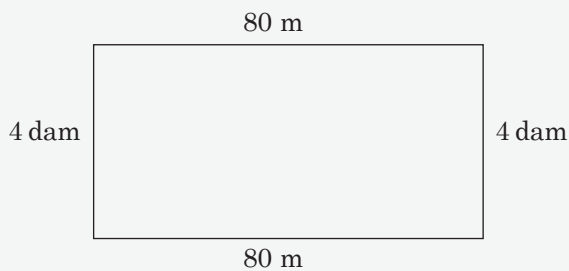
Assessing knowledge

1) **Convert** the following units:

- (i) 25 dm = ____ hm
- (ii) 300 mm = ____ dam
- (iii) 3.45 t = ____ kg
- (iv) 30 dl = ____ l

Assessing skills

- 2) In an athletic competition, an athlete ran a distance of **80 dam**. If her step length was **80 cm**, calculate how many strides she took. Show your work.
- 3) While walking along a road, you counted **91 trees** planted in a straight line with equal **8 m** spacing. Determine the total distance covered by the trees. Justify your answer.
- 4) Look at the piece of land below.



It was to be fenced using posts at intervals of 3 m. How many posts were to be used? Present your work. What materials do you need for fencing?

Assessing attitudes and values

- 5) **Reflect:** Why is it important to use standard units of measurement in real-life situations like construction or sports? Give an example.
- 6) **Ethical Consideration:** If a farmer is fencing land near a wildlife habitat, what factors should they consider to minimize environmental impact?

SOLVING PROBLEMS INVOLVING TIME INTERVALS

7h45min

Start time

End time

Converting Days to Hours.

Multiply the number of days by the expression in red.

$$\text{Days} \times \frac{24 \text{ Hours}}{1 \text{ Day}}$$

Key Unit competence: You will be able to solve real life problems that involve finding time intervals and conversion of units.

Learning activities:

By the end of this unit, you should be able to:

Knowledge and understanding

Explain the various units of time.

State the units used in time measurements.

Describe how to solve mathematical problems involving time.

Explain the meaning of time intervals.

Skills:

Convert the measurement of time (hours, minutes and seconds).

Find the duration of a time interval.

Add and subtract time.

Solve real life problems involving time intervals.

Attitudes and values:

Show the spirit of hard work to meet the deadline.

Show the spirit of punctuality.

Be confident and accurate when working out problems involving time.



8.0 Introduction

Time is something we use every day, from knowing when to wake up, when we go to school in morning, when we go back home in afternoon or evening. In class students attend lessons for a period of time. This time can be expressed in hours, minutes. How long the students stay in a class depends on the timetable. In this unit, you will learn how to solve problems that involve time intervals, or the amount of time between two events.

Understanding time helps us to be organised, plan and manage our day better.



Introductory Activity

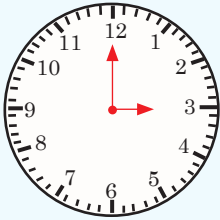
- 1) How many hours do students use in the morning session of lessons from 7:20 am to 9:20 am? How many minutes are these?
- 2) If students go for break time from 9:20am and go back to class at 9:40am:
How many minutes do they use in break time?
How many seconds do they use in break time?

8.1 Converting hours into minutes



Activity 8.1

- 1) From the clock face below, identify the hour and minute hands.



- a) From this time, where will be the two hands after one hour?
 - b) How many minutes are in one hour? Explain.
- 2) Convert the following hours into minutes. Then present your findings.
(a) 3 hours (b) 4 hours



Summary

1 hour is equal to 60 minutes.

1 h = 60 min

Example:

A tourist took 6 hours in her visit to the animal park. How many minutes did the tourist take in the park?

Solution

1 h = 60 minutes. 6 h = 6×60 min = 360 min.



Application Activity 8.1

- 1) Convert the hours given below into minutes.
a) 2 hours b) $6\frac{1}{2}$ hours c) 14 hours
- 2) In a cross-country, an athlete run for 2 hours. How many minutes did the athlete run? Discuss the importance of participating in athletics.

- 3) A traditional music festival lasted for $3\frac{1}{2}$ hours. How many minutes did the festival last? Why is it important to participate in traditional music festival?
- 4) A teacher took her pupils to visit Akagera National Park for 6 hours. How long did their visit last in minutes? Explain your answer.

8.2 Converting minutes into hours



Activity 8.2

Convert the following minutes into hours. Discuss the steps followed.

- (a) 360 minutes (b) 480 minutes



Summary

60 minutes equal to one hour.

60 minutes = 1 hour

Examples:

- (a) A National drama festival lasted for 180 minutes. How many hours is this?

Solution

60 min = 1 h.

180 min = ____ h.

180 min = $(180 \div 60)$ h = 3 h

$$\begin{array}{r} 3 \text{ h} \\ 60 \overline{)180} \text{ min} \\ - 180 \text{ min} \\ \hline 00 \end{array}$$

- (b) In a school, an annual general meeting lasted for 200 minutes. Calculate the number of hours and minutes it took. Explain your working steps.

Solution

We know, 60 min = 1 h

200 min = ____ h ____ min

So 200 min is $(200 \div 60)$ h

By dividing 200 by 60, we get 3 h and 20 min remained.

Thus, 200 min = 3 h 20 min



Application Activity 8.2

- 1) Convert the following into hours.
(a) 120 min (b) 360 min (c) 840 min
- 2) Convert the following into hours and minutes. Explain your answers.
(a) 72 min (b) 130 min (c) 90 min
- 3) A football match took 100 min. How many hours and minutes did the match take? Discuss your steps.
- 4) In a certain village, community work of cleaning the road took 425 minutes. How many hours and minutes did the work take? Why should we participate in community work? Explain.

8.3 Converting hours into seconds



Activity 8.3

1. Get a real clock face.
2. Identify the seconds hand (the longest hand)
3. Carefully observe the time it takes the second hand to make one minute.
4. Finally observe the number of seconds that it takes to make one hour. Convert the following into seconds: (a) 2 hours (b) 4 hours

What steps have you followed? Explain



Summary

From the previous activity,

$$1 \text{ h} = 60 \text{ min}$$

$$1 \text{ min} = 60 \text{ seconds.}$$

$$\text{Therefore } 1 \text{ h} = (60 \times 60) \text{ s} = 3\,600 \text{ s.}$$

Conversion fact: **1 hour = 3 600 seconds**

Examples:

a) Convert 3 hours into seconds.

Solution

$$1 \text{ h} = 3600 \text{ s}$$

$$3 \text{ h} = (3 \times 3\,600) \text{ s} = 10,800 \text{ s}$$

(b) A public meeting took 6 h 50 s. How many seconds was the public meeting?

Solution

$$6 \text{ h} = 6 \times 3\,600 \text{ s} = 21\,600 \text{ s}$$

$$\text{The public meeting took } 6 \text{ h } 50 \text{ s. Thus, } 6 \text{ h } 50 \text{ s} = 21,600 \text{ s} + 50 \text{ s} = 21,650 \text{ s}$$



Application Activity 8.3

1. Convert the following into seconds.

(a) $1\frac{1}{2}$ hours (b) 5 hours (c) $10\frac{1}{4}$ hours

2. Convert the following into seconds. Discuss your steps.

(a) 3 hours (b) 12 hours (c) $2\frac{3}{4}$ hours

3. A bus took 10 h 30 s to travel from Town A to Town B. Find the time in seconds that the bus travelled between the two towns.

8.4 Changing days into hours



Activity 8.4

1. Discuss the number of hours that are there from Tuesday midnight up to Wednesday midnight.

2. I went to camp on Monday 8:00 a.m. I came back on Friday at 8:00 a.m.

(a) How many days did I spend in the camp?

(b) How many hours did I spend in the camp? Discuss your answer.



Summary

There are 24 hours in one day.

1 day = 24h

To convert the number of days to hours,

take the number of days then, multiply by 24 ours.

Number of hours = Number of days $\times \frac{24 \text{ hours}}{\text{day}}$

Examples:

(a) How many hours are in 5 days?

(b) Workers took $10\frac{1}{2}$ days to make terraces for controlling soil erosion. How many hours did the workers took?

Solution

a) 1 day = 24 hours. So, 5 days = $5 \times 24 \text{ h} = 120 \text{ h}$

Therefore, 5 days have 120 hours.

(b) 1 day = 24 hours

$$\text{So, } \frac{1}{2} \text{ day} = \frac{1}{2} \times 24 \text{ hour} = 12 \text{ hours}$$

$$10 \text{ days} = 10 \times 24 \text{ h} = 240 \text{ h}$$

$$\text{Workers took } 10\frac{1}{2} \text{ days} = 12 \text{ h} + 240 \text{ h} = 252 \text{ hours}$$



Application Activity 8.4

1) Find the number of hours in the following:

(a) 10 days (b) 15 days (c) 6 days (d) 13 days (e) 11 days

2) Joyce became the best in her class performance. As a reward, her parents then permitted her to visit and stay at her uncle's place for 14 days. How many hours did Joyce spend at her uncle's place? Why should you work hard?

3) A National music festival took place over $5\frac{1}{2}$ days. How many hours is this? Justify your answer.

8.5 Changing hours into days



Activity 8.5

(a) How many hours are there from midnight to midday?

- How many hours are there from midday to midnight?
- How many hours are there from one midnight to the next midnight?

Then, how many hours are in 1 day? Discuss your findings.

(b) Now discuss and convert the following into days.

(i) 24 h (ii) 48 h (iii) 120h

(c) Present your findings.



Summary

A day has 24 hours. A day starts at midnight up to the next midnight.

$$24 \text{ h} = 1 \text{ day}$$

To convert hours in days, divide the number of hours by 24 hours per day.

Examples:

a) How many days are in 144 hours?

Solution:

a) 24 hours = 1 day

168 h = ____ days

168 h = (168 ÷ 24) days

$$\begin{array}{r} 6 \text{ days} \\ 24 \overline{) 144 \text{ h}} \\ \underline{- 144 \text{ h}} \\ 00 \end{array}$$

144 hours = 6 days

b) A marriage counsellor took 182 hours to counsel two couples and solve marriage problem.
How many days and hours were these?

Solution:

(b) 24 hours = 1 day

182 h = ____ days ____ hours

$$\begin{array}{r} 7 \text{ days} \\ 24 \overline{) 182 \text{ h}} \\ \underline{- 168 \text{ h}} \\ 14 \text{ h} \end{array}$$

182 hours = 7 days 14 hours.



Application Activity 8.5

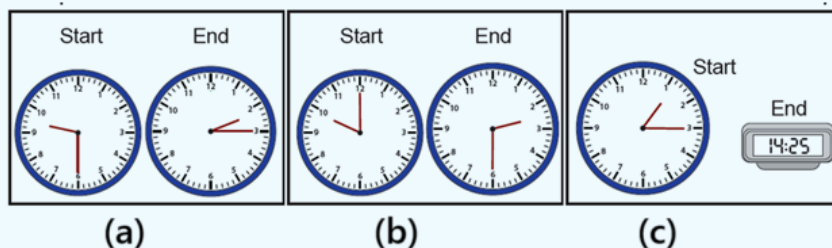
- 1) How many days are in (a) 120 hours? (b) 612 hours? (c) 720 hours?
- 2) A youth camp took place over 312 hours. How many days did the camp take? Explain your answer.
- 3) A tour took 249 hours to visit East African countries by bus. How many days did the tour take? Explain your answer. Why should you take time to tour other countries?

8.6 Finding time intervals or time taken by an event: Using clock face, watch and calendar



Activity 8.6

- 1) What time do you go to school in the morning? What time do you go back home in the evening from school? Find the length of time between the two events.
- 2) An event starts at a given time start time and it ends at the end time. Find the duration of the event:





Summary

Duration: Length of time in which a particular event takes place.

$$\text{Duration} = \text{Ending time} - \text{Starting time}$$

Starting and Ending time can be given different names. For example;

(a) For a journey starting time is departure time. Ending time is arrival time.

(b) For a match of football, starting time is kick-off time. Ending time is stoppage time.

Examples:

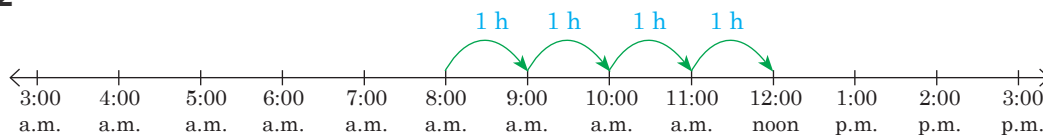
1) A doctor was on duty in hospital from 8:00 a.m. to 12:00 noon. How long was she on duty?

Solution

Method 1

We subtract Duration = 12:00 – 8:00 = 4 hours

Method 2



The time is 4 hours.



Application Activity 8.6

1) How many hours are there from

(a) 9:00 a.m. to 3:00 p.m.? (b) 4:00 a.m. to 1:00 p.m.? (c) 6:00 a.m. to 4:00 p.m.?

2) A baby slept at 8:15 a.m. and woke up at 10:15 a.m.

How long did the baby sleep? Explain your answer.

3) A train left from one station at 10:45 a.m. It arrived at the next station at 12:45 p.m.

Calculate the time it took from one station to the next.

8.7 Real life problems that involve addition and Subtraction of time



Activity 8.7

1) Study the questions below. Then answer the questions correctly.

A car travelled from town A to town B in 2 hours. It then travelled to town C in 3 hours. Find the total duration of travel.

2) A cross-country race ended at 12:35 p.m. The duration of the race was 2 h 10 min.

At what time did the race begin? Present your answers.



Summary

Adding and **subtracting** time is an important skill used in our everyday lives. These problems help us understand how long things take, when something will happen, or how much time we have left.

Examples:

- 1) A train travelled from town P to Q over 7h 55min. Then it travelled to town R over 2h 15 min. Find the total time taken to travel from P to Q to R.

Solution

Time taken = 7 h 55 min + 2 h 15 min

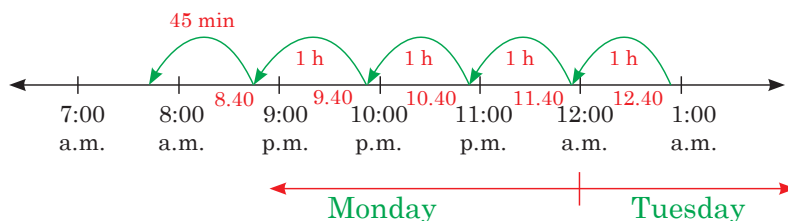
= (7 h + 2 h) + (55 min + 15 min)

= 9 h + 70 min. (Now 70 min = 1 h 10 min)

= 9 h + 1 h 10 min = (9 h + 1 h) + 10 min = 10 h 10 min

- 2) An aircraft left Kigali for Kenya on Monday. It then came back to Kigali and arrived at 12:40 a.m. on Tuesday. The journey had taken total time of 4 h 45 min. At what time did the aircraft leave Kigali for Kenya?

Solution



We subtract 4 h from 12:40 a.m..

We stop at 8:40 p.m. previous day (Monday).

We now have 8:40 p.m. – 45 min = 7:55 p.m.

The aircraft left Kigali at 7:55 p.m. on Monday.



Application Activity 8.7

1) Complete:

a) 14 h 18 min + 12 h 32 min = ____ day ____ h ____ min

b) 6 days 12 h – 3 days 9 h = ____ days ____ h

2) The winner took 1 h 20 min to reach the second town. He then drove to a third town in 1 h 45 min. Calculate the total time the winner drove.

3) We went for lunch at 12:45 p.m. We took a 1 h 15 min lunch break. When did the lunch break end? Discuss your steps.



Assessing Knowledge

1) Convert the following into minutes:

a) 5 hours = _____ minutes

b) $\frac{1}{2}$ hour = _____ minutes

2) Convert the following into hours and minutes:

a) 150 minutes = _____ hours _____ minutes

b) 90 minutes = _____ hours _____ minutes

3) How many seconds are in:

a) 3 hours?

b) 10 minutes?

4) Calculate the time interval:

a) From 7:30 a.m. to 11:45 a.m. = _____ hours _____ minutes

b) From 1:15 p.m. to 4:50 p.m. = _____ hours _____ minutes

Assessing skills

5) I woke up at 3:00 a.m. and two hours later the cock crowed. What time did the cock crow? Discuss.

6) A competence mathematics activity started at 8:40 a.m. It ended at 9:55 a.m. Find the duration of the activity.

7) A bus left Kigali at 6:45 a.m. and arrived in Musanze at 10:20 a.m.

a) How long was the journey?

b) If the bus stopped for 5 minutes at Nyirangarama center, what was the actual travel time?

Assessing attitudes and values

8) Why is it important to manage time well in daily life?

9) How does knowing how to calculate time intervals help you in school?

10) If you finish a test early, how can you use the extra time wisely?



UNIT 9

MONEY AND ITS FINANCIAL APPLICATIONS

Key Unit competence: You will be able to explain money and its financial applications.

Learning activities:

By the end of this unit, you should be able to:

Knowledge and understanding

Explain how to change money and give examples.

Define: Buying price, selling price, profit and loss.

Skills:

Receive an amount of money and provide the change using coins and notes.

Calculate the selling price, buying price.

Compare the buying price and the selling price and decide if there is a loss or a profit.

Attitudes and values:

Appreciate the importance of money in our daily life.

Be honest, faithful and trustworthy in any situation when you are entrusted with money.

Be confident and accurate when changing the amount of money.



9.0 Introduction

In this unit, learners will explore the concept of money. They will engage in real-life scenarios involving changing money, buying and selling, budgeting and simple financial planning. They will develop competence in performing basic financial calculations involving profit, loss, discount, and change.

This unit empowers learners with lifelong financial skills and encourages values such as responsibility, honesty, and critical thinking in financial matters.



Introductory Activity

Consider the following situation and answer the related questions:

A student has just finished university and has got a job in Rwanda Revenue Authority. This student earns 500,000Frw per month.

- (a) List down at least three ways this student would use this first salary.
- (b) Why do you think people should have money in life?
- (c) Can you give different ways in which your parents use money?

9.1 Changing Rwandan currency: coins and notes



Activity 9.1

Money Game: Role play the changes of Rwandan coins and notes.

Here you have Play money (coins and notes: 5Frw, 10Frw, 20Frw, 50Frw, 100Frw, 500Frw, 1000Frw, 2000Frw, 5000Frw)



One pupil is going to act as a **shopkeeper**, the other as a **customer** in a small shop setup.

The customer will present a note or coin, and the shopkeeper will provide the changes. The third learner will work with the second to **verify if the change is correct**: if the total makes the note or coin presented.

After few rounds, **switch roles**.

- a) How did you organize change for 500Frw? What combinations of coins and notes did you use?
- b) How did you organize change for 1000Frw? What combinations of notes did you use?
- c) How did you organize change for 2000Frw? What combinations of notes did you use?
- d) If the shopkeeper wants to include only 2 notes of 500Frw, what combinations of other notes and coins can he/she use to change 5000Frw?



Summary

Money is **a current medium of exchange in the form of coins and banknotes**. Rwandan currency is made by coins and notes:

Rwandan currency coins: **1Frw** (One franc), **5Frw** (Five francs), **10Frw** (Ten francs), **20Frw** (Twenty francs), **50Frw** (Fifty francs), **100Frw** (One hundred francs).

Rwandan currency notes: **500Frw** (Five hundred francs), **1000Frw** (One thousand francs), **2000Frw** (Two thousand francs) and **5000Frw** (Five thousand francs)

When you change money, make sure the sum of the changes equals the amount of money presented.

How to Exchange Money:

1) Breaking Down Larger Notes:

Example: A **5000Frw** note can be exchanged for:

- a) $5 \times 1000\text{Frw}$ notes
- b) $10 \times 500\text{Frw}$ notes
- c) A combination (e.g., $2 \times 2000\text{Frw} + 1 \times 1000\text{Frw}$)

2) Using Coins for Smaller Change:

Example: To make **100Frw**, you can use:

- $1 \times 100\text{Frw}$ coin
- $2 \times 50\text{Frw}$ coins
- $10 \times 10\text{Frw}$ coins

3) Common Exchanges:

Examples:

$$500\text{Frw} = 5 \times 100\text{Frw} \text{ coins}$$

$$200\text{Frw} = 4 \times 50\text{Frw} \text{ coins}$$

$$50\text{Frw} = 5 \times 10\text{Frw} \text{ coins.}$$



Application Activity 9.1

- 1) If you have a **1000Frw** note, how many **500Frw** notes can you get?
- 2) How many **100Frw** coins make **2000Frw**?
- 3) Give two ways to make **250Frw** using coins.
- 4) You buy a snack for **350Frw** and pay with a **500Frw** note. How much change should you get?
- 5) A book costs **1200Frw**, and you pay with a **2000Frw** note. What is your change?
- 6) If you have one **5000Frw**, two **1000Frw**, and five **100Frw** coins, what is the total amount of money you have?

9.2 Buying and selling prices

9.2.1 Finding Cost/Buying Price (C.P)



Activity 9.2.1

Mutuyimana sold a land for 500,000Frw. He made a profit of 150,000Frw. What was the cost/buying price?



Summary

When people **buy and sell** goods, they usually do it to make a **profit** or to offer a **service**. To understand this, we need to learn about **cost/buying price**, **selling price**, **profit**, and **loss**. Here are the key terms:

Cost Price (CP) or Buying Price: It is the price the seller paid to get the product.

Profit: When the **selling price is more** than the **cost price**.

$$\text{Profit} = \text{Selling Price} - \text{Cost Price} = \text{SP} - \text{CP}$$

Loss: When the **cost price is more** than the **selling price**.

$$\text{Loss} = \text{Cost Price} - \text{Selling Price} = \text{CP} - \text{SP}$$

Cost or buying Price = Selling Price - Profit or

Cost or buying price = Selling Price + loss when a loss is realised.

Examples:

- 1) Mukaruziga sold a radio for 50,000Frw. She made a profit of 1,300Frw. What was the cost/buying price?
- 2) A business lady made a loss of 1,000Frw after selling 100kg of potatoes at 120Frw per kg. What was the cost/buying price of 100kg of potatoes?

Solution

$$1) \text{C.P} = \text{S.P} - \text{Profit} = 50,000\text{Frw} - 1,300\text{Frw} = 48,700\text{Frw}$$

The **cost/buying price** was **48,700Frw**

$$2) \text{S.P} = 100 \times 120\text{Frw} = 12,000\text{Frw}$$

when a loss is realised, $\text{C.P} = \text{S.P} + \text{Loss} = 12,000\text{Frw} + 1,000\text{Frw} = 13,000\text{Frw}$

The **cost/buying price** was **13,000Frw**.



Application Activity 9.2.1

1. Angelo sold a water bottle for 4,000Frw. He made a profit of 800Frw. What was the cost/buying price for the water bottle?
2. Martine sold a piglet for 30,000Frw. She made a loss of 3,000Frw. What was the cost/buying price for the piglet?

9.2.2 Finding Selling Price (S.P)



Activity 9.2.2

Kalimu bought a bicycle for 72,000Frw. After selling it. He made a profit of 4,000Frw. What was the selling price?



Summary

Selling Price (SP): It is the price at which the customer buys the product.

Selling Price = Cost/Buying Price + Profit or

Selling price = **Cost/Buying price – loss** when the loss was realised.

Examples:

1. Claire bought a cooker for 80,000Frw. She later sold it and made a profit of 14,700Frw. What was the selling price?
2. Butera bought a calculator for 9,500Frw. He later sold it making a loss of 2,200Frw. What was the selling price?

Solution

1. $S.P = C.P + \text{Profit} = 80,000\text{Frw} + 14,700\text{Frw} = 94,700\text{Frw}$

The **selling price** was **94,700Frw**

2. $S.P = C.P - \text{Loss} = 9,500\text{Frw} - 2,200\text{Frw} = 7,300\text{Frw}$

The **selling price** was **7,300Frw**



Application Activity 9.2.2

- 1) Manzi bought a bicycle for 38,000Frw. After selling it, he made a profit of 4,000Frw. What was the selling price?
- 2) Faina bought a lamp for 3,500Frw. After selling it, she made a loss of 500Frw. What was the selling price?

9.2.3 Profit or loss



Activity 9.2.3

Mr. Gatete bought mangoes at 400Frw each. He later sold 1 mango to Mrs Aline for 600frw.

Did Gatete get a profit? Or loss? Discuss this in a pair.



Summary

Profit: When the **selling price is more** than the **cost price**.

$$\text{Profit} = \text{Selling Price} - \text{Cost Price}$$

Loss: When the **cost price is more** than the **selling price**.

$$\text{Loss} = \text{Cost Price} - \text{Selling Price} = \text{CP} - \text{SP}$$

Example 1: Profit

A trader buys a radio at 13,000Frw and sells it at 15,000 RWF.

$$\text{Profit} = 15,000 - 13,000 = 2,000 \text{ RWF}$$

Example 2: Loss

A shopkeeper buys shoes for 21,000Frw but sells them at 17,500 RWF.

$$\text{Loss} = 21,000 - 17,500 = 3,500 \text{ RWF.}$$



Application Activity 9.2.3

1) The table below shows the cost/buying and selling prices for a trader for different items.

a) Calculate the profit or loss in each case.

Items	Cost/buying price	Selling price	Profit/Loss
Television set	80,000	60,000	
Shirt	6,000	5,000	
Bed sheets	10,000	12,000	

b) List the items in which the trader gained a profit and the items in which s/he gained the loss.

2) Paul bought a shirt for 4,500Frw and sold it for 3,800Frw. What loss did he make?

9.3 Simple budgeting and operation on Rwandan francs



Activity 9.3

Needs are things that are required or essential for someone to live. These include food, shelter and clothes.

Wants are items or experiences we desire, but they are not essential for our survival or daily living. This may include toys, video games, ice creams and radio. Money should be spent on the most important things first. After our needs are taken care of, we can choose to spend on wants. This is called **setting priorities**.

Now, Study the needs and wants below:

Food, clothes, car, television set, ice cream, shelter, school fees, shoes, education, water.

- (i) Which are needs?
- (ii) Which are wants?
- (iii) List the needs and wants in order of priority.
- (iv) Explain why you classified the items as wants or needs.



Summary

Budgeting is the process of planning how to spend your money. It helps you know:

- How much money you have.
- What your needs and wants are.
- How to spend wisely and avoid waste.

Example:

A family got 50,000 Frw from sales at their shop. The family had the following projects they wanted to do:

Painting house at the cost of 12 000 Frw.

Buying family food for 20 000 Frw.

Buying clothes for 12 000 Frw.

Paying school fees worth 16 000 Frw.

- a) Order the family needs according to priority.
- (b) How much money does the family require to meet their budget?
- (c) Do they have the required amount?
- (d) What can the family do?
- (e) State the item that can be done later and explain why.

Solution

- (a) Buying family food, buying clothes, paying school fees, Painting house
- (b) $(12\ 000 + 20\ 000 + 12\ 000 + 16\ 000)$ Frw = 60 000 Frw.
- (c) They don't have enough money to do everything.
- (d) Budget money according to priority.
- (e) Painting house at the cost of 12 000 Frw. The family can wait and do it once they have more money.



Application Activity 9.3

1. List three most important needs.
2. Nzigiyimana plans to do the following: Buy food for his family; Buy a television set; Pay for holiday trip; Buy a car; Buy clothes; Construct a house.
 - (a) List Nzigiyimana's needs in order of priority.
 - (b) List Nzigiyimana's wants in order of priority.
3. Suppose your family has 200,000 Frw. List three things your family can spend the money in order of priority.

9.4 Role play the buying and selling



Activity 9.4

Assume you created a role play of buying and selling at school. Kanyana went to the market to buy 3 items; 1 notebook, 2 bottles of water and 1 bottle of Fanta while she has 5,000Frw. How much money the seller would give her back as a change?



Summary

Role play in the context of buying and selling is a training and practice method where individuals simulate real-world sales transactions.

One person acts as the seller and another as the buyer, often switching roles.

Examples:

In a school shop, pupils practice selling and buying one day a week. Last week, it was Gatesi's round as a seller and Marc as buyer. Marc went to the market with 5,000Frw to buy 1 mathematical set, 1 ruler and 1 pen. How much Gatesi will give Marc as change?

Mathematical set is 1,000Frw, Ruler is 500Frw and Pen is 200Frw.

Solution

Marc will pay $1,000 \text{ Frw} + 500 \text{ Frw} + 200 \text{ Frw} = 1,700 \text{ Frw}$

Thus, Gatesi will give to March $5,000 \text{ Frw} - 1,700 \text{ Frw} = 3,300 \text{ Frw}$



Application Activity 9.4

1. Simulate a classroom market and make the playing notes and coins using sheets of paper. Then John has 3 notes of 2000Frw, He goes to buy 4 bottle of juice, one bottle costs 1,000Frw. He buys also one yogurt that costs 1,000Frw. Find the change the seller will give back to John.



2. There is a market near the school.



Batamuliza goes to buy bananas. one banana costs 100Frw and she has 1,200Frw. How many bananas can Batamuliza buy?

9.5 Converting currencies



Activity 9.5

Study these currencies. They are from different countries. Name these currencies (Values and Country)



Present your answers to the class.



Summary

Different currencies have different values in relation to our currency (Frw).

For example, the following are values in Rwandan francs;

It was displayed in a Forex shop in Kigali on 06/05/2025 at 8 a.m.

Currency	Frw
1 USD	1419.00
1 EUR	1606.70
1 KSh	10.95
1 UGSh	0.39

You can convert Frw to any other currency but always be sure to use current exchange rates accepted by the National Bank.

Example:

Using exchange rate displayed in a forex shop in Kigali on 06/05/2025 at 8 am

Convert 1000 Frw into:

- (i) US dollars (USD)
- (ii) Euros (EUR)
- (iii) Kenya shillings (KSh)
- (iv) Uganda shillings (UGSh)

Solution

(i) 1419 Frw = 1 USD

Thus, 10 000 Frw = $(10\ 000 \div 1419)$ USD = **7.047 USD**

(ii) 1606.7 Frw = 1 EUR

Thus, 10 000 Frw = $(10\ 000 \div 1606.7)$ EUR = **6.223EUR.**

(iii) 10.95 Frw = KSh 1

Thus, 10 000 Frw = KSh $(10\ 000 \div 10.95)$ = **913.242KSh**

(iv) 1 Frw = 0.39 UGSh

Thus, 10 000 Frw = $10\ 000 \times 0.39$ UGSh = **3,900 UGSh**



Application Activity 9.5

1. Use the exchange rates given in Example above in this activity 1. Convert 5 000 Frw into
 - (i) Kenyan shillings (iii) Euros (ii) Ugandan shillings (iv) US dollars
2. Convert in Rwandan Francs using exchange rate of 06/05/2025 given here above.
 - a) 100 USD
 - b) 50 EUR
 - c) 20,000 UGSh
 - d) 2,000 KSh



End unit assessment

Assessing knowledge and understanding

- 1) Below are some sources of money. Which one is not a good way of getting money?
Explain your answer.
Working for pay on a farm.
Salary from employment.
Stealing from a friend.
- 2) The following are needs and wants. State and explain the most important need.
Food
Television
Clothes
House
- 3) Dalia has the following needs and wants. List them from the most important to the least important. Then explain your reasoning.
Holiday camp
Food
School fee
Clothes
- 4) Explain why you should budget before spending.

Assessing skills

5) Study the currency exchange table below. It was observed on 06/5/2025 at 9 a.m in a Forex shop.

Currency	Frw
1 USD	1419.00
1 EUR	1606.70
1 KSh	10.95
1 UGSh	0.39

(a) Convert 15,000 Frw into

(i) EUR (ii) USD (iii) KSh (iv) UGSh

(b) Convert KSh 500 into Frw. Discuss your answer.

(c) Convert 20 000 UGSh into Frw. Discuss your answer.

6) If you have one **5000Frw**, state 3 different ways of changing it using a combination of notes of **1000Frw**, and coins of **100Frw**.

7) Manzi went to the market with one note of 5000Frw. He bought 1 mathematical set at 1000Frw, 1 ruler at 500Frw and 1 pen at 150Frw. How much money will the seller give back to Manzi as change.

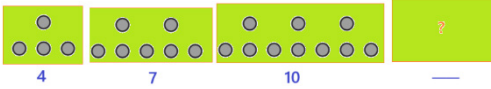
8) Complete the following table:

Item	SP in Frw or selling price	CP in Frw or Buying price	Profit in Frw	Loss in Frw
Laptop	?	150,000	50,000	
Telephone	100,000	130,000		?
Cow	400,000	?	102,500 Frw	

Assessment of attitudes and values

9) Is stealing from a friend a good way of getting money? Explain your answer

10) Explain the importance of budgeting?



SEQUENCES THAT INCLUDE WHOLE NUMBERS, FRACTIONS AND DECIMALS

Key Unit competence: You will be able to explore sequences of whole numbers, fractions and decimals.

Learning activities:

By the end of this unit, you should be able to:

Knowledge and understanding

Explain how to order whole numbers according to their size in increasing and decreasing order.

Explain sequences of whole numbers, fractions and decimals.

Show the patterns used.

Skills:

Find the missing numbers in the sequence.

Complete consecutive terms of a sequence with a constant difference

Complete the consecutive terms of a sequence with constant ratio.

Describe how whole numbers are arranged in different patterns.

Attitudes and values:

Appreciate the importance of orderliness when writing number patterns.



10.0 Introduction

A sequence is an ordered set of numbers, shapes, or objects that follow a specific pattern or rule. In mathematics, sequences can involve whole numbers, fractions, decimals, or even geometric shapes, each progressing in a logical way. Learning sequences helps students recognize patterns, predict future terms, and develop critical problem-solving skills. Whether counting by twos, alternating fractions and decimals, or following more complex rules, sequences form the foundation for algebra, coding, and data analysis.

In nature, sequences appear in the arrangement of leaves and even weather cycles. Financial planning, such as saving money over time (e.g 10; 20; 30; ...), also depends on recognizing sequences.



Introductory Activity

(a) Explain how you can arrange the following numbers in decreasing order.

43,250; 42,420; 43,502; 40,352

(b) Discuss the sequence given below and the pattern used, then find the next two numbers:

$10, 11\frac{1}{2}, 13, 14\frac{1}{2}, 16, 17\frac{1}{2}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}.$

10.1 Ordering whole numbers according to their size in increasing order



Activity 10.1

Rwandan athletes participated in the national athletics trials. They participated in the following races: 10,000 m, 800 m, 100 m, 200 m, 400 m and 5,000 m. Arrange the races in order of increasing distance. Justify your answer.



Summary

We can arrange numbers in increasing order. This is done by ordering/ arranging them from the smallest to the largest.

Examples:

1) Describe how the following numbers can be arranged in increasing order.

46,295; 45,690; 68,925

Solution

By checking the digits in the highest place value to the digits in the lowest place value. The numbers 46,295; 45,690; 68,925 arranged in increasing order are

45,690; 46,295; 68,925.

2) Complete the sequence:

15, 20, 25, __, 35, __

Solution

We see that we are adding 5 to find the next number.

15, 20, 25, $25+5$, 35, $35+5$.

The sequence becomes: 15; 20; 25; 30; 35; 40.

3) Count backwards in 6s starting from 42:

42, __, __, __, __

Solution

To count backwards in 6s **starting from 42**, we have: 42, 36, 30, 24, 18.



Application Activity 10.1

Arrange the numbers below in increasing order.

- 1) 5,000m; 4,000m; 9,000m
- 2) 637,045; 705,365; 673,045; 637,450
- 3) 491,279; 137,004; 397,080; 491,792

10.2 Ordering whole numbers according to their size in decreasing order



Activity 10.2

- 1) The table below shows the number of soccer fans who attended the CECAFA tournament. The data is for the first four matches.

Matches	Number of fans (attendance)
1st match	9,284
2nd match	10,150
3rd match	10,020
4th match	12,345

- A) Which match had
 - i) The lowest attendance?
 - ii) The highest attendance?
 - B) Now arrange the match attendance from the highest attendance to the lowest attendance. Justify your answer.
- 2) Tell examples where you can arrange quantities in decreasing order.



Summary

We can arrange numbers in decreasing order: from the greatest number to the smallest number.

Examples:

Explain how to arrange the following in decreasing order.

43,250; 42,420; 43,502; 40,352

Solution

By checking the numbers from the digit with the highest value to the lowest value.

The numbers arranged in decreasing order are 43,502; 43,250; 42,420; 40,352.



Application Activity 10.2

1) Arrange the following numbers in decreasing order.

(a) 213,456; 213,564; 213,546; 213,645

(b) 23,451; 23,514; 23,145; 23,415

(c) 860,720; 806,720; 860,270; 860,027

2) Arrange the following in decreasing order. Discuss your steps.

(a) 602,097; 632,097; 602,039; 600,397

(b) 708,540; 785,040; 780,504.

(c) 234,567; 243,567; 235,467.

3) Four farmers sold their farm products. The money they got are recorded in the table below.

Farmer	1 st	2 nd	3 rd	4 th
Money (Frw)	150,150	155,100	115,50	150015

(a) Order the money of the four farmers in decreasing order.

(b) Which farmer got the highest amount of money. Explain your answer.

10.3 Simple sequences



Activity 10.3

Work in pairs to examine the following sequences and describe the rule using: add..., subtract ..., or multiply by ... etc.

1) 2, 4, 6, 8, ____, ____: *to get the next number, we add 2.*

2) 5, 10, 20, 40, ____, ____

3) 100, 90, 80, ____, ____

4) 1, 4, 9, 16, ____, ____



Summary

A **sequence** is an ordered list of numbers that follows a particular pattern or rule.

To complete a sequence,

Identify the rule,

Use the rule to find the next term(s).

Check your pattern by verifying if it fits the previous terms.

Examples:

a) 3, 6, 9, 12, ... (**Add 3 each time**)

b) 2, 4, 8, 16, ... (**Multiply by 2 each time**)

c) 1, 4, 9, 16, 25, ... (**Each number is a square number**)



Application Activity 10.3

1) What are the next 2 numbers in the sequence

a) 10, 15, 20, 25, ____, ____

b) 7, 14, 21, 28, ____, ____

c) 4, 25, 36, 49, 64, ____, ____

2) Identify the pattern in the following sequences and fill in the missing numbers.

a) 15, 26, 37, 48, 59, 70, 81, ---, ---, ---

b) 2, 109; 2, 210; 2, 311; 2, 412; 2, 513; ---; ---; ---

10.4 Simple sequences that include fractions



Activity 10.4

Observe the following sequence given below. What is the number added to the first term to get the next number? Use it to complete the next two missing numbers.

10; $11\frac{1}{2}$; 13; $14\frac{1}{2}$; 16; $17\frac{1}{2}$; ____; ____

Write more numbers on flash cards with a pattern that follows the rule "add $1\frac{1}{2}$ " in increasing order.

Explain the pattern used.



Summary

Example:

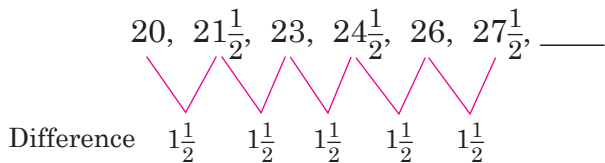
Study the sequence below.

20; $21\frac{1}{2}$; 23; $24\frac{1}{2}$; 26; $27\frac{1}{2}$; _____

Explain the steps involved to find the next number.

Solution

Find the differences in between the numbers.



(To get the next number, you add $1\frac{1}{2}$ to the previous number).

The next number will be: $27\frac{1}{2} + 1\frac{1}{2} = 29$



Application Activity 10.4

1) Find the next numbers in the sequences below.

a) 15; $16\frac{1}{2}$; 18; $19\frac{1}{2}$; 21; ____; ____

b) 36; $34\frac{1}{2}$; 33; $31\frac{1}{2}$; 30; ____; ____

2) Explain the steps involved in calculating the next numbers.

a) 50; $52\frac{1}{2}$; 55; $57\frac{1}{2}$; 60; ____; ____

b) $25\frac{1}{4}$; 25; $24\frac{3}{4}$; $24\frac{1}{2}$; $24\frac{1}{4}$; ____

3) Create Your Own Sequence. Each student invents a 5-term number sequence and writes:

The rule in words

The next two terms

A challenge question for a partner: "What comes next?"

10.5 Simple sequences that include decimals



Activity 10.5

Discuss the sequence given below.

10; 10.5; 11; 11.5; 12; 12.5; ____; ____

Complete: The pattern used is: add ____

Form your own sequences involving decimals. Make presentation to the class.



Summary

A **simple decimal sequence** is an ordered list of numbers that increase or decrease by adding or subtracting a constant **decimal value**.

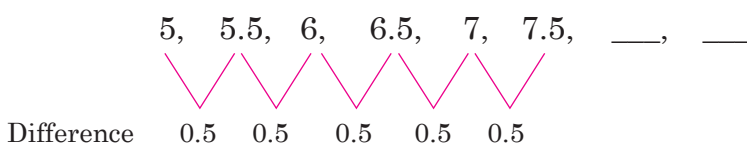
Example:

Find the next numbers in the sequence below.

5; 5.5; 6; 6.5; 7; 7.5; ____; ____

Solution

Find the difference in between the numbers.



The numbers are increasing by 0.5.

We find the next number in the sequence by adding 0.5 to 7.5. Then find the following number.

$$7.5 + 0.5 = 8$$

$$8 + 0.5 = 8.5$$

The next numbers in the sequence are 8 and 8.5.

The sequence is 5; 5.5; 6; 6.5; 7; 7.5; **8; 8.5**.



Application Activity 10.5

1) Find the next numbers in the sequences below.

a) 1; 1.5; 2; 2.5; 3; 3.5; 4; ____; ____

b) 14.5; 13; 11.5; 10; 8.5; 7; ____; ____

2) Find the next number in the sequences below. Explain your steps.

a) 70; 73.5; 77; 80.5; 84; 87.5; ____; ____

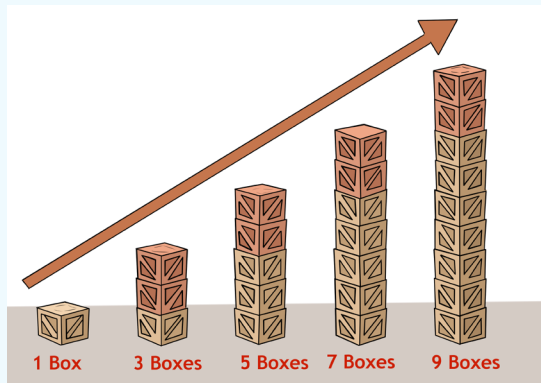
b) 19; 18.5; 18; 17.5; 17; 16.5; 16 ____

10.6 Sequence with constant differences



Activity 10.6

1) observe the 5 piles of boxes.



How many boxes will be at the 6th pile?

2) Observe the following sequence: 25, 28, 31, 34, 37, ____

Complete: The pattern used is: add ____.

Now, form your own sequences with constant differences. Then make a presentation to the class.



Summary

The sequences with constant difference **d** is called **arithmetic progression**.

When you have the first term, find the second term by adding the common difference **d**.

Find the third term by adding **d** to the second term.

Examples:

1) Express the steps involved in getting the next number in the sequence below.

2, 4, 6, 8, ____

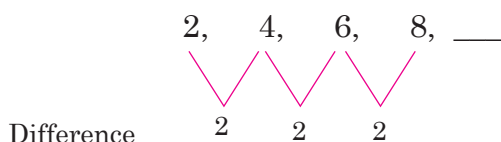
Solution

Method 1

Steps:

Find the difference between two consecutive numbers.

Observe the pattern of the differences.

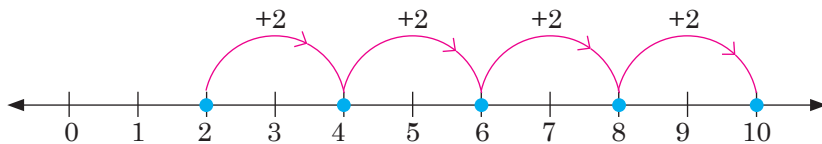


Observation: The common difference is 2. Therefore, we add 2 to 8. So, $8 + 2 = 10$

The sequence is 2, 4, 6, 8, **10**.

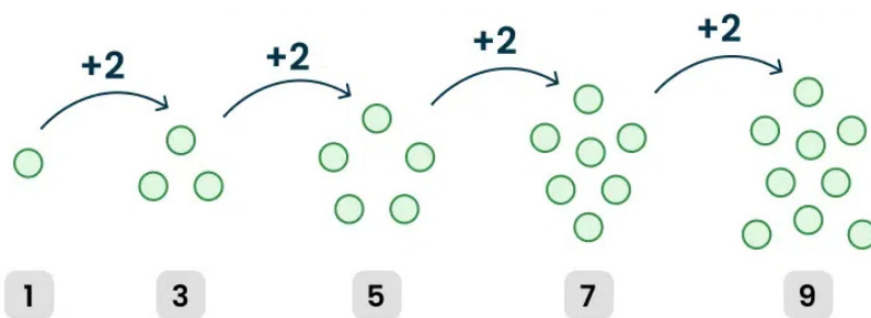
Method 2

We find the missing number using a number line. On a number line, we have;



Moving in +2 steps. The pattern is 2, 4, 6, 8, **10**.

2) Observe the sequence made by the piles of counters. Provide the number of counters for two next piles.



Solution

The common difference is: $3 - 1 = 2 = 5 - 3 = 7 - 5 = 9 - 7$

Therefore, the next pile will have $9 + 2 = 11$ counters.



Application Activity 10.6

1) Find the missing numbers in the sequences below.

a) 35, 37, 39, 41, 43, ____, ____

b) 7, 11, 15, 19, 23, ____, ____

2) Find the missing numbers in the sequences below. Explain your steps.

a) 40, 43, 46, 49, ____, ____

b) 60, 64, 68, 72, ____

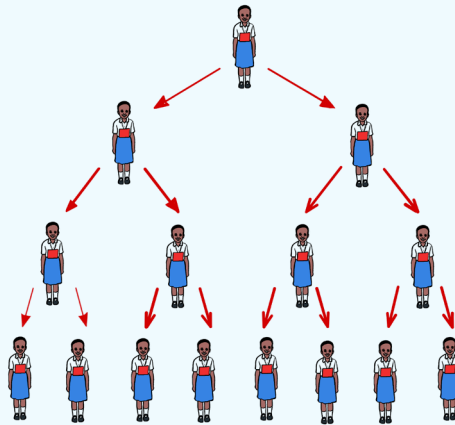
10.7 Sequences with constant ratios



Activity 10.7

- 1) Consider that **K-virus** is spreading in a way such that at any stage two new persons get affected from an affected person.

At first stage, one person is affected, at second stage two persons are affected and is spreading to four persons and so on.



Then, complete the number of affected persons at each stage. There are 1, 2, 4, 8, __; __

How do you get the next number of affected persons from a given stage to the next stage?

- 2) Look at the following sequence: 3, 9, 27, 81, ____

- Find the missing numbers.
- To describe the pattern you have discovered, complete: To find the next number, I multiply by ____.
- Form your own sequences with the pattern you discovered. Then make a presentation to the class.



Summary

A sequence with common ratio is a sequence for which each term after the first is obtained by multiplying the previous term by a fixed, non-zero constant number.

The common number is called a **common ratio**.

A sequence with constant ratio is also called **geometric sequence** or geometric progression.

Example:

Discuss the steps involved to extend the sequences below. 5, 15, 45, ____

Solution

Find the constant ratio by dividing each number by the previous one. $\frac{15}{5} = \frac{45}{15} = 3$

The constant ratio is 3. Multiply $45 \times 3 = 135$ Therefore the sequence is 5, 15, 45, 135.



Application Activity 10.7

1) Find the next numbers in the following sequences.

a) 1, 2, 4, 8, ____, ____

b) 16, 32, 64, ____

2) Find the missing numbers in the sequences below. Explain your steps.

a) 1, 5, 25, 125, ____

b) 10, 100, 1,000, ____

c) 176, 88, 44, 22, ____, ____

10.8 Sequences where the changing difference is a geometric sequence



Activity 10.8

Look at the sequence: 11, 23, 47, 95, ____.

1) Find the difference between consecutive numbers.

2) What pattern does the differences make?

3) Then, how do you find the next number?

4) Now, find the next number. 11, 23, 47, 95, ____

Form your own sequences like the sequence above. Make posters for your sequences and present to the class.



Summary

For some sequences the difference is not common. It can change regularly.

Sequences where the changing difference is geometric means that the differences of consecutive terms make a geometric sequence.

Examples:

Consider the following sequence:

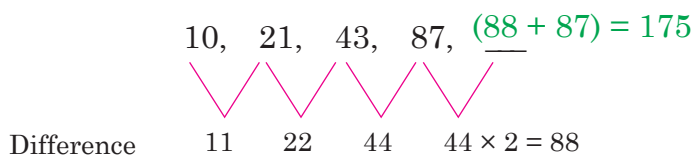
10, 21, 43, 87, ____

Find the differences between all consecutive numbers.

Observe the pattern of the differences.

Find the next number using the obtained pattern with differences.

Solution



The difference follows a geometric pattern. This is 11; 22; 44; ...

The difference is changing. To obtain the next difference, we multiply the previous difference by 2. The next difference is $44 \times 2 = 88$.

So, add $88 + 87 = 175$.

Thus, the sequence is: 10, 21, 43, 87, **175**.



Application Activity 10.8

1) Find the next numbers in the sequences below.

a) 1, 3, 7, 15, ____

b) 2, 5, 11, 23, ____, ____

2) Find the next number in the sequences below. Discuss your steps and present.

a) 5, 11, 23, 47, ____

b) 8, 17, 35, 71, ____

c) 20, 41, 83, 167, ____



End unit assessment

Assessing knowledge

1. Arrange the following from the smallest to the largest.

(a) 2,300; 3,200; 2,003; 3,002

(b) 5,732; 7,532; 735; 5,372

2. Arrange the following in decreasing order.

(a) 9,481; 9,841; 9,148; 9,099

(b) 23,452; 23,425; 23 245; 25,254

Assessing skills

3. Analyse the constant difference and find the next number in the sequence below. Explain your steps.

(a) $5; 6\frac{1}{2}; 8; 9\frac{1}{2}; \underline{\hspace{1cm}}$

(b) $30; 31\frac{1}{2}; 33; 34\frac{1}{2}; 36; \underline{\hspace{1cm}}$

4. Use a number line to find the next number in the following sequences. Explain your pattern.

(a) 3, 6, 9, 12, $\underline{\hspace{1cm}}$

(b) 1, 5, 9, 13, $\underline{\hspace{1cm}}$

5. (a) Write an increasing number sequence where each number increases by 7, starting at 4 (write next 3 terms).

- (b) Write a decreasing sequence where each number reduces by 0.5, starting at 10 (write next 4 terms).

6. A farmer plants trees in rows following a pattern: Row 1 = 3 trees, Row 2 = 6 trees, Row 3 = 9 trees.

- (a) How many trees will be in Row 5?

- (b) If the farmer has 12 rows, how many trees are planted in total?

Assessing attitudes and values

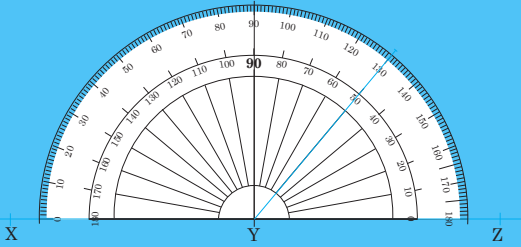
7. Why is recognizing patterns useful in everyday life? (Give two examples.)

8. Explain how teamwork helped you in learning sequences.

9. Which sequence type (increasing or decreasing) was easiest for you? Which was most challenging? How did you overcome the difficulty?

UNIT 11

CONSTRUCTING ANGLES AND DESCRIBING ANGLES FORMED BY INTERSECTING LINES



Key Unit Competence: You will be able to construct different angles and describe properties of angles formed by intersecting lines.

Learning activities:

By the end of this unit, you should be able to:

Knowledge and understanding:

Describe the process of drawing angles and the process of constructing angles using a ruler and a pair of compasses.

Explain the concept of parallel, transversal, intersecting and perpendicular lines.

Explain how to solve problems involving angles.



Skills:

Measure angles using a protractor.

Construct angles using a ruler and a pair of compasses.

Carry out the process of bisection of an angle using a ruler and compass.

Solve problems involving angles around a point, angles made by intersecting lines and the angles formed by a transversal line of two parallel lines.

Find the angle sum of a triangle.

Attitudes and values:

Appreciate the importance and use of lines and angles in daily life.

Demonstrate confidence and accuracy in drawing angles and lines.

11.0 Introduction

In real life, drawing skills is very much used. Carpenters, architects and designers use such skills. Many objects have plane shapes with many sides, angles and vertices. Some have same size of sides and angles while others don't. The concept of drawing and constructing angles is used in making different objects in our daily life.



Introductory Activity

- 1) Look around in or out of your classroom and identify plane shaped objects and try to name them basing on the number of sides and angles.
- 2) (a) Is there any object of 3 sides and 3 angles in your class?
(b) Is there any object of 4 sides and 4 angles?
(c) How are the sides of the observed objects?
(d) Find out if they have all equal sides, and equal angles.
3. What do you use when constructing angles?

11.1 Construction of different angles with a protractor



Activity 11.1

Draw a straight line segment, $XZ = 10$ cm.

Mark its centre Y. Have centre Y meet with the centre of your protractor. Mark a point where the angle is 50 degrees from left side.

Repeat the steps above but do it from the right side of protractor.

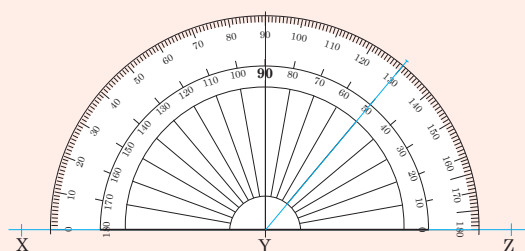
What is the right direction of using a protractor?



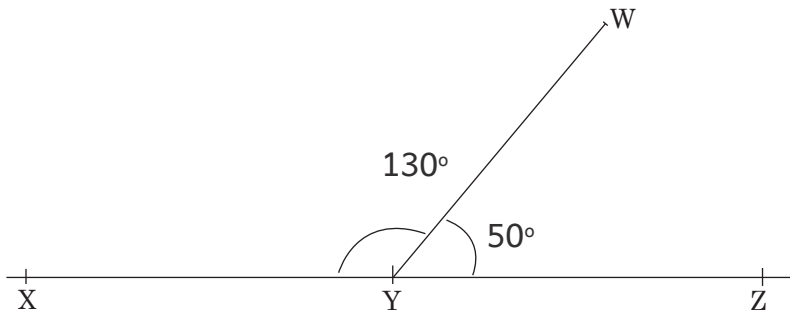
Summary

Here's a simple summary of the steps to draw an angle:

1. **Draw a straight line XZ:** This will be one side (arm) of the angle.
2. **Mark a point on the line:** Name it, for example, point Y.
3. **Choose a point for the vertex:** Usually, this is the same as point Z, or a different point not on the line.
4. **Use a protractor:** Place it so that its center Y (the baseline) is at the vertex.
5. **Measure the desired angle,** for example 50 degrees is measured on the protractor from Z to W: Mark the angle's opening W on the protractor.
6. **Draw the second arm:** From the vertex Y, draw a line passing through the mark W you made on the protractor.
7. **Label the angle:** Use letters like $\angle ZYW = 50$ degrees.



Example:



Angle $WYX = 130^\circ$

The line YW also passes through the 50° mark of the inner scale.

Therefore, angle $ZYW = 50^\circ$.



Application Activity 11.1

Use a protractor and a ruler to draw the following angles. Justify whether you have used the inner or outer scale.

- 1) 120° 2) 60° 3) 70° 4) 110° 5) 80° 6) Angle $ABC = 100^\circ$ 7) Angle $DEF = 85^\circ$

11.2 Bisection of angles (Using folding)



Activity 11.2

Draw angles 120° , 90° , 80° on a paper.

Cut out the angles. Fold each of the paper angles into two equal halves.

Cut out the angles along the line created from folding.

Measure the angles of each of the halves.

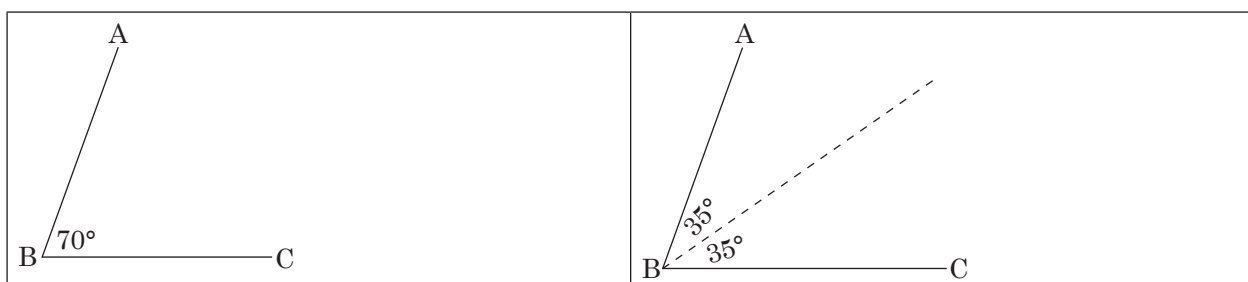
Present your findings to the class.



Summary

To bisect an angle, we divide its size into 2 equal parts. When you bisect 90° , you get 45° .

Example: Bisect the angle given below by folding:



Step I: Draw angle CBA on a paper. Make a paper cutout for the angle.

Fold the angle CBA into two equal parts.

Step II: Unfold the angle. Draw a line along the folded part as shown by the dotted line.

Step III: Measure the size of the angle on each of the two pieces. What do you notice?



Application Activity 11.2

Draw the following angles. Then explain how to bisect them using folding.

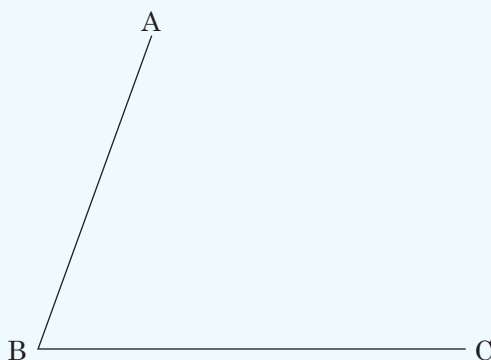
1) 60° 2) 100° 3) 80° 4) 120° 5) 90° 6) 130°

11.3 Bisecting angles using a pair of compasses and a ruler



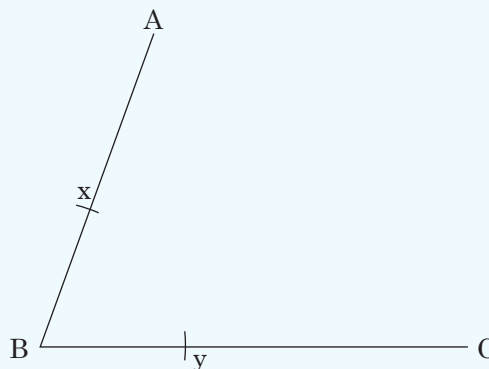
Activity 11.3

a) Make a paper cutout of the angle CBA below. Fold it in half. What do you get?

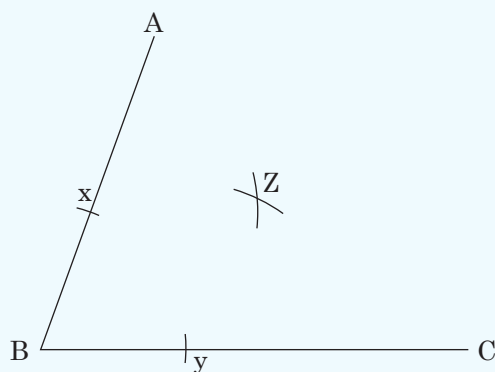


b) Follow the following steps to bisect the angle:

Step I: At point B, make an arc of any radius to cut line AB and BC at point x and y respectively.



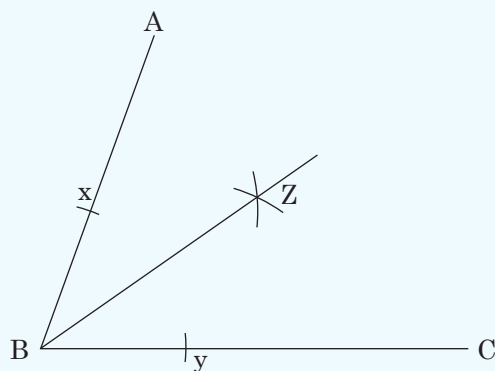
Step II: With points x and y as the centres, make arcs to intersect at point Z .



Step III: Using a ruler, draw a line from point B to Z .

Line BZ divides angle ABC into two equal parts.

Angle ABZ is equal to angle CBZ

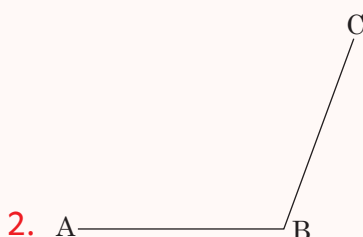
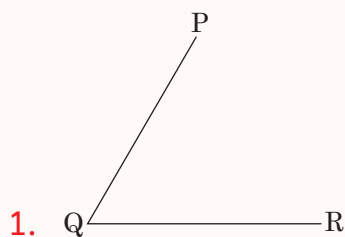


Application Activity 11.3

1) Draw the following angles. Then explain how to bisect them.

- a) 60° b) 100° c) 80° d) 90° e) 180°

2) Using a pair of compass and a ruler, bisect the following angles



11.4 Constructing 90° , 45° and 22.5° angles



Activity 11.4

(a) Make a paper cutout for 90° .

Fold it to make an angle of 45° .

Fold the paper cutout for 45° to form 22.5°

(b) Now construct an angle of 90° . Use a pair of compasses and a ruler.

Bisect 90° to have 45° .

Bisect 45° to have 22.5° .

Example:

Explain the steps involved in constructing 90° using a ruler and a pair of compasses only.

Solution

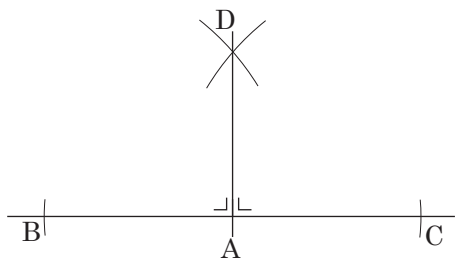
Step I: Mark an arc A on a straight line. Using A as the centre, make two other arcs on both sides of point A. Label them as B and C.



Step II: Increase the radius and use points B and C as centres to draw arcs intersecting at point D above the line.



Step III: Join point A to D.



Angle DAB = 90°

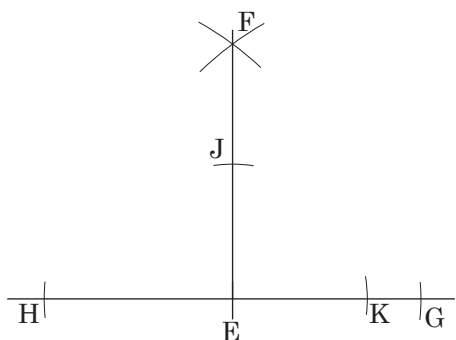
Angle CAD = 90°

Example

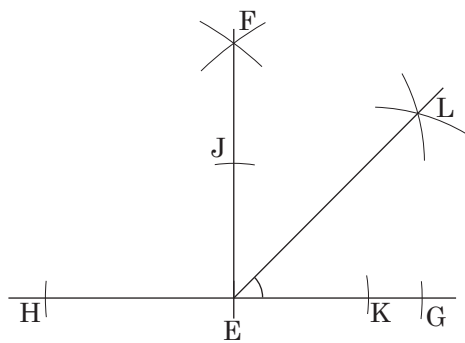
Describe the steps involved in constructing 45° using a pair of compasses and a ruler.

Solution

Step I: Construct 90° . With point E as the centre, mark arcs to cut line EF and EG at point J and K respectively.



Step II: With J and K as the centres, mark arcs to intersect at point L.



Angle GEL = 45° . Angle FEL = 45° .

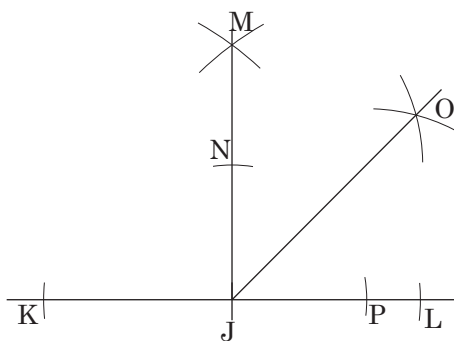
Example:

Explain the steps involved in constructing 22.5° using a ruler and a pair of compasses.

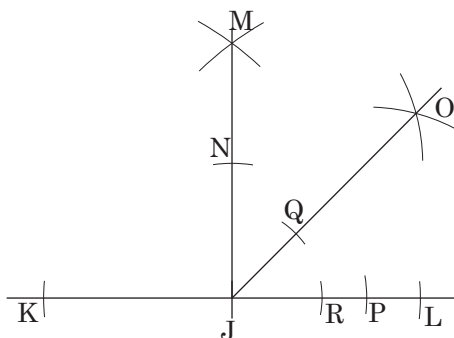
Solution

Construct 90° then bisect the angle to have 45° . Bisect 45° to have 22.5° .

Step I: Construct 90° . Bisect angle 90° to have $\text{OJM} = \text{LJO} = 45^\circ$.

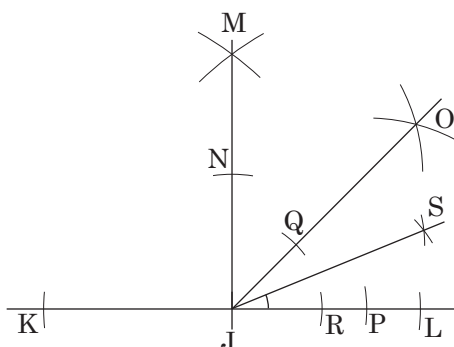


Step II: With point J as the centre, mark arcs. The arcs cut line OJ at Q and line JL at R.



Step III: With Q and R as the centres, mark arcs to intersect at point S.

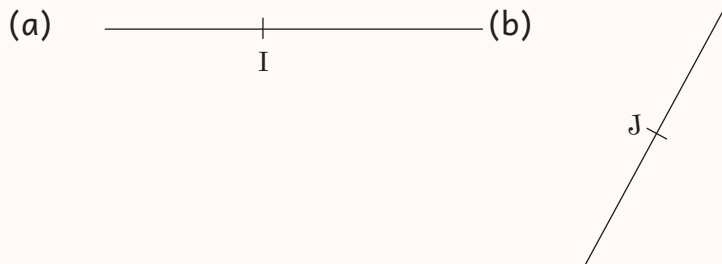
Angle LJS is half of 45° . Angle LJS = 22.5° .





Application Activity 11.4

1. Construct 90° angle at the points marked with letters on the lines below.
2. Explain the steps involved in constructing a 45° angle at the points marked with letters.
3. Explain the steps involved in constructing a 22.5° angle at the points marked with letters.



11.5 Constructing 60° , 30° and 15° angles



Activity 11.5

- (a) Construct the 60° angle using a protractor.
 - b) Construct 60° angle using a pair of compasses and a ruler.
State the steps involved in constructing a 60° angle.
 - (c) Construct a 30° angle using a pair of compass.
- Discuss the steps involved in constructing a 30° angle.



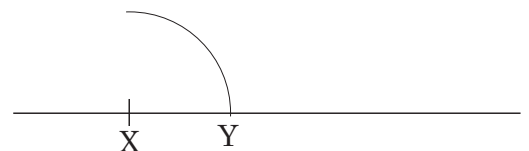
Summary

After constructing the angle of 60 degrees, you can bisect it to get the angle of 30 degrees. Then, you can bisect the angle of 30 degrees to get the angle of 15 degrees.

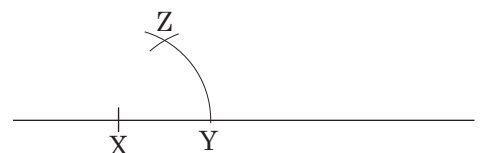
Example:

- (a) Construct a 60° angle at point X using a ruler and a pair of compasses

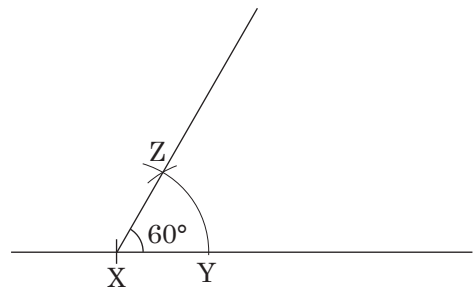
Step I: Use point X as the centre to mark an arc of any radius to cut the line at Y.



Step II: Keep the same radius and use Y as the centre. Draw another arc to intersect the first arc at Z.



Step III: Draw a line through XZ. Angle $YXZ = 60^\circ$.



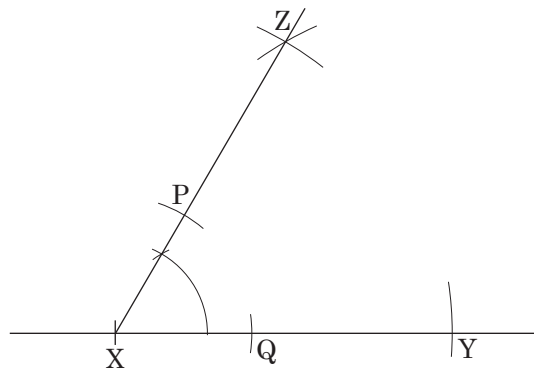
(b) Use a pair of compasses and a ruler to construct a 30° angle at X. Discuss your steps.



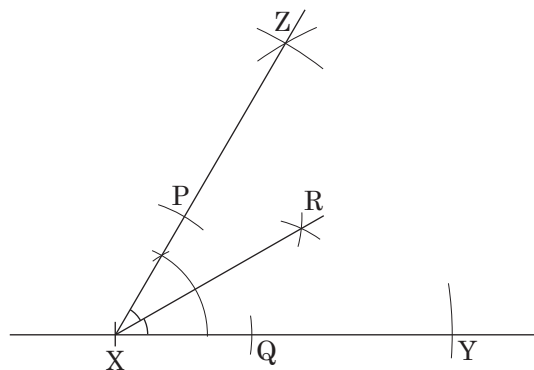
Solution

Construct a 60° angle then bisect it to get 30° .

Step I: Construct 60° at X. With X as the centre, draw arcs to cut line XZ and XY at point P and Q. See the diagram below.



Step II: With points P and Q as the centre, draw arcs to intersect at point R. Join X to R. Measure angle YXR. Angle $YXR = RXZ = 30^\circ$.



Example

Construct a 15° angle at point X using a ruler and a pair of compasses.

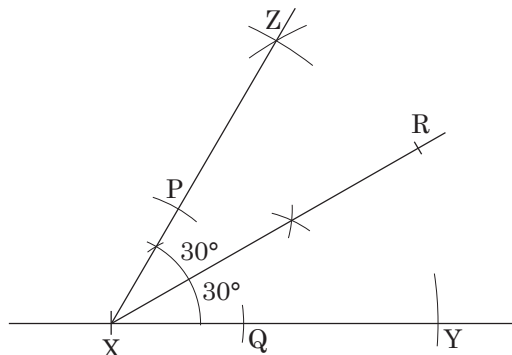


Discuss your steps.

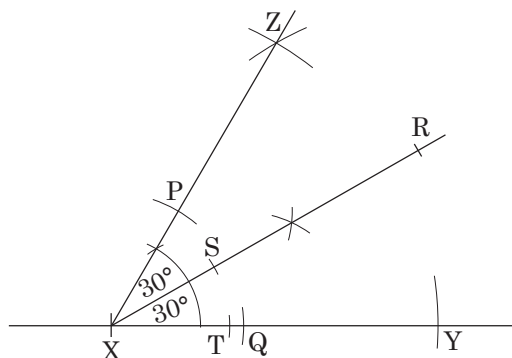
Solution

Construct a 60° angle. Bisect 60° to get 30° . Bisect 30° to get 15° .

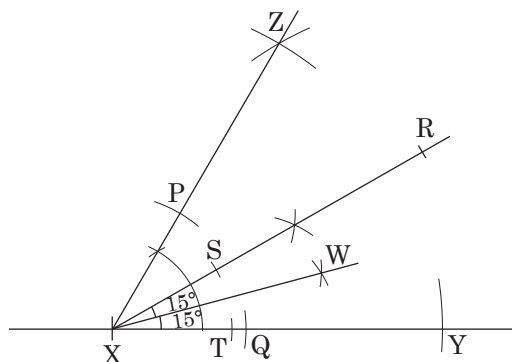
Step I: Construct a 60° angle at X. Bisect 60° to get 30° .



Step II: With point X as the centre, draw arcs to cut XR at S and XY at T.



Step III: With S and T as the centres, draw arcs to intersect at point W. Then join XW. Measure angles WXR and YXW.



Angle WXR = YXW = 15° .



Application Activity 11.5

1. Construct a 60° angle at the points marked with letters.
2. Construct a 30° angle using a ruler and a pair of compasses at the points marked with letters. Discuss your steps.

(a)

(b)

(c)

3. Construct a 15° angle using a ruler and a pair of compasses at the points marked with letters. Explain your steps.

11.6 Constructing angles 120° and 150° angles



Activity 11.6

Basing on the angle of 180 degrees, can you construct the angle of 60 degrees from it?

Now explain how to construct 120° and 150° angles using a ruler and a pair of compasses. Discuss your steps



Summary

After constructing the angle of 180 degrees, you can construct the angle of 30 degrees from it and get 150 degrees. You can also construct the angle of 60 degrees from the angle of 180 degrees to get the angle of 120 degrees.

Example:

Construct the following angles using a ruler and a pair of compasses.

Describe your steps.

(i) 120°

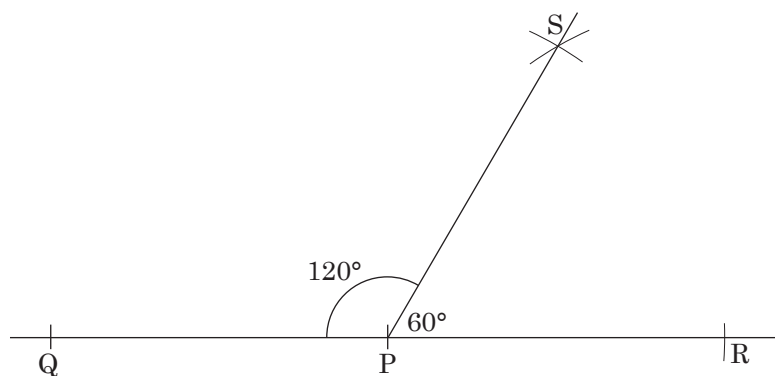
(ii) 150°

Solution

(i) Constructing a 120° angle at point P.

Considering that $120^\circ = 180^\circ - 60^\circ$. We construct a 60° angle on a straight line. The supplementary of 60° is 120° .

Step I: Construct a 60° angle on one side of point P on line QPR.

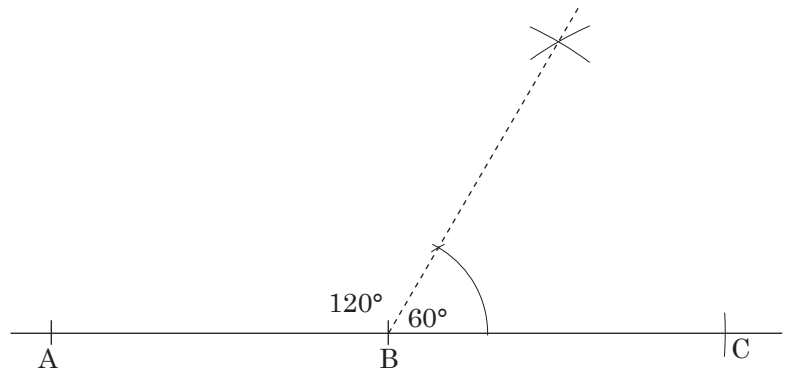


Angles $SPQ = 120^\circ$.

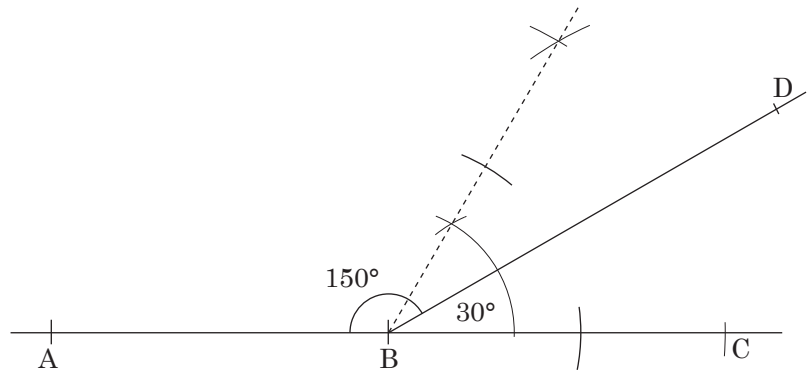
(ii) Constructing a 150° angle: Consider that $150^\circ = 180^\circ - 30^\circ$.

We construct a 30° angle on a straight line. The supplementary of 30° is 150° .

Step I: Construct a 60° angle at point B on line ABC.



Step II: Bisect a 60° angle to get 30° .



Since angle $ABC = 180^\circ$ and angle $CBD = 30^\circ$,
then angle $DBA = 150^\circ$. Measure it to confirm.



Application Activity 11.6

- Construct a 120° angle at the points marked with letters. Use a pair of compass and a ruler.
- Discuss the steps involved in constructing a 150° angle at the point marked with letters. Use a pair of compasses and a ruler.

(a)

(b)

(c)

(d)

11.7 Identify different lines and angles formed on real objects



Activity 11.7

Observe the tables, desk, chairs and walls of the class.



You can observe other objects in your class like boxes or cartons.

Identify different lines such as;

- (i) straight lines.
- (ii) lines that meet.
- (iii) lines that do not meet.
- (iv) Present your findings.



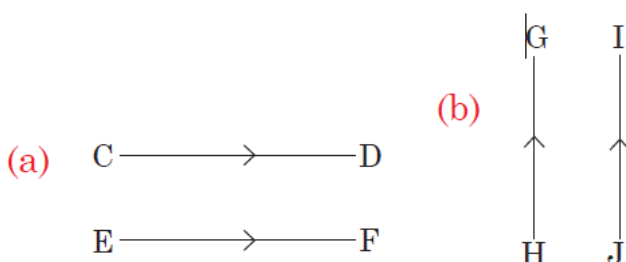
Summary

A point is a specific location or position in space. It has no size, shape, or width, it is just a position.

A line is a continuous mark that goes on forever in both directions. It has no ends. A line joins two points on a flat surface

A ————— B

When the lines do not meet, they are said to be parallel.



Line CD and line EF are parallel. We write $CD \parallel EF$. Line GH and line IJ are parallel. We write $GH \parallel IJ$.

When two lines meet, we say they have intersected.

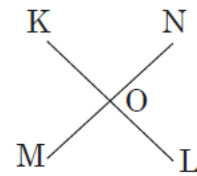
When straight lines intersect they form angles.

Depending on the position of lines, we have:

Parallel lines: They do not meet. $CD \parallel EF$



Intersecting lines: Intersecting lines KL AND MN meet at a point. Point O is intersection point.

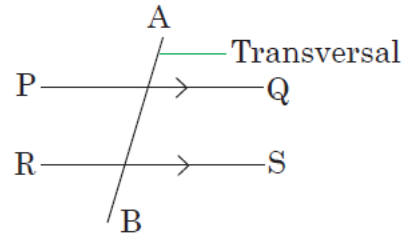


(iv) The transversal:

The transversal cuts parallel lines.

$PQ \parallel RS$

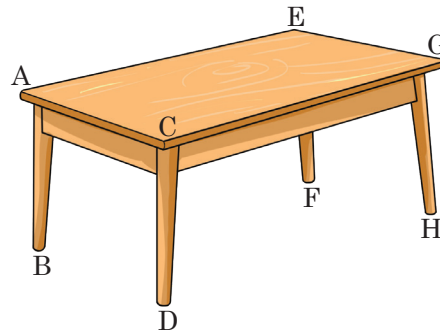
AB is transversal to PQ and RS.



Example:

Below is a picture of a table. Study and answer the following questions.

- (i) Identify the lines that do not meet.
- (ii) Identify the lines that meet.
- (iii) Identify lines that cut parallel lines.



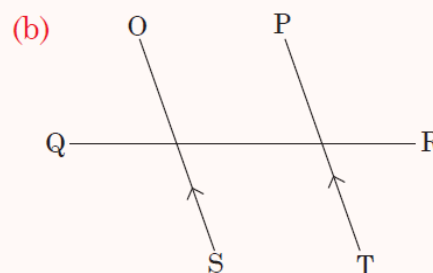
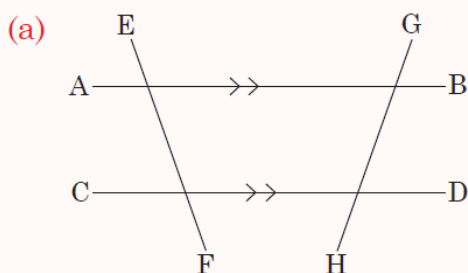
Solution

- | | | |
|---|--|---|
| <p>(i) • Lines AB, CD, EF and GH do not meet.</p> <p>• Lines AC and EG do not meet.</p> <p>• Lines AE and CG do not meet.</p> | <p>(ii) • Lines AB, AE and AC meet at A.</p> <p>• Lines AC, GC and DC meet at C.</p> <p>• Lines AE, GE and FE meet at E.</p> <p>• Lines CG, HG and EG meet at G.</p> | <p>(iii) Lines that cut parallel lines are:</p> <p>• CG cutting DC and HG.</p> <p>• EG cutting FE and HG.</p> <p>• AE cutting AB and FE.</p> <p>• AC cutting AB and DC.</p> |
|---|--|---|

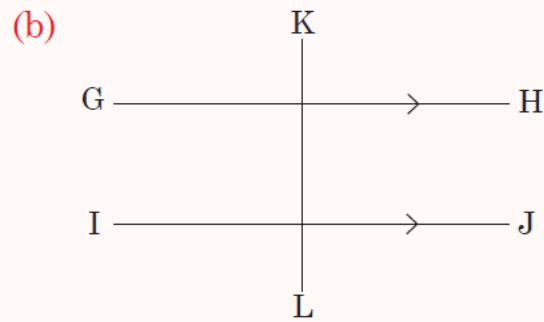
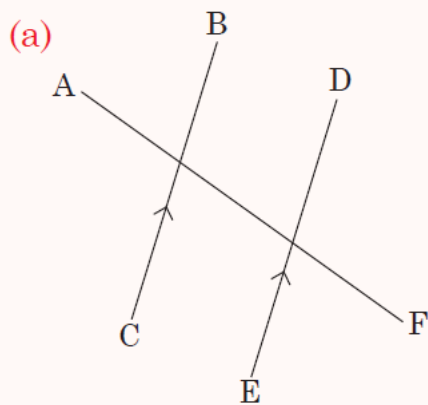


Application Activity 11.7

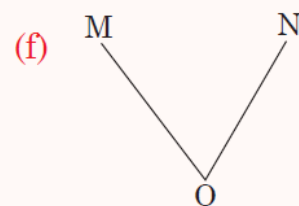
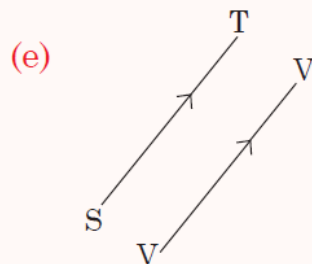
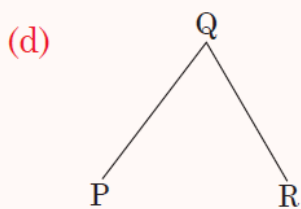
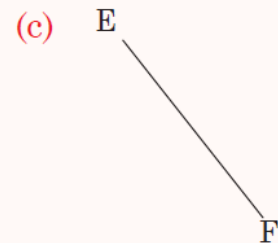
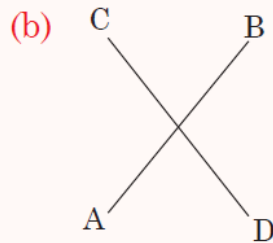
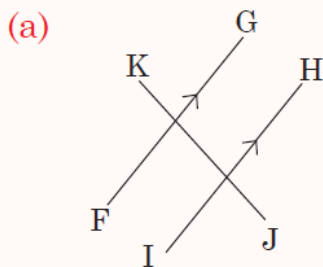
- 1) Draw your own: (i) parallel lines (ii) intersecting lines (iii) transversal
- 2) Name lines that are parallel to each other from the diagrams below.



2. Identify the transversals in the diagrams below.



3) Explain how you would identify intersecting lines from the lines drawn below.



11.8. Perpendicular lines



Activity 11.8

Look at the walls of the classroom. Use a protractor or the set square to measure the angle where the walls are meeting. What do you notice? Present your observation.

Observe the windows of the classroom. Look at the corners where the frames join each other. Measure the angle. What is the angle?



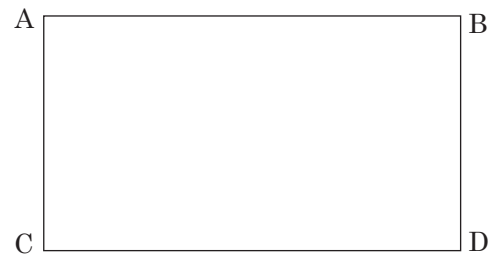
Summary

Lines AB meet line BD forming a right angle.

A right angle is 90° .

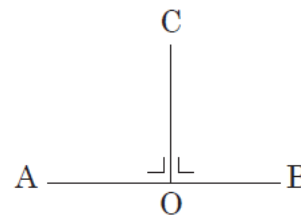
(a) From the rectangle, we note:

- Lines that intersect at 90° are called perpendicular lines.
- Line AB is perpendicular to line BD.
- Line BD is perpendicular to line DC.
- Line BA is perpendicular to line AC.



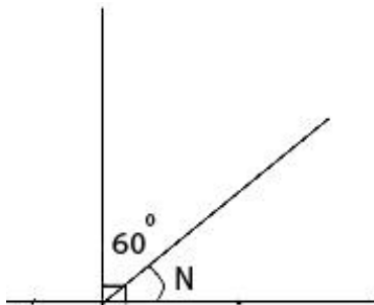
b) Line CO and line AB intersect in O. They form a right angles BOC and COA. So Line CO and Line AB are perpendicular.

The symbols $\perp$ in the diagram show that there are right angles with 90° .



Example:

When the sum of two angles make 90 degrees, they are called **complementary angles**. Find the value of angle N, if N is the Complementary angle of 60 degrees.



Solution:

As N and 60 degrees are complementary angles, we have:

$$N + 60 \text{ degrees} = 90 \text{ degrees.}$$

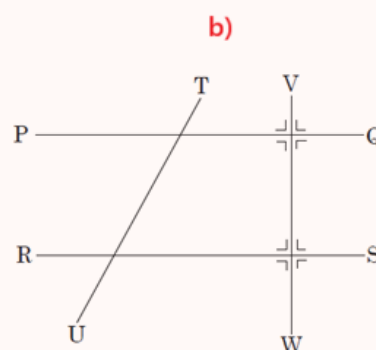
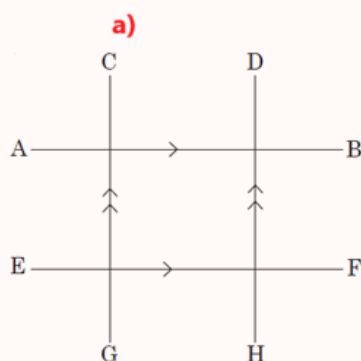
Therefore,

$$N = 90 \text{ degrees} - 60 \text{ degrees} = 30 \text{ degrees.}$$



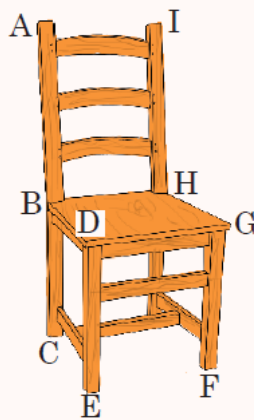
Application Activity 11.8

1. Identify the perpendicular lines in the following diagrams.



2. Observe the frame of the chair below.

Which lines are perpendicular to each other? Explain

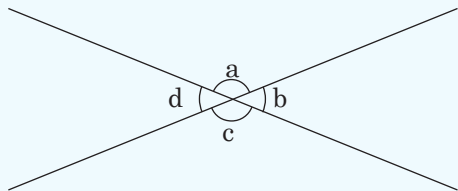


11.9 Properties related to angles formed by intersecting lines



Activity 11.9

Study the figure below. Draw any two intersecting lines. Note angles a , b , c and d .



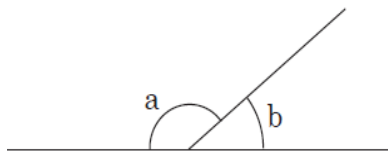
- Use a protractor to measure the angles a , b , c , d . What do you notice about: a and c , and d and b ?
- Add angle; $a + b$, $c + d$, $a + b + c + d$ and give the sum. Present your results.



Summary

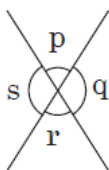
(i) **Angles on straight line add up to 180° .**

They are called **supplementary angles**.



$$a + b = 180^\circ$$

(ii) **Angles at a point add up to 360° .**



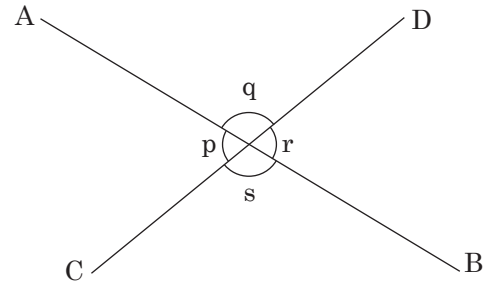
Angles p and r are equal.

Angles s and q are equal.

Angles p and r or s and q are called **opposite angles**.

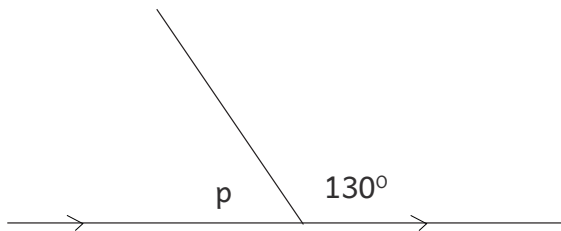
Example:

- 1) Two straight lines AB and CD intersect as shown on the figure.
 - (i) Use a protractor to measure the angles p, q, r and s.
 - (ii) Find the sum of $p + q$, $r + s$.
 - (iii) What do you notice about angle q and s?
 - (iv) What do you notice about angle p and r?
 - (v) Find the sum of angle p, q, r and s.



Solution

- (i) Angle $p = 70^\circ$ Angle $q = 110^\circ$ Angle $r = 70^\circ$ Angle $s = 110^\circ$
 - (ii) • Angle $p + \text{angle } q$: $p + q = 70^\circ + 110^\circ = 180^\circ$
• Angle $r + \text{angle } s$: $r + s = 70^\circ + 110^\circ = 180^\circ$
 - (iii) Angles q and s are equal.
 - (iv) Angles p and r are equal.
 - (v) Sum of angles p, q, r, s
 $p + q + r + s = 70^\circ + 110^\circ + 70^\circ + 110^\circ = 360^\circ$
- 2) Find the value of angle p in the figure below if p is the supplementary angle of 130 degrees.



Solution:

As P and 130 degrees are supplementary angles, $P + 130 \text{ degrees} = 180 \text{ degrees}$.
Therefore, $P = 180 \text{ degrees} - 130 \text{ degrees} = 50 \text{ degrees}$.



Application Activity 11.9

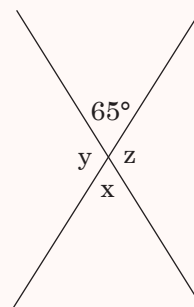
- 1) In the diagram below the value of one of the angles is given.

- (a) Explain steps you follow to find the size of the following angles:

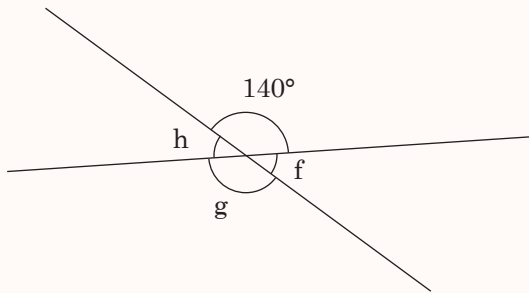
- (i) Angle $x =$
- (ii) Angle $y =$
- (iii) Angle $z =$

- (b) Angle y is opposite to angle ____.

- (c) Angle $x + \text{angle } z =$ ____



2) Find the size of angles marked f, g and h. Explain your steps

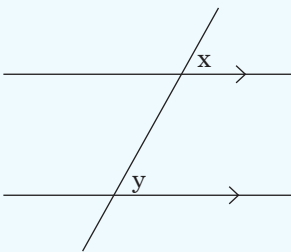


11.10 Angle properties of parallel lines: Corresponding angles



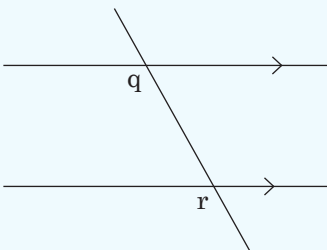
Activity 11.10

i) Measure and discuss the angles marked with letters given in the diagrams below.



What do you notice about the size of angle x and y ?

ii) Measure and discuss the angles marked with letters given in the diagrams below.



Explain what you can notice about the size of angle q and r .



Summary

Basing on the activity, we find:

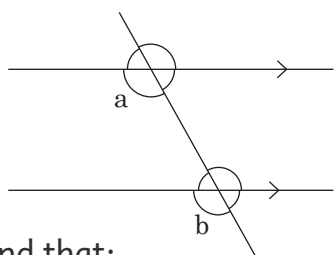
Corresponding angles are pairs of angles that are in the same position when two lines are crossed by a transversal.

When two parallel lines are crossed by a transversal, corresponding angles are equal.

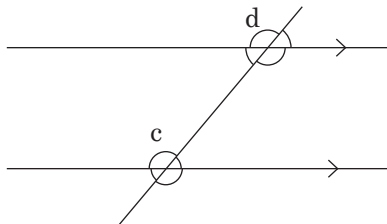
Example:

When we use a protractor to find the angles marked with letters:

(a)



(b)



We find that:

At the figure (a), the angle $a = 120^\circ$ and $b = 120^\circ$

At the figure (b), the angle $c = 130^\circ$ and $d = 130^\circ$.

Angles a and b are equal and Angles c and d are equal.

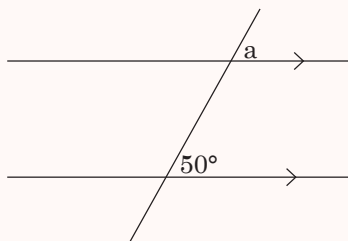
We can say angle a corresponds to angle b . Angle c corresponds to angle d .



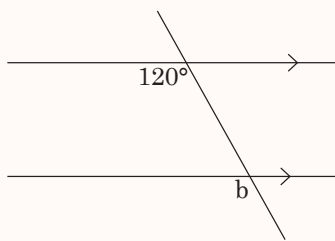
Application Activity 11.10

Find the value of the angles marked with letters.

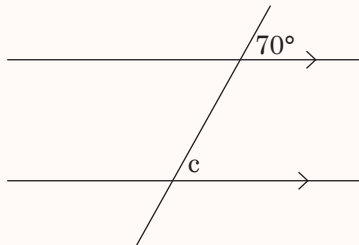
1.



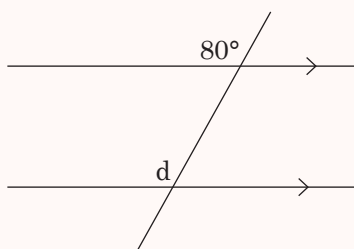
2.



3.



4.



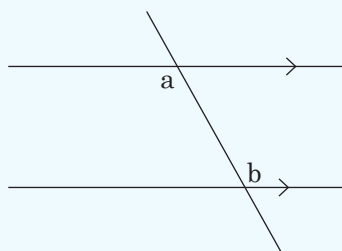
11.11 Angle properties of parallel lines: Alternate angles



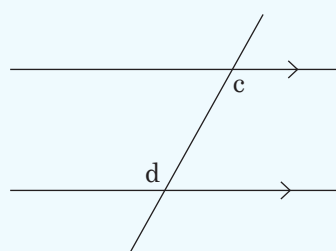
Activity 11.11

Use a protractor to measure and discuss the angles marked with letters on the diagram.

(a)



(b)



Measure angles a , b , c and d .

What do you notice about angle a and angle b ?

Explain the relationship between angles c and angle d .



Summary

Alternate angles are pairs of angles that are on opposite sides of a transversal line crossing two other lines.

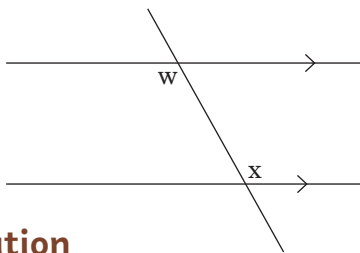
When two parallel lines are crossed by a transversal, **alternate angles are equal**.

Example,

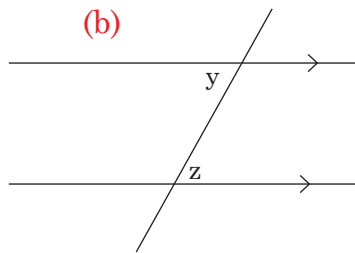
Using a protractor to measure the angles marked with letters. We find that $w = 120$ degrees and $z = 45$ degrees. Find the size of the angle x and the size of angle y . Use a protractor to verify.

Discuss the relationship between angles w and x ? What is common between y and z ?

(a)



(b)



Solution

(a) Using a protractor, we see that $x = 120^\circ$.

Angles w and x are equal because they are **alternate angles** made by one transversal on two parallel lines.

(b) Using a protractor, we see that $y = 45^\circ$.

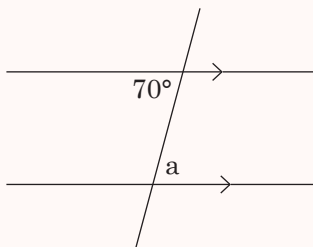
Angles y and z are equal because they are **alternate angles** made by one transversal on two parallel lines.



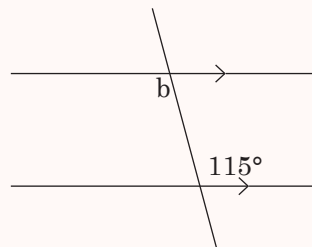
Application Activity 11.11

Find the value of the angles marked with letters and explain the relationship

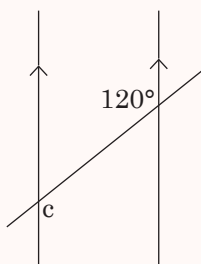
1.



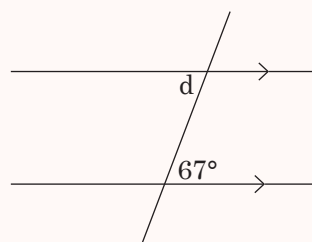
2.



3.



4.



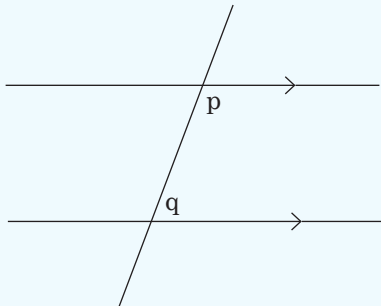
11.12 Angle properties of parallel lines: Co-interior angles



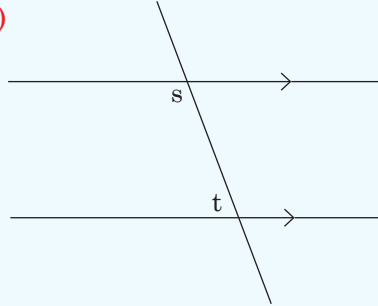
Activity 11.12

Use a protractor to measure the angles marked with letters.

(a)



(b)



Add angles $p + q$; Then add $s + t$.

What do you notice with the sum of angle p and angle q ?

What do you notice with the sum of angle s and t ? made by the transversal of two parallel lines? Explain.



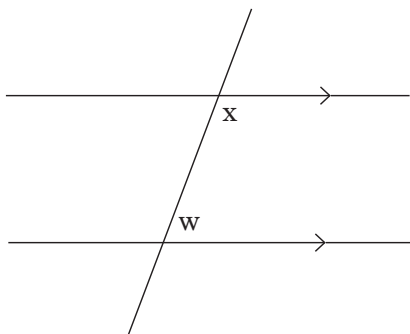
Summary

Co-interior angles are the angles that are on the same side of a transversal and inside the two lines it crosses.

When the transversal crosses two parallel lines, the co-interior angles are supplementary: they add up to 180 degrees. $P + q = 180$ degrees.

Example:

(a) In the figure below, $w = 70$ degrees. Use the protractor to measure angles x when two lines are parallel.



Solution

By measuring using a protractor, angle $w = 70^\circ$

angle $x = 110^\circ$

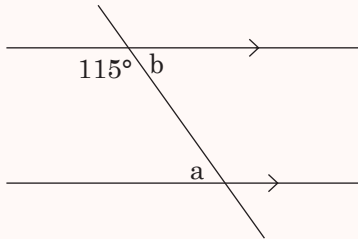
It is clear that $x = 180 \text{ degrees} - 70 \text{ degrees} = 110 \text{ degrees}$.



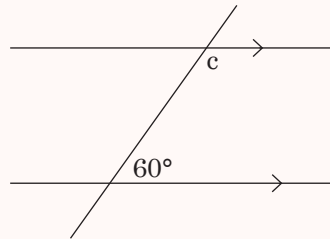
Application Activity 11.12

Find the value of the angles marked with letters

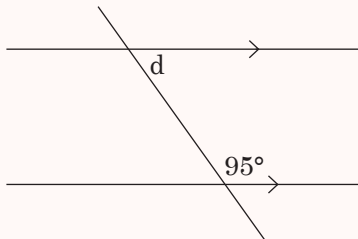
1.



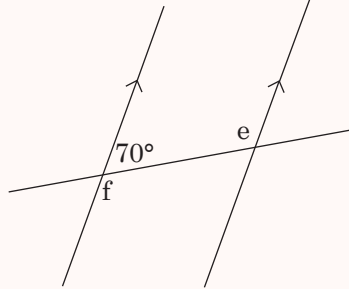
2.



3.



4.



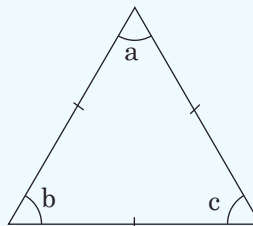
11.13 Sum of Angles of a triangle



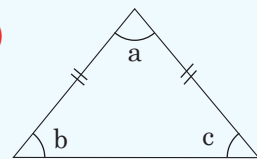
Activity 11.13

- Study the following triangles. We have labelled its angles with letters a, b, c.
- In each case, use a protractor to measure angles a, b and c.
- In each case, find the sum of $a + b + c$.
What do you notice?
- Present your findings in class.

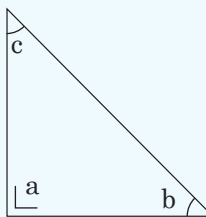
(i)



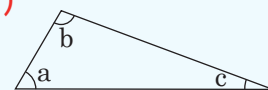
(ii)



(iii)

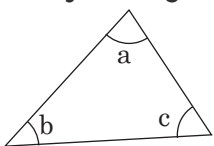


(iv)



Summary

For any triangle, the sum of its interior angles is 180° .



- a, b, c are the interior angles of the triangle.
- $a + b + c = 180^\circ$

Example:

- 1) In triangle PQR below, two of the angles are given. Find the value of angle x .

Solution

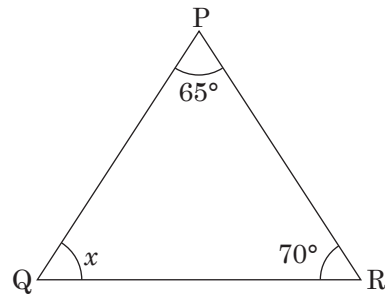
The interior angles of a triangle add up to 180° .

$$\text{Therefore } x + 65^\circ + 70^\circ = 180^\circ$$

$$x + 135^\circ = 180^\circ$$

$$x = 180^\circ - 135^\circ$$

$$x = 45^\circ$$



- 2) Find the value of the angle y in the triangle ABC.

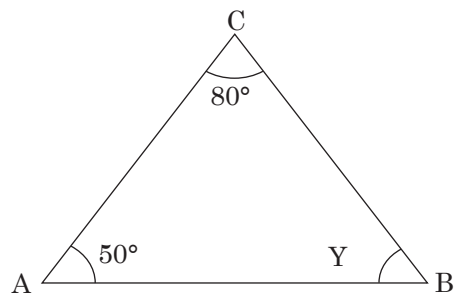
Solution

The sum of all angles of the triangle ABC is 180 degrees.

$$y + 50 \text{ degrees} + 80 \text{ degrees} = 180 \text{ degrees}$$

$$y + 130 \text{ degrees} = 180 \text{ degrees}$$

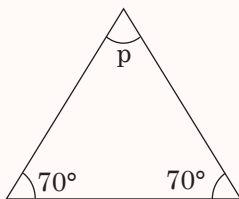
$$y = 180 \text{ degrees} - 130 \text{ degrees} = 50 \text{ degrees.}$$



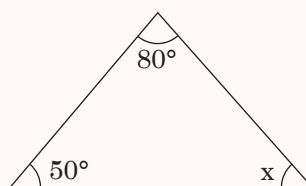
Application Activity 11.13

Study each triangle below. Then find the value of the angles marked with letters.

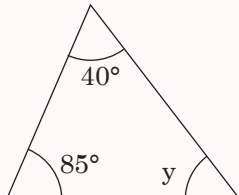
1.



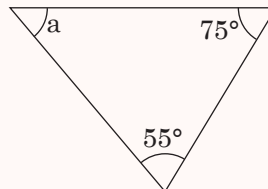
2.



3.



4.





Assessing knowledge and understanding

1. Multiple Choice

- a) Two angles that add up to 90° are called:
(i) Supplementary (ii) Complementary (iii) Opposite (iv) Corresponding
- b) When two parallel lines are cut by a transversal, alternate angles are:
(i) Equal (ii) Supplementary (iii) Complementary (iv) equilateral
- c) The sum of angles in a triangle is:
(i) 90° (ii) 180° (iii) 360° (iv) 270°

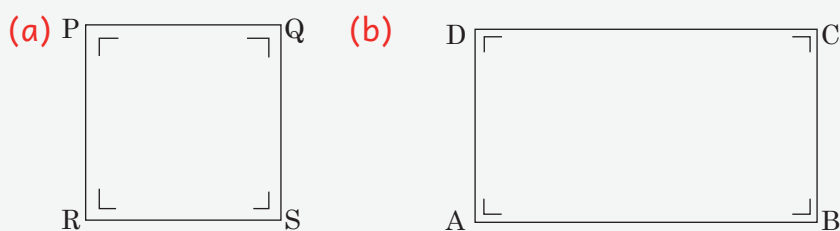
2. True or False

- a) A 45° angle can be constructed by bisecting 90° .
- b) Co-interior angles add up to 90° .

Assessing skills

3. Constructing Angles (6 marks)

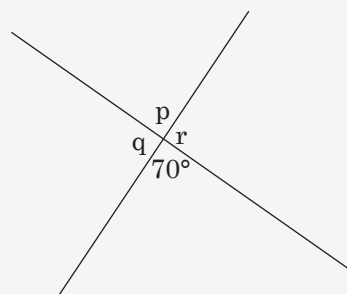
- a) Use a protractor to construct a 120° angle. Label the vertex A and arms AB and AC.
- 4) In the diagram below, identify lines that are parallel and lines that are perpendicular



- 5) The diagram below represents intersecting lines.

Find the value of the angles marked with letters

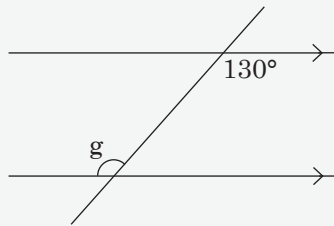
(i) $p =$ _____ (ii) $q =$ _____ (iii) $r =$ _____



- b) Using a ruler and compasses, construct a 30° angle. Show all construction arcs.

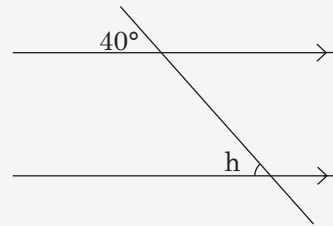
6) Study the transversals and parallel lines below. In each case, find the size of the marked angle. Explain your steps.

(a)



angle $g =$ _____

(b)



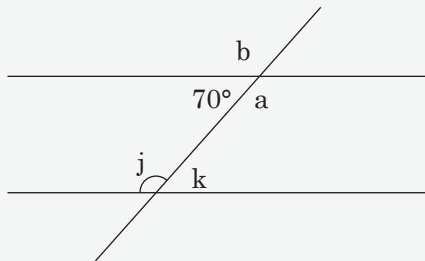
angle $h =$ _____

Assessing attitudes and Values

7.a) Give **two** examples of objects with perpendicular lines in your classroom.

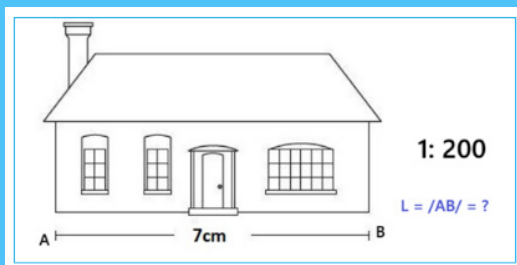
b) Explain two cases in which a house builder uses angle properties.

8. Study the diagram below in which two parallel lines cut by a transversal with one angle = 70°



i) Find all other angles: a , b , j and k .

ii) Explain why corresponding angles are equal.



UNIT 12

INTERPRETING AND CONSTRUCTING SCALE DRAWINGS

Key Unit Competence: You will be able to interpret and construct scale drawings.

Learning activities:

By the end of this unit, you should be able to:

Knowledge and understanding

Explain the concept of scale drawing.

Explain how to find the actual distance on the ground.

Explain how to find the scale of drawings and maps.

Skills:

Calculate the scale of a map.

Calculate the actual distance on the ground.

Find the distance represented on a map.

Solve problems involving scale drawings.



Attitudes and values:

Appreciate the importance of scale drawing in daily life.

Be confident and accurate when calculating scales.

12.0 Introduction

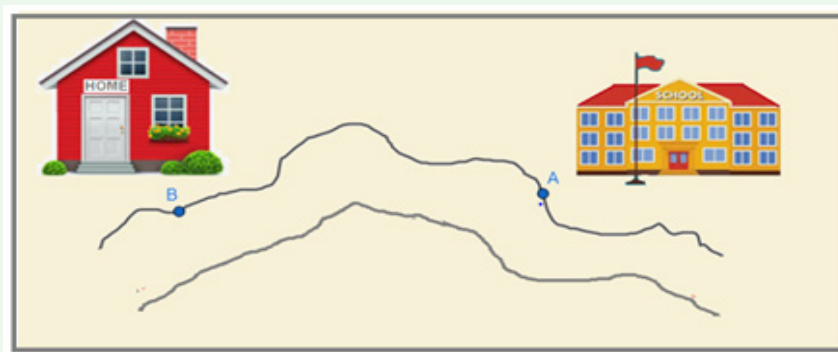
This unit provides information on scale drawing. Actual length of real objects may not be possible to fit on a piece of paper while drawing them. It is necessary to draw them to the size of the paper using shorter distances.

Drawing on the size of a paper is called scale drawing and a scale is usually expressed using small units where for example 1 cm can represent 1 km on drawings or it can be expressed using ratio / fraction without mentioning units, for example the ratio of 1: 100 000 can be used. The scale drawing is used when drawing maps of countries or specific region, plans of houses to be constructed, people or any real object.



Introductory Activity

Now, consider the following image and try to answer the related questions.



- 1) Estimate the distance from your school (in point A) to your home (in point B) and write it.
- 2) To write the real distance from school to your home, are you using big or small length unit?

On a piece of paper, can you represent 10 km? How can you proceed to represent such a distance? Explain your idea

12.1 Concept of scale drawing of actual size and the size on a map



Activity 12.1

Draw diagrams to represent the following in your books.

- (a) Your classroom.
- (b) The distance between your classroom and the office.
- (c) The chalkboard.

Measure and record actual distances of real objects you have drawn.

Measure the length of your drawings.

Compare the actual measurements with their drawing measurements. Which distances are bigger? Make a presentation to the class.



Summary

Actual distances may not be possible to fit in a drawing on a paper.

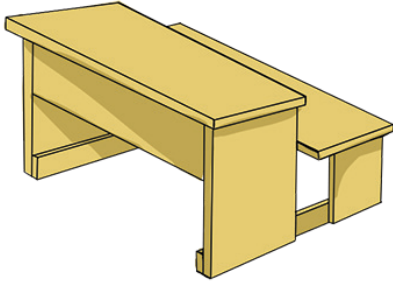
We draw them to the size of the paper using a shorter distance.

When we draw, we reduce objects to a scale. For example, a distance of 12 km road can be drawn as 12 cm on paper. We say that we have drawn to **a scale**.

A **scale drawing** is representation of an object or space, where its size is reduced or enlarged by the same ratio called the **scale**.

Example

Measure the length of the top of the desk drawn below. Record its length in cm.



Now measure the length of your actual desk in cm.
Compare the drawing length with the actual length.

Solution

The length of the top of the desk drawn is 3 cm.

The length of the real desk in the classroom is 150cm.

We see that 3cm on the paper represent 150 cm of actual desk. Therefore, we can find the scale that indicates the number of cm represented by 1 cm on a paper.



Application Activity 12.1

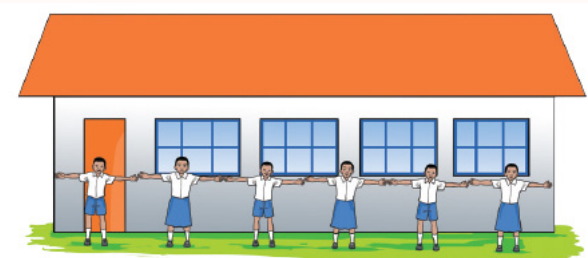
Study the following objects:



Section of road



Pencil



Classroom

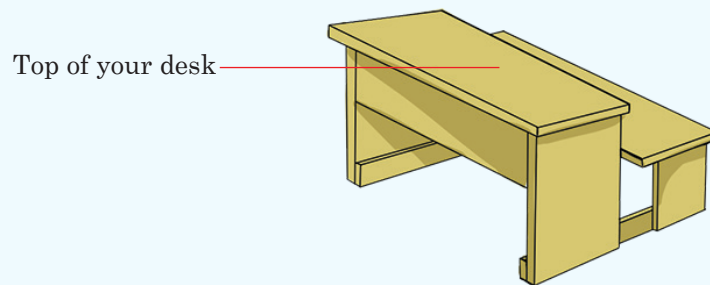
- What are their actual lengths?
- Measure their drawing lengths.
- Explain how diagram of a large object can fit on a small sheet of paper

12.2 Finding the scale to which a real object will be drawn on a sheet of paper



Activity 12.2

Use a ruler to draw the length of the top of your desk. Record it in cm.



Try to draw the top of your desk. Use your ruler to measure the length of the top of your drawing. Record it in cm.

To find the scale you have used, find how many cm of the desk represented by 1cm of the paper.

Find it and complete: 1 cm of the drawing on the paper represents _____ cm of the real desk.

This is: **1:** _____

Make presentations of your findings to the class.



Summary

To find a scale;

- (i) Measure drawing length
- (ii) Measure the actual length of the object in the same unit as the drawing length.
- (iii) Calculate the scale

$$\text{Scale} = \frac{\text{Drawing length}}{\text{Actual length}} = \text{Drawing length} \div \text{Actual length}.$$

The scale does not have units.

Actual length = Drawing length $\div$ Scale.

Example:

- (1) 12 sheets of paper fit on top of a desk. The top of a desk is drawn on one piece of paper. Find the scale.

Solution

- 12 sheets covering top of desk.

- To draw one sheet, I divide actual lengths by 12.

The scale is 1:12

(2) The actual length of a path is 20 m. It is drawn using a line 5 cm long. What scale has been used?

Solution

Drawing length= 5cm

Actual length= 20m = 2000 cm

Working in same units, we have:

$$\text{Scale} = \frac{\text{Drawing length}}{\text{Actual length}} = \frac{5\text{cm}}{200\text{cm}} = \frac{1}{400} = 1:400$$

The scale used is 1:400.



Application Activity 12.2

1) The actual lengths for various items were measured. Their drawing lengths were recorded as follows.

	Item (s)	Actual length	Drawing length	Scale used
(a)	Length of a desk	2 m	2 cm	2:200 = 1:100
(b)	Width of a table	1.5 m	5 cm	_____
(c)	Length of a ruler	30 cm	10 cm	_____
(d)	Diameter of a cup	7 cm	3.5 cm	_____
(e)	Length of a road	40 km	10 cm	_____
(f)	Perimeter of a classroom	6 dam	30 cm	_____

Calculate the scale used and fill in the table above accordingly.

- 2) The actual distance for a section of road is 25 km. It is drawn on a map using a 5 cm line. Explain how to find the scale of the map.
- 3) A flag post is drawn to scale. Its drawing height is 5 cm. Suppose the actual height is 10 m. Find the scale used to draw the flag post.
- 4) The actual length of the Nyabarongo River is 300 km. On a map, it is represented by a 30 cm long line. Discuss how to find the scale used of the map.

12.3 Construction of scale drawings



Activity 12.3

Measure the actual length of your classroom. Use a scale to draw a line to represent the length of your classroom.

Measure the width of your classroom. Use the same scale to draw the width on the shape of the floor.

What is the actual distance from

(i) your classroom to assembly?

(ii) your classroom to the office?

Draw a simple map showing distances for:

Classroom point to office point and assembly point.

Use a suitable scale.



Summary

To make a scale drawing,

(i) Know the actual distances

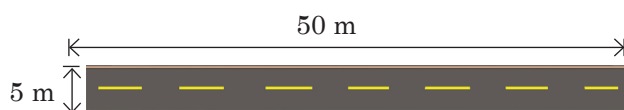
(ii) Choose a good scale that will help you to fit the drawing on the size of a paper you have.

(iii) Find drawing distances

(iv) Draw the diagram.

Example:

Look at the sketch of a section of a road. The actual size is mentioned on the diagram.



Explain how you would use a scale of 1:500 to show the size of the road on paper.

Solution

The scale 1: 500 indicates that 1cm represents 500 cm. This means that 1cm represents 5 m.

Therefore, let us find the drawing length that represents the length of 50 m:

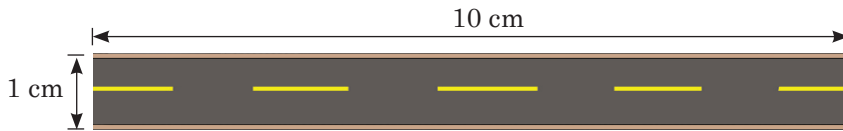
5m are represented by 1cm

1m is represented by $1\text{cm}/5$

50 m are represented by $\frac{1\text{cm}}{5} \times 50 = 10\text{cm}$

For the width, 5m are represented by $\frac{1\text{cm}}{5} \times 5 = 1\text{cm}$

Therefore, the drawing has the following size:



- 2) A dining hall measures 40 m long and 35 m wide. Using a scale 1:1 000, explain how to make a scale drawing of the hall

Solution

Length = 40m, width = 35m

The scale 1:1 000, means 1 cm represents 1 000 cm. This also means 1 cm represents 10m (because $1000\text{cm} = 10\text{m}$)

10m are represented by 1cm

1m is represented by $1\text{cm} \div 10$.

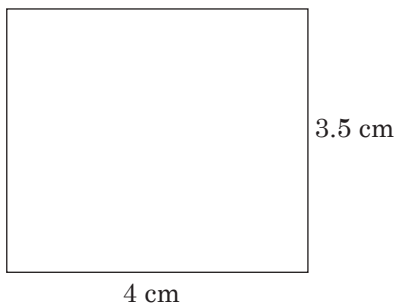
40m of the hall are represented by $(1\text{cm} \div 10) \times 40 = 4\text{cm}$

Drawing length of dining hall is represented by 4cm.

The width of the hall is 35m, it is represented by $(1\text{cm} \div 10) \times 35 = 3.5\text{cm}$.

The dining hall will be represented by a rectangle. The rectangle is 4 cm long and 3.5 cm wide.

The rectangle is drawn below.



Application Activity 12.3

- Using a scale of 1:100, make a scale drawing of the following.
 - A rectangle measuring 7 m by 3 m.
 - An equilateral triangle with 5 m sides.
 - A square with 4 m sides.
- Using a scale of 1:500 000, draw lines to represent the following distances.
 - 15 km
 - 25 km
 - 45 km
 - 10 km
- Using a scale 1:1 000, describe how to draw lines to represent the following length.
Present to the class.
 - 7 200 cm
 - 8 000 cm
 - 6 800 cm

12.4 Finding actual distance



Activity 12.4

1) Measure the length of the line below. It represents a ruler.



The scale used is 1:10. Find the actual length of the ruler.

2) Consider 15 cm ruler, 30 cm ruler and a metre-rule with 100cm.

Which of the three rulers has its length represented by AB?

Explain your answer.



Summary

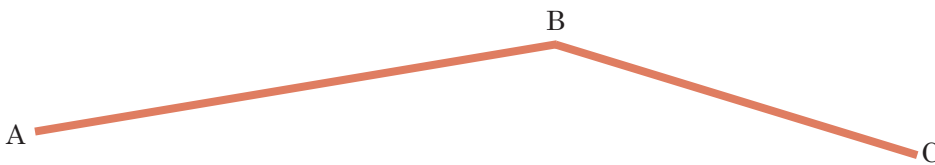
To find actual distance,

- (i) Measure drawing length.
- (ii) Explain or interpret the scale.
- (iii) Use formula,

$$\begin{aligned}\text{Actual distance} &= \frac{\text{drawing length}}{\text{scale}} \\ &= \text{drawing length} \times \text{value of scale (represented by 1 cm)}.\end{aligned}$$

Example:

In a map, a section of road is represented by the line below:



The scale used is 1:10 000

- (a) Measure line AB, BC. What distance is AC through B?
- (b) Interpret the scale.
- (c) Find the actual distance AB and AC.

Find the actual distance of the section of the road.

Solution

(a) Drawing lengths: $AB = 7 \text{ cm}$, $BC = 5 \text{ cm}$.

$AC \text{ through } B = (7 + 5) \text{ cm} = 12 \text{ cm}$.

(b) The scale 1:10 000 means that 1 cm drawing length represents 10 000 cm or 100 m actual distance on the road.

(c) Actual distance; $AB = 7 \times 100 \text{ m} = 700 \text{ m}$ $BC = 5 \times 100 \text{ m} = 500 \text{ m}$

Distance of the road = $700 \text{ m} + 500 \text{ m} = 1\,200 \text{ m} = 1.2 \text{ km}$.



Application Activity 12.4

1) The drawing length for a section of a river is 10 cm. The scale used was 1:2 500. Find the actual length of the section of the river (in m).

2) Given the scale 1:200 000, explain how to find the actual lengths of the following.

(a) 5 cm (b) 2.5 cm (c) 3.2 cm (d) 4.9 cm

12.5 Finding the drawing length



Activity 12.5

Measure the actual distance from your classroom to the assembly ground.

Use a metre rule or tape measure.

Measure the actual lengths of your classroom. Record your results in the table below.

Use a scale 1:1 000 to find drawing lengths for each object.

	Actual distance	Drawing length
(a) Classroom to assembly		
(b) Length of classroom		
(c) Width of classroom		

Present your findings to the whole class.



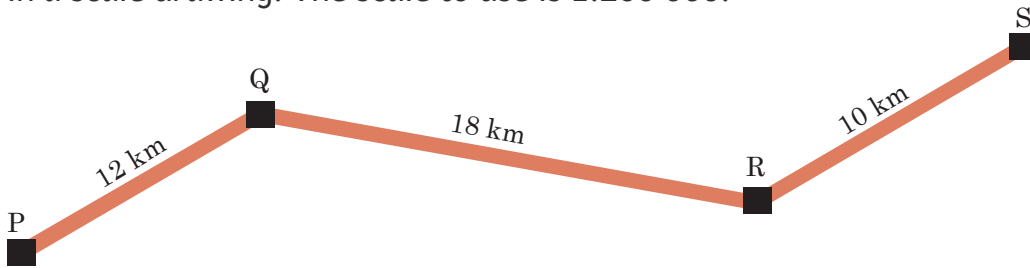
Summary

The drawing size is the size used to draw a real object on the paper;

The drawing length = scale $\times$ actual size = Actual size / value represented by one unit.

Example:

The sketch below shows actual distances between towns P, Q, R and S. It is to be represented in a scale drawing. The scale to use is 1:200 000.

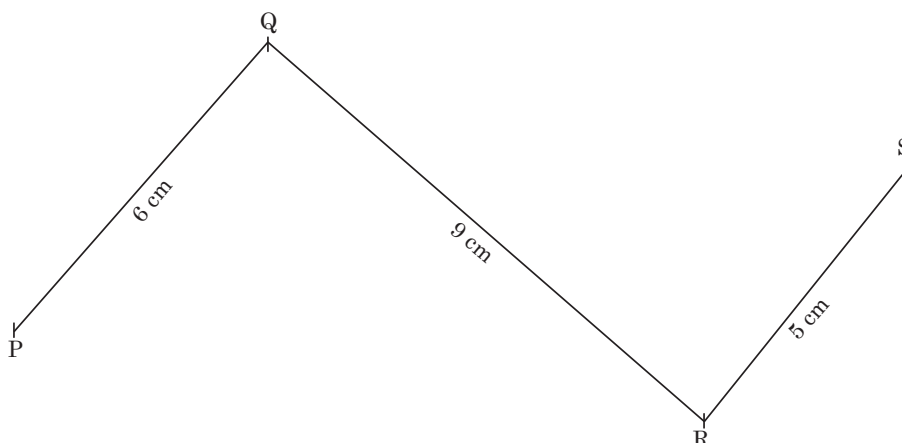


- (a) Interpret the scale.
- (b) Explain how to get the drawing measurements between the towns.
 - (i) PQ (ii) QR (iii) RS
- (c) Make a scale drawing of the distances between towns.

Solution

- (a) The scale 1:200 000 means 1 cm represents 200 000 cm or 2 km. Thus, 1 cm on the drawing represents 2 km of actual distance.
- (b) (i) 1 cm represents 2 km. So PQ = 12 km.
12 km is represented by $(12 \div 2)$ cm = 6 cm.
 - (ii) Actual distance QR = 18 km. Drawing length for QR = $(18 \div 2)$ cm = 9 cm.
 - (iii) Actual distance RS = 10 km. Drawing length for RS is $(10 \div 2)$ cm = 5 cm.

c)





Application Activity 12.5

1. Use the scale 1:10 000. Find the drawing lengths for these:
 - (a) A section of river that is 350 m
 - (b) A section of road that is 820 m
 - (c) The length of school path that is 225 m
2. Use the scale 1: 100 000. Find the drawing length for each of the following:
 - (a) A road joining towns PQ = 60 km
 - (b) A railway line joining towns XY = 225 km
 - (c) A length of river joining two provinces = 200 km.
3. The distance between two towns is 120 km. Use a scale of 1:300 000.
Find the drawing length for the two towns.
4. Using the scale of 1:2 000, make the following scale drawings.
 - (a) Rectangular field measuring 80m by 60m.
 - (b) A path which is 240 m long.Then explain your work.
5. Using the scale of 1:30 000, make scale drawings of the following. Then, discuss your work.
 - (a) square field with sides of 1200m.
 - (b) A section of a road which is 2 700 m.



End unit assessment

Assessing knowledge and understanding

Choose the correct answer (A, B, C, or D).

1. What is a scale drawing?

- A) A drawing made with a ruler
- B) A drawing that reduces or enlarges real objects proportionally
- C) A freehand drawing
- D) A colorful artwork

2. If the scale is 1:100, what does this mean?

- A) 1 cm on the drawing = 1 cm in real life
- B) 1 cm on the drawing represents 100 cm in real life.
- C) 100 cm on the drawing represent 1 cm in real life
- D) The drawing is 100 times bigger.

3. Which of these is a reason for using scale drawings?

- A) To make drawings look more colorful
- B) To fit large objects onto small paper
- C) To avoid using rulers
- D) To make drawings 3D

Assessing skills:

- 4. A road is **25 km** long. On a map, it is **5 cm** long. Calculate the scale used.
- 5. A classroom is **8 m** long and **6 m** wide. Using a scale of **1:200**, find the drawing dimensions.

A building measures 24 m by 10 m. It is to be drawn to scale on a paper measuring 30 cm by 20 cm. Discuss the appropriate scale to be used.

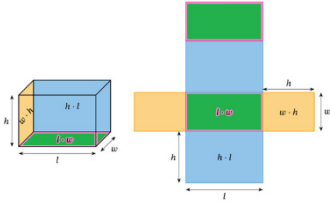
- 6. A football field is **90 m** long. Using a scale of **1:1,000**, what will be its drawing length?
- 7. A map uses a scale of **1: 25,000**. Explain how you would find the real distance if the map distance is **8 cm**.
- 8. A section of river is 1 720 m. It is drawn on a map. The drawing length is 17.2 cm. Find the scale used in the map.
- 9. Using the scale 1:1 000, make scale drawings of the following and discuss your answers.
 - (a) Rectangle measuring 40 m by 20 m.
 - (b) Square field whose sides are 50 m.
- 10. In a map, a scale of 1:300 000 is used. Discuss the steps to follow to calculate the drawing length for these distances:
 - (a) 21 km
 - (b) 27 km

Assessing attitudes and values

- 11. Give **two** examples of where scale drawings are used in real life.
- 12. Why is teamwork helpful when making a school map?

UNIT 13

CALCULATING CIRCUMFERENCE OF A CIRCLE, SURFACE AREA AND VOLUME OF CUBES AND CUBOIDS



Key Unit Competence: You will be able to calculate circumference of a circle, surface and volume of cubes and cuboids.

Learning activities:

By the end of this unit, you should be able to:

Knowledge and understanding

Explain the concept of circumference.

Describe the process of finding the circumference of a circle.

Establish the relationships between cubes and cuboids.

Explain how to find the volume of cubes or cuboids.



Skills:

Practically calculate the value of pi (π)

Calculate the surface area and the volume of cubes and cuboids

Solve mathematical problems involving the circumference of a circle.

Solve mathematical problems related to the finding the volume of cuboids and cubes.

Attitudes and values:

Show confidence when exploring the circumference of circular objects from daily life.

Appreciate the importance of surface area and volume of cubes and cuboids in daily life.

13.0 Introduction

In this unit, you will learn how to calculate the circumference of a circle, as well as the surface area and volume of cubes and cuboids. These skills are not just for the classroom; they are used in many real-life situations.

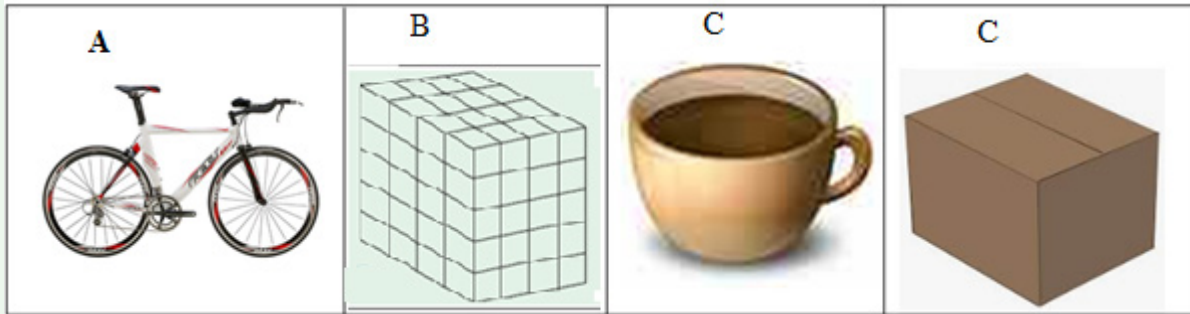
The circumference of a circle is applied when finding the length of a fence wire for fencing a garden.

The surface area of cube and cuboids is used to determine the size of a wrapping paper required to cover a gift box. The volume of cubes and cuboids can be applied when finding for example the quantity of water a fish tank can hold.



Introductory Activity

In nature, there are objects that are in circular, cube and cuboids shapes. Refer to the images below and answer the related questions:



- 1) Identify objects with circular shapes in the above images. Why do you think these objects have circular shapes? Discuss.
- 2) Take the box in turns. Count its faces, vertices and edges. Record the counted numbers in a table and discuss your results.

A box	Number of faces	Number of Vertices	Number of edges

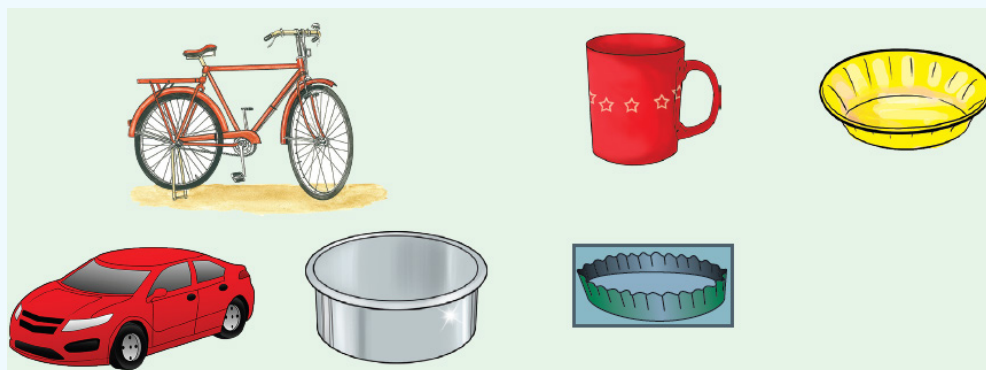
- 3) In your own words, how can you describe the volume of a box?

13.1 Circumference of circular objects



Activity 12.5

- 1) Study the diagrams below.



- (i) Trace the circular path on each of them.
- (ii) Which objects with circular paths have you brought?
- (iii) Wrap a string around the circular path of the object. Untie the string and measure its length using a ruler. How do you call the length obtained?

2) Conduct this practical activity:

Step 1: Consider a near-by round about in the school compound, and then trace the circular path of the roundabout.



Step 2: Wrap a rope along the circular path of the roundabout (following the red decorations).

Step 3: Now straighten the rope and measure it using a measuring tape.

Step 4: Record the length of the rope that traced the circular path as the circumference of the roundabout.

Step 4: Measure the radius R of the circle round about and complete the following table.

Radius	Diameter	Measured length of the roundabout	Circumference = (Length of diameter x 3.14)

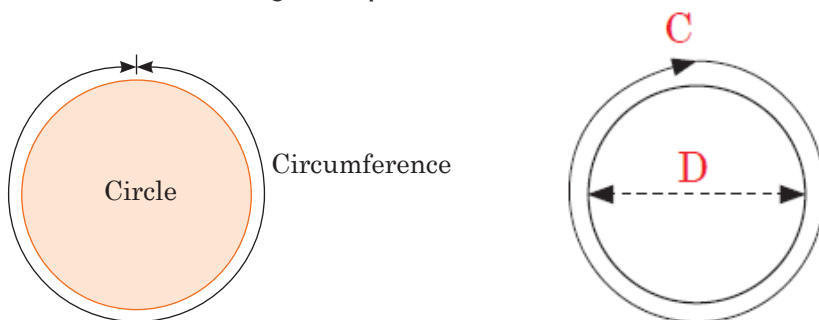
Step 5: Compare the Measured length of the roundabout and the product “Length of diameter x 3.14”. What is your observation? How do we find the circumference of a circular object if we have its diameter?



Summary

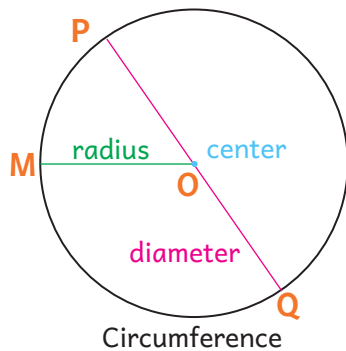
The circumference of a circular path is the total distance around that path.

We obtain it by tracing their paths, wrapping a rope or string around the path and then measure the string or rope with a ruler or by finding the product of the diameter by 3.14.



$$\text{Circumference} = \text{diameter} \times 3.14$$

A **circle** is a perfectly round two-dimensional shape where every point on its edge (called the **circumference**) is the same distance from a fixed point inside it, known as the **center**.



Key Parts of a Circle:

Center – The fixed point O in the middle of the circle.

Radius R: is the distance from the center O to any point M on the circle. $R=OM$

Diameter (D) – A straight line passing through the center, connecting two points on the circle. It is twice the radius: $D=PQ=2R$.

Circumference (C) is the total distance around the circle.

Circles are everywhere in real life: wheels of vehicles, clocks, plates, and even the shape of a pizza. Understanding circles helps us measure round objects and solve problems in construction, engineering, and daily activities.

To calculate the **circumference of a circle** using the formula:

$$C = 3.14 \times D \text{ or}$$

$$\text{Given that } D = 2 \times R, \text{ we have } C = 2 \times R \times 3.14$$

Examples:

(1) Find the circumference of a circle whose diameter is 14 cm.

Solution

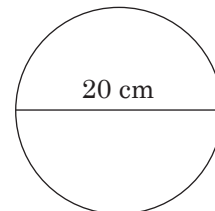
$$\text{The circumference } C = D \times 3.14 = 14\text{cm} \times 3.14 = 43.96\text{cm}.$$

(2) Observe the following circle. Find its circumference.

Solution

The circumference

$$C = D \times 3.14 = 20\text{cm} \times 3.14 = 62.8 \text{ cm}.$$



Application Activity 13.1

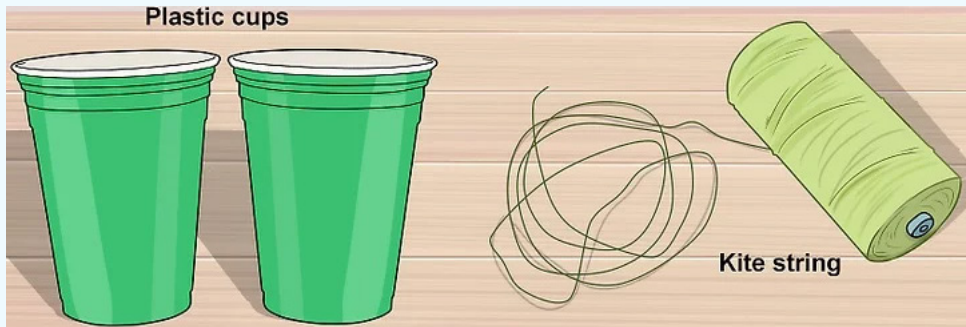
- 1) A bicycle wheel has a circumference of 220 cm. How far will the bicycle travel if the wheel completes 50 full rotations?
- 2) A Ferris wheel with a circumference of 62.8 m completes 5 rotations (revolutions) in 10 minutes. What is the total distance traveled by a rider?
- 3) A coin rolls along a cylindrical bar with circumference 7.54 cm without slipping. How many complete rotations does it make if it travels 100 m?

13.2 The number pi (π)



Activity 13.2

Step 1: Wrap a string around the top of the plastic cup one time



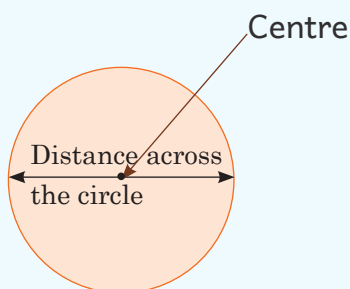
Step 2: Untie the string and use a tape measure to measure the length of the string from the plastic cup and record its length.



Note: The length of the string from the cup is the distance around the cup called **Circumference (C)**

Step 3: Use a tape measure to measure the distance from one side of the cup through the center to the other end and record it.

Note: This distance that passes through the center of the cup is called **Diameter (D)**
See the figure below:



Step 4: Draw a table on a manila chart as shown below to record in other measurements of circular objects.

Object	Circumference (C)	Diameter (D)	$C \div D$



Summary

Observation:

The number obtained from **Circumference (C) ÷ Diameter (D)** of each circular object is a constant number called **Pi** symbolically written as π .

For every circular object;

Pi (π) = Circumference (C) ÷ Diameter (D) = approximately 3.14 as a constant value.

Definition:

Pi (π) is a mathematical constant that represents the ratio of a circle's circumference to its diameter.

In other words, it tells us how many times the diameter fits around the circumference of any circle.”

Mathematically, this is expressed as:

$$\pi = \frac{C}{D} = \frac{C}{2R}$$

where: C is the **circumference** of the circle, D is the **diameter** of the circle. And R is the radius of the same circle.

Key Properties of π :

The number π cannot be expressed as a simple fraction, and its decimal representation never ends or repeats $\pi \approx 3.1415926...$ but we simply write $\pi \approx 3.14$.

Note: In some books, authors use $\pi = \frac{22}{7}$ but this is not true, it is a way of helping pupils to easily calculate without decimals.

It is a universal Constant: This ratio ($\pi = C/D$) is the same for all circles, regardless of size.

Examples

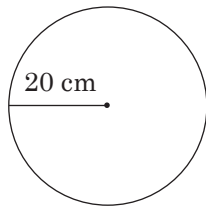
(1) What is the circumference of a circle whose radius is $10\frac{1}{2}$ cm. Take $\pi = \frac{22}{7}$.

Solution:

$$R = 10\frac{1}{2} \text{ cm} = \frac{21}{2} \text{ cm}$$

$$\text{The circumference } C = 2 \times R \times \pi = 2 \times \frac{21}{2} \times \frac{22}{7} = 66 \text{ cm}$$

(2) Find the circumference of the circle below:



Solution:

The circumference

$$C = 2 \times R \times \pi$$

$$= 2 \times 20\text{cm} \times 3.14 = 125.6 \text{ cm}$$

(3) A circular flower garden has a radius of 7 m. It was fenced all around the circumference. What is the length of the fence?

Solution

Length of the fence = $D \times \pi$

As $D = 2 \times R$, $D = 2 \times 7\text{m} = 14\text{m}$.

The circumference of garden = Length of fence

Length of fence = $D \times \pi = 14\text{m} \times 3.14 = 43.96\text{m}$.

(4) The circumference of a circle is 88 cm. **Solution**

a) Find its diameter. Take $\pi = 22/7$.

$$(a) \quad d = \frac{c}{\pi}$$

$$d = 88 \div \frac{22}{7}$$

$$= 88 \times \frac{7}{22} = 28 \text{ cm}$$

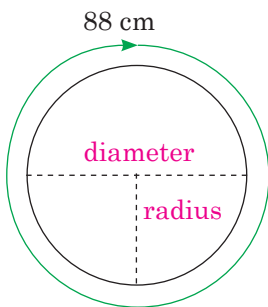
b) Find its radius.

$$(b) \quad 2r = d$$

$$\frac{2r}{2} = \frac{28}{2} \text{ cm}$$

To get the radius, divide the diameter by 2.

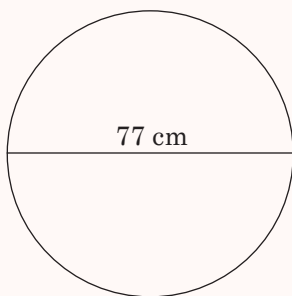
$$r = 14 \text{ cm}$$



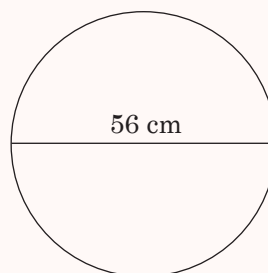
Application Activity 13.2

1) Find the circumference of the circle. Take $\pi = 22/7$.

1.



2.



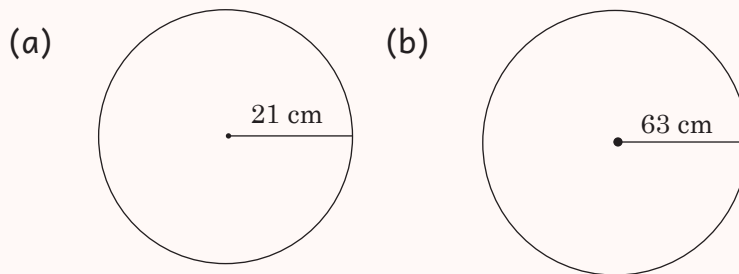
2) Discuss and find the answers to the following questions. Present your findings.

- a) The diameter of a circular ring is 21 cm. What is its circumference?
- b) A bicycle wheel has a diameter of 98 cm. What is the circumference of the wheel?
- c) The diameter of the circle at the centre of a football field is 9.8 m.

i) What is the circumference of the circle?

ii) A player ran round the circle three times. What distance did he cover?

3) What is the circumference of each of the circles below? Take $\pi = 22/7$.



4) The radius of a circle is 15 cm. What is its circumference?

5) The radius of the lid of a bucket is 45 cm. Calculate the circumference of the lid.
Discuss your steps to answer.

6) The circumference of a circle is 110 m. Calculate its diameter.

7) The circumference of a roundabout is 30.8 m. Calculate its radius.
Take $\pi = 22/7$ Discuss your steps.

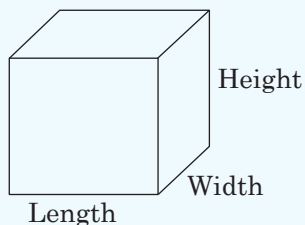
8) The circumference of a circle is 439.6 cm. What is its diameter? Take $\pi = 3.14$.
Discuss your steps.

13.3 Properties of cube and cuboids



Activity 13.3

1) Discuss and identify the length, width and height of different carton boxes.



Prepare a chart like one shown below.

Measure the length, width and height of different boxes.

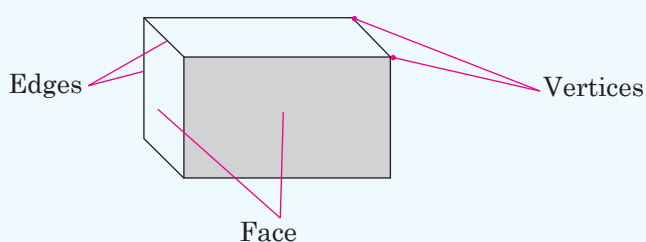
Record your results in your chart

Cubes and cuboids chart			
Box	Length	Width	Height

2) For which boxes the length, width and height have the same size? Those boxes are called **cubes**.

For which boxes the length, width and height have the different sizes? Those boxes are called **cuboids**.

3) Observe the faces, edges and vertices for cubes and cuboids,



Take the cubes and the cuboids in turns. Count their vertices, faces and edges.

Record them in your chart. Discuss your results.

	Faces (f)	Edges (e)	Vertices (v)
Cube			
Cuboids			

For each solid, verify if $f + v = e + 2$

4) Now, make a chart as shown below. Study cubes and cuboids then fill in your chart appropriately:

Properties	Cube	Cuboid
Opposed faces are equal		
Opposed edges are equal		
Have 8 vertices		
Have 12 edges		
All faces are equal		

(a) Discuss the similar properties between cubes and cuboids.

(b) List the difference between cubes and cuboids.



Summary

Properties of a Cube

Faces: 6 square faces (all equal in size).

Edges: 12 edges (all equal in length).

Corners (Vertices): 8 corners.

Examples: Dice, chalk box, sugar cubes.

Properties of a Cuboid

Faces: 6 rectangular faces (opposite faces are equal).

Edges: 12 edges (grouped into 3 lengths: length, width, height).

Corners (Vertices): 8 corners.

Examples: Books, shoeboxes, bricks.



Application Activity 13.3

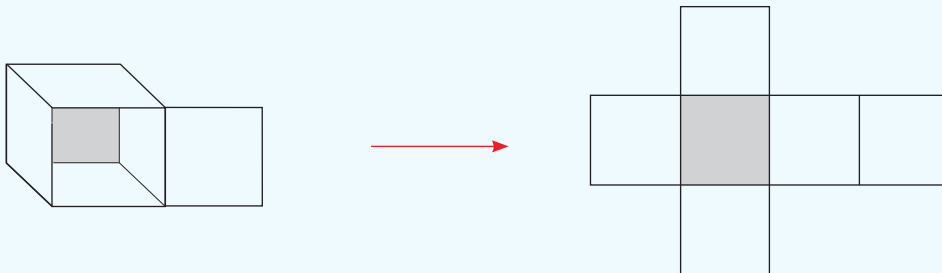
- 1) How many vertices does a cube have?
- 2) How many edges are there in a cuboid?
- 3) How many faces are there in a cube?
- 4) How many faces are there in cuboid?
- 5) What is the product of edges and vertices in a cube?
- 6) Observe the shapes of objects in the classroom.
 - (a) Which ones are cubes? Why are they cubes?
 - (b) Which ones are cuboids? Explain why they are cuboids.

13.4 Use of nets to calculate the surface area of cubes



Activity 13.3

- 1) Take a cubic box. Open it as shown below:



The completely opened shape of the box is called a net.

To make a cube, we must first prepare 'nets'.

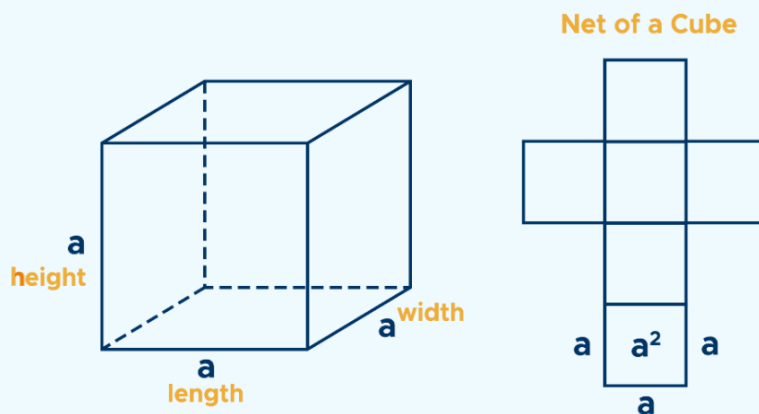
Open other cubes to form different nets.

How many faces are in each net?

Fold a net to form a cube.

Fold nets to form a cube or cuboid. Present your model

2) Draw the net of a cube of sides $a = 5$ units. Use your net to calculate the total surface area of the cube. Compare your answer with those of other groups in your class.

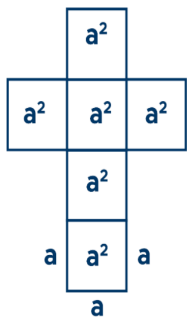


Summary

By definition, a cube has six faces that are identical. The faces are all squares. Therefore, total surface area = area of one face $\times$ 6.

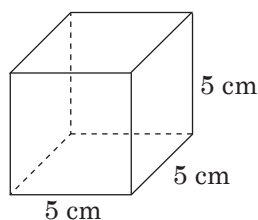
If the length of each face is a unit.

Area = $6 \times a \times a$ square units



Examples

1) Find the surface area of a cube of side 5 cm

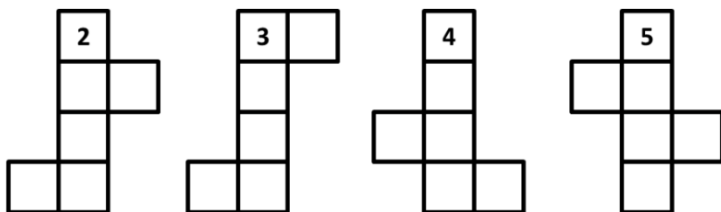


Solution

The side $a = 5\text{ cm}$

$$SA = 6 \times a \times a = 6 \times 5\text{ cm} \times 5\text{ cm} = 150\text{ cm}^2.$$

2) Observe the following nets. Use sheets of paper to make them and verify if they are nets of a cube. Try to make others.



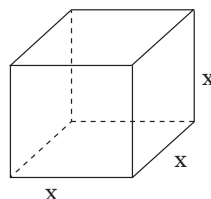
Solution

They are nets of cubes because they have the same size of the side or edge. It depends on how they were attached.

A cube can have 11 different forms of nets. Make sure they have the same size of the edge.

3) A cube has a total surface area of 96 square cm.

Find the length of the side of the cube



Solution

Start with the formula:

$$\text{Surface area of a cube} = 6 \times (\text{side} \times \text{side}) = 96\text{ cm}^2$$

Divide both sides by 6:

$$\text{side} \times \text{side} = 96\text{ cm}^2 / 6 = 16\text{ cm}^2$$

Find the number that was multiplied by itself to get 16:

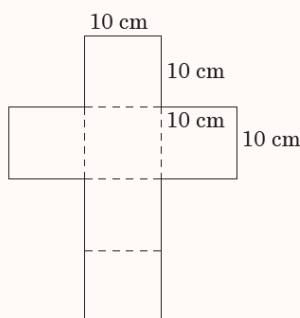
This number is 4 because $4 \times 4 = 16$

Answer: The side of the cube is 4cm.

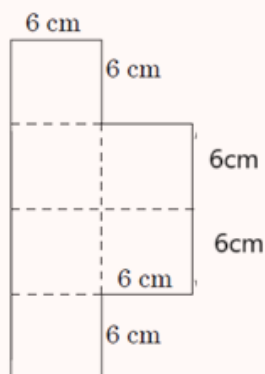
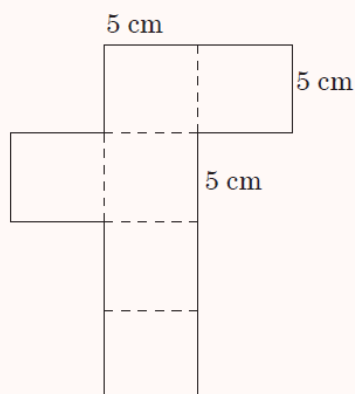


Application Activity 13.4

1) Find the surface area of the cube if its nets are the following:



- 2) Sarah is wrapping a gift box shaped like a cube with 3 cm sides. How much wrapping paper (in cm^2) does she need to cover the entire box?
- 3) A cube has a total surface area of 216 cm^2 . Find the length of the side of the cube.
- 4) Draw the following on manila paper. Cut the nets out. Fold them along the dotted lines.

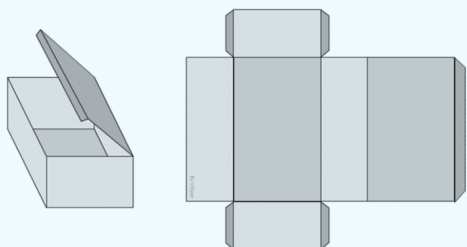


13.5 Use of nets to calculate the surface area of cuboids



Activity 13.5

- 1) Take a cubic box. Open it as shown below:

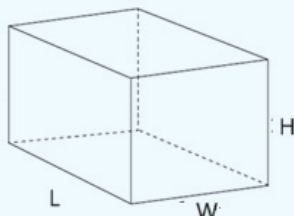
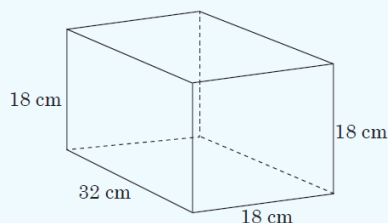


The completely opened shape of the box is called a net.

To make a cuboid, we must first prepare 'nets'. Open other boxes to form different nets.

- How many faces are in each net?
- Fold a net to form a cuboid.
- Fold nets to form cuboid. Present your model

- 2) Draw the net of a sides $L=32\text{cm}$, $w= 18\text{cm}$ and $h=18\text{cm}$.



Draw a well labeled net of the cuboid above.

- How many faces does the cuboid have from the net?
- How would you calculate the area of the cuboid using the net?
- Use the net to calculate its area in terms of length (L), width (W), and height (H).

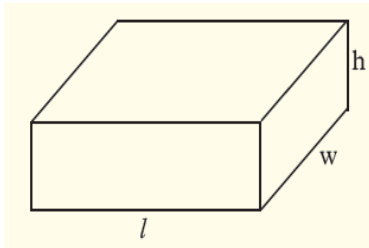


Summary

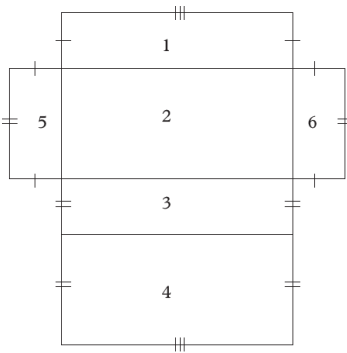
From Activity, you have learnt that:

The surface area of a cuboid of length L , width W and height H , is given by the surface of 6 faces considering that 2 parallel faces are equal. This means that total surface area is:

$$A = 2(L \times W) + 2(L \times H) + 2(W \times H)$$

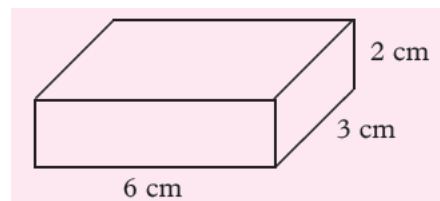


The net of the cuboid will be as follows:



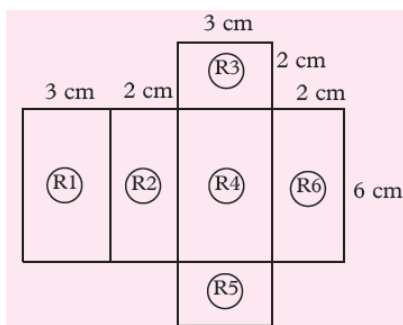
Examples

The net of a cuboid consists of a series of rectangles. How many rectangles are there? What is the surface area of the cuboid if it measures 6 cm by 3 cm by 2 cm?



Solution

See the possible net of a cuboid which measures 6 cm by 3 cm by 2 cm. It is composed of three pairs of rectangles i.e. 6 rectangles.



The surface area of the cuboid = sum of the areas of all the rectangles that comprise the net.

$$\text{Area of rectangle } R1 = 6 \text{ cm} \times 3 \text{ cm} = 18 \text{ cm}^2$$

$$\text{Area of rectangle } R2 = 6 \text{ cm} \times 2 \text{ cm} = 12 \text{ cm}^2$$

$$\text{Area of rectangle } R3 = 3 \text{ cm} \times 2 \text{ cm} = 6 \text{ cm}^2$$

$$\text{Area of rectangle } R4 = 6 \text{ cm} \times 3 \text{ cm} = 18 \text{ cm}^2$$

$$\text{Area of rectangle } R5 = 3 \text{ cm} \times 2 \text{ cm} = 6 \text{ cm}^2$$

$$\text{Area of rectangle } R6 = 6 \text{ cm} \times 2 \text{ cm} = 12 \text{ cm}^2$$

$$\text{Total surface area} = 18 \text{ cm}^2 + 12 \text{ cm}^2 + 6 \text{ cm}^2 + 18 \text{ cm}^2 + 6 \text{ cm}^2 + 12 \text{ cm}^2 = 72 \text{ cm}^2$$

$$\text{Surface area of the cuboid} = 72 \text{ cm}^2$$

This is the same as :

$$S = 2 \times (\text{Surface of Rectangle } R1) + 2 \times (\text{surface area of rectangle } R2) + 2 \times (\text{surface area of rectangle } R3).$$

2) Observe the following nets. Use sheets of paper to make them and verify if they are nets of a cuboid. Try to make others.

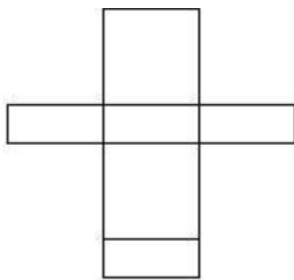


Figure 1

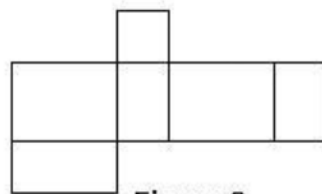


Figure 2

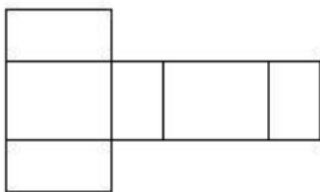


Figure 3

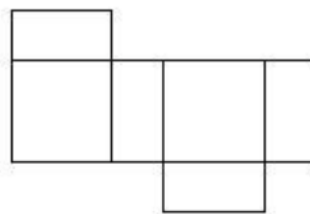


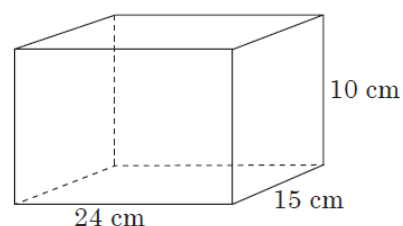
Figure 4

Solution

They are nets of a same cuboid because in each one, two parallel faces are equal.

When making nets of a cuboid, make sure nets are made by 6 faces and 2 parallel faces are equal.

3) Find the total surface area of the box illustrated on the diagram.



Solution

Start with the formula:

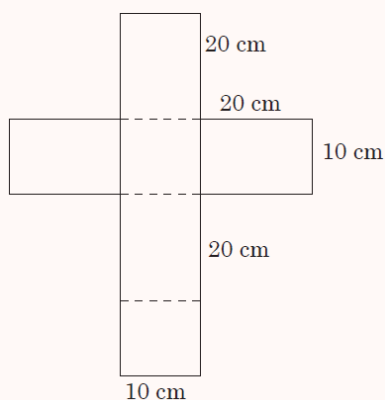
$S = 2 \times (\text{Surface of Rectangle R1}) + 2 \times (\text{surface area of rectangle R2}) + 2 \times (\text{surface area of rectangle R3}).$

$$= 2 \times (24\text{cm} \times 15\text{cm}) + 2 \times (15\text{cm} \times 10\text{cm}) + 2 \times (24\text{cm} \times 10\text{cm}) = 1500\text{ cm}^2$$

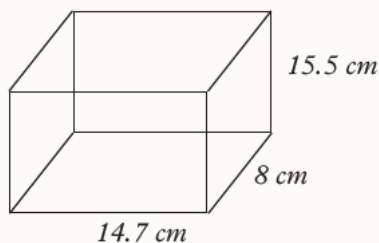


Application Activity 13.5

1) Find the surface area of the cuboid if its nets are the following:

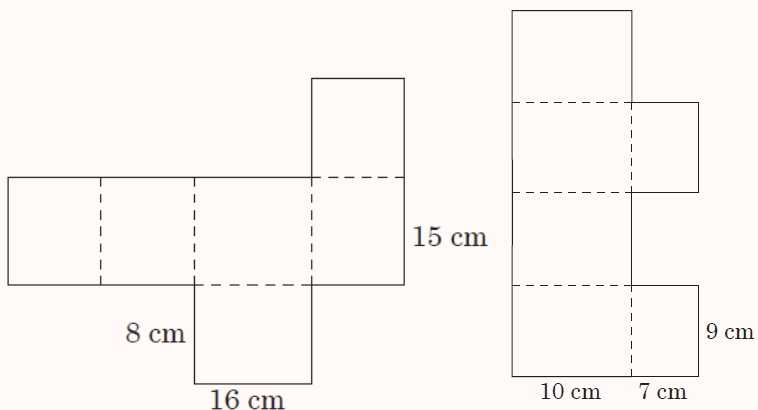


2) Jane is wrapping a gift box shaped like a cuboid of the following form:



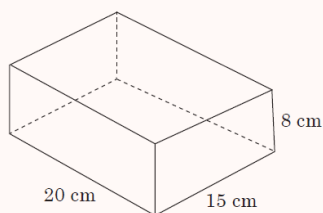
How much wrapping paper (in cm^2) does she need to cover the entire box?

3) Draw the following on manila paper. Cut the nets out. Fold them along the dotted lines.



4) A building brick is 20 cm long, 15 cm wide and 8 cm high. Calculate its total surface area

Discuss your steps.

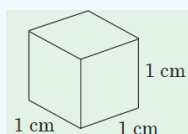


13.6 Calculating the volume of a cube and a cuboid

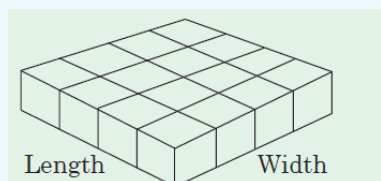


Activity 13.6

Make several cubes like one shown below. This is a unit cube.

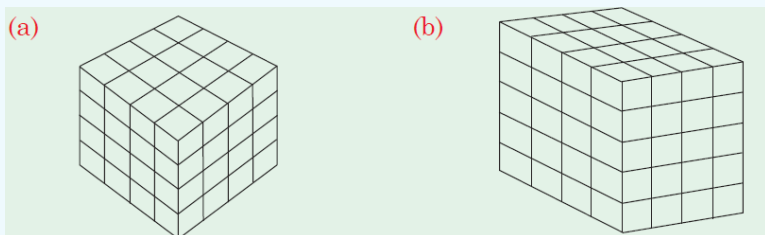


Make a layer like the one below using unit cubes



- | | |
|--|--|
| (i) How many cubes are there along the length? | (iii) How many cubes are there along the height? |
| (ii) How many cubes are there along the width? | (iv) Count the number of cubes in the layer. |
| | (v) Calculate the number of cubes in the layer. |

By adding similar layers on top, make the following:



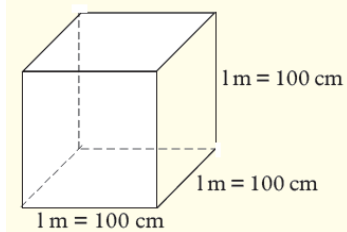
- (i) How many cubes are along the length?
(ii) How many cubes are along the width?
(iii) How many layers are in the stack? Explain.
(iv) How many cubes form each stack? This is the volume of the stack. Discuss your results.
(v) Now compare your results in (iv) with the following product:
Cubes along length x cubes along width x cubes along height. This is the volume.



Summary

Volume is the amount of space occupied by an object. A unit cube is used as the basic unit of volume. So volume is expressed in terms of cubic units.

Volume of cuboid = $L \times W \times H$



For a cuboid

Volume of a cuboid = Base surface area x Height

$$= \text{Length} \times \text{breadth} \times \text{Height} = L \times W \times H$$

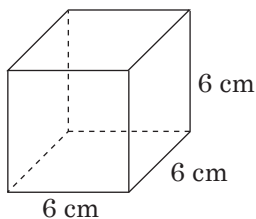
For a cube

Volume of a cube = Base surface area x Height

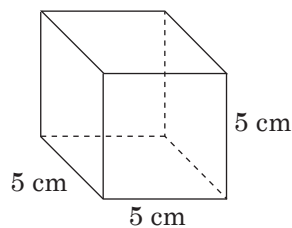
$$= \text{Side} \times \text{side} \times \text{side} = L \times L \times L = L^3$$

Examples

1) Find the volume of the cube illustrated on the diagram below:



(a)



(b)

Solution

a) $L = 6\text{cm}$, $W = 6\text{cm}$, $H = 6\text{cm}$

As the length is the same for all edges, The diagram represents a cube.

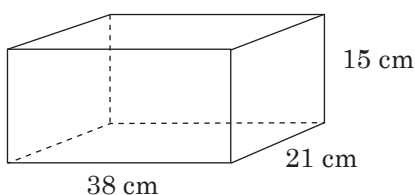
The volume = Length $\times$ breadth $\times$ Height = $L \times W \times H = 6\text{cm} \times 6\text{cm} \times 6\text{cm} = 216 \text{ cm}^3$

b) $L = 5\text{cm}$, $W = 5\text{cm}$, $H = 5\text{cm}$

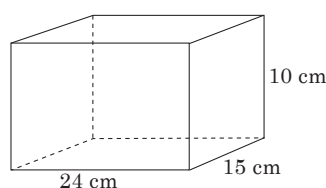
As the length is the same for all edges, The diagram represents a cube.

The volume = Length $\times$ breadth $\times$ Height = $L \times W \times H = 5\text{cm} \times 5\text{cm} \times 5\text{cm} = 125 \text{ cm}^3$

2) Find the volume of the cuboids illustrated on the diagram below:



(a)



(b)

Solution

a) $L = 38\text{cm}$, $W = 21\text{ cm}$, $H = 15\text{ cm}$

As the edges are not equal, we have the cuboids.

The volume = Length $\times$ breadth $\times$ Height = $L \times W \times H = 38\text{cm} \times 21\text{cm} \times 15\text{cm} = 1,1970\text{ cm}^3$

b) $L = 24\text{cm}$, $W = 15\text{ cm}$, $H = 10\text{ cm}$

As the edges are not equal, we have the cuboids.

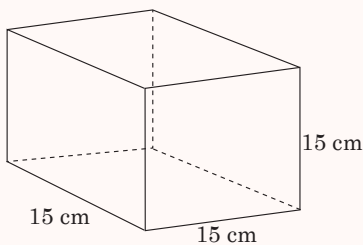
The volume = Base surface area $\times H = (L \times W) \times H$

$$= (24\text{cm} \times 15)\text{cm} \times 10\text{cm} = 360\text{ cm}^2 \times 10\text{cm} = 3600\text{ cm}^3$$

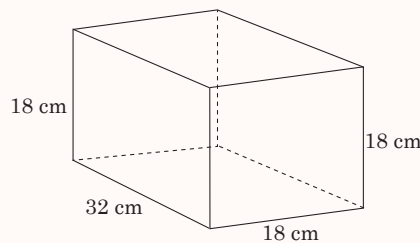


Application Activity 13.6

1) Calculate the volume of each of the following:

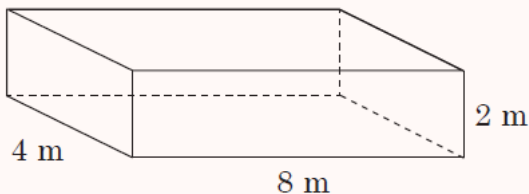


(a)



(b)

- 2) A rectangular tank measures 4.3 m long, 2.4 m wide and 1.5 m high. Calculate the volume of the tank.
- 3) A large carton measures 64 cm long, 32 cm wide and 30 cm high. What is the volume of the carton? Where do we use a carton? Discuss.
- 4) The figure below represents a swimming pool. How much water in cubic meter are in it when it is half full? Explain how you found your answer. Tell importances of a swimming pool.



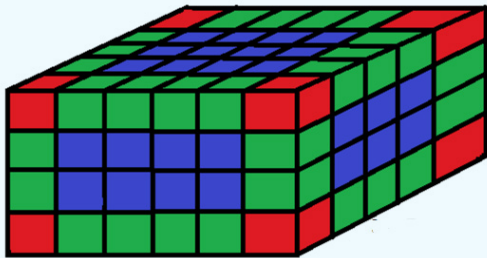
- 5) What is the volume of a cube whose sides are 20 cm. Explain how you arrive at the answer.
- 6) A rectangular water tank (cuboid) measures 4 m long, 3 m wide and 2 m high. What is its volume?
- 7) A building brick is 20 cm long, 15 cm wide and 8 cm high. What is its volume? Discuss your steps.

13.7 Finding one dimension of a cuboid



Activity 13.7

Consider the pile of 120 small cubes below, each one has 1 cubic centimeter.



- Complete one missing dimension
- If the total number of small cubes is 48, complete the length.
- If the total number of small cubes is 42, complete the height.

	Total number of cubes (Volume)	Number of cubes along length (L)	Number of cubes along width (W)	Number of cubes along height (H)
a)	120	6	5	
b)	48		4	2
c)	42	7	2	

Basing on the observation made from the activity,



Summary

Given that the volume V of a cuboid = Base surface area $\times$ Height = $L \times W \times H$,

We can find one dimension if 3 are given.

$$1) V = L \times W \times H$$

$$2) L = \frac{V}{W \times H}$$

$$3) W = \frac{V}{L \times H}$$

$$4) H = \frac{V}{L \times W}$$

Example:

1) The volume of a cuboid is 420 cm^3 . It has a length of 10 cm and width 7 cm. What is its height?

Solution

$$V = 420 \text{ cm}^3,$$

$$L = 10 \text{ cm}$$

$$W = 7 \text{ cm}$$

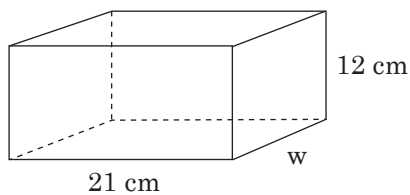
$$H = ?$$

$$\text{As } V = L \times W \times H,$$

$$H = \frac{V}{L \times W} = \frac{420 \text{ cm}^3}{10 \text{ cm} \times 7 \text{ cm}} = 6 \text{ cm}$$

The height is 6 cm.

2) Study the cuboid below. Its volume is 4536 cm^3



Find its width.

Solution

$$V = 4536 \text{ cm}^3,$$

$$L = 21 \text{ cm}$$

$$H = 12 \text{ cm}$$

$$W = ?$$

$$\text{As } V = L \times W \times H,$$

$$W = \frac{V}{L \times H} = \frac{4536 \text{ cm}^3}{21 \text{ cm} \times 12 \text{ cm}} = 18 \text{ cm}$$

The width is 18 cm.

3) A rectangular tank has a volume of 7 m^3 . It is 2 m wide and 1.4 m high.

Find the length of the tank

Solution

$$V = 7 \text{ m}^3,$$

$$H = 1.4\text{m}$$

$$W = 2\text{m}$$

$$L = ?$$

$$\text{As } V = L \times W \times H,$$

$$L = \frac{V}{W \times H} = \frac{7\text{m}^3}{1.4\text{m} \times 2\text{m}} = 2.5\text{m}$$

Find the length of the tank is 2.5m.



Application Activity 13.7

1) Copy and complete the table below.

	Length	Width	Height	Volume
1.	4 m	5 m	12 m	
2.	12 m	6 m		288 m ³
3.	36 cm		50 cm	25 200 cm ³
4.		50 cm	50 cm	125 000 cm ³

2) A carton has a volume of 142 560 cm³. Its width is 36 cm and its height is 72cm. Calculate its length.

3) The volume of a rectangular water tank is 414 720 cm³. Its length is 144 cm while its width is 36 cm. Calculate its height. Explain how you arrive at your answer.

4) A log of wood is in the shape of a cuboid. It is 35 cm long and 30 cm high. Its volume is 25 200 cm³. How wide is the log?

5) A container is 40 cm wide and 18 cm high. Its volume is 21 600 cm³. What is its length?

13.8 Find the height of a cuboid given its volume and base area

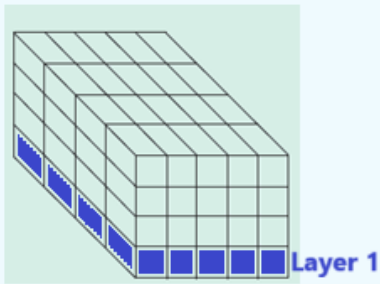


Activity 13.8

Play the game of finding the number of cubes that make a cuboid.

Player 1 makes a cuboid and states how many cubes are in it. Player 2 states how many unit cubes are in one layer. Player 3 states how many layers make the height.

Follow the following example:



Player 1: There are 80 cubes in the cuboid.

Player 2: There are 20 cubes in the layer 1

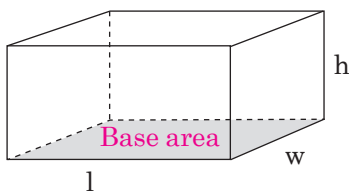
Player 3: *There are 4 cubes along the height.*

Question:

- How do you find the answer to be provided by player 3 when player 1 and player 2 gave their numbers?
- Can you find the height of a cuboid when its volume V and the base surface area $S = (L \times W)$ are given?

Summary

When the volume V and the base surface area $S = (L \times W)$ of a cuboid are given, It is possible to calculate its height H .



$$H = \frac{V}{L \times W} = \frac{V}{S} \text{ where } S \text{ is the base surface area.}$$

Example:

The volume of the cuboid is $72\,000 \text{ cm}^3$. Its base area is $2\,400 \text{ cm}^2$. What is its height?

Solution

$$V = 72\,000 \text{ cm}^3$$

$$S = 2\,400 \text{ cm}^2$$

$$H = ?$$

$$\text{As } V = L \times W \times H = S \times H, \quad H = \frac{V}{L \times W} = \frac{V}{S}$$

$$\text{Therefore, } H = \frac{V}{S} = \frac{72000 \text{ cm}^3}{2400 \text{ cm}^2} = 30 \text{ cm}$$

The height is 30cm.



Application Activity 13.8

1) Copy and complete the table below.

Volume	Base area	Height
$3\,150\text{ cm}^3$	315 cm^2	
576 cm^3	144 cm^2	
250 m^3	50 m^2	

2) The volume of a cuboid is $18\,000\text{ cm}^3$. Its base is a square of side 30cm. What is

(a) its base area? Explain how you got answer.

(b) its height?

2) The base of a tank is a rectangle whose length is 90 cm and width 50 cm. Its volume is 10 000 cubic centimeters. Calculate its height.

3) The volume of a rectangular water tank is 8 000 cubic centimeters. It has a base area of 160 square centimeters. What is its height? Explain your answer.

4) The floor of a classroom measures 8m long and 7 m wide. The volume made by walls of the classroom is 168 cubic meters. What is the height of the classroom? Compare your answer with the height of your classroom. Discuss your results.

13.9 Finding the area of a face of cuboids

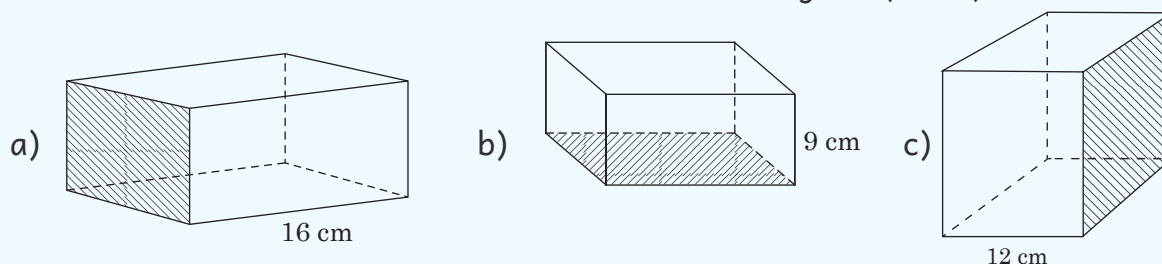


Activity 13.9

Study the following cuboids. Discuss and find the area of their shaded faces. The volume of each cuboid is $1\,728\text{ cm}^3$.

The shaded face becomes the base and the given length is the height.

Remember that the Volume = Base surface area $\times$ Height = $(L \times W) \times H$.



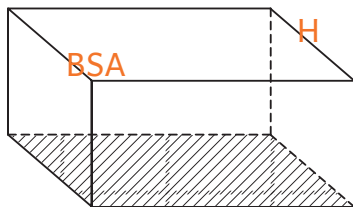
Present your findings and how you got your answer.

Summary

Given that the volume of a cuboid = Base surface area $\times$ Height,

$$V = BSA \times H,$$

$$\text{So, } BSA = V/H$$

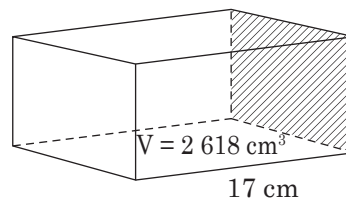


$$\text{Base surface area} = \frac{V}{H}$$

Note that, every face of a cuboid can be taken as a base and the perpendicular edge becomes the height H .

Example:

Calculate the area of the shaded face of the cuboid below. Its volume is 2 618 cubic centimeters.



Solution

Volume of the cuboid = $2\,618\text{ cm}^3$, $H = 17\text{ cm}$.

Requested: Area of the shaded face = ?

$$\text{Calculation: Area of the shaded face} = \frac{V}{H} = \frac{2\,618\text{ cm}^3}{17\text{ cm}} = 154\text{ cm}^2$$

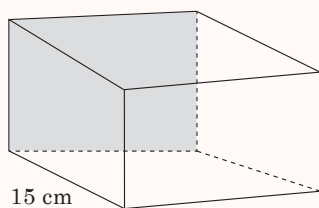
Area of the shaded face is 154 cm^2



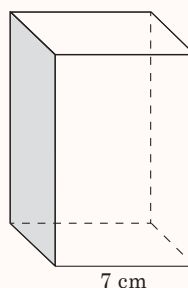
Application Activity 13.9

1) Find the area of the shaded faces of the given cuboids. The volume (V) and one dimension have been given in each case.

(a) $V = 3\,120\text{ cm}^3$



(b) $V = 1\,428\text{ cm}^3$

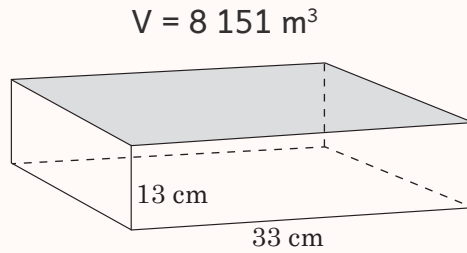


2) Study the figures below:

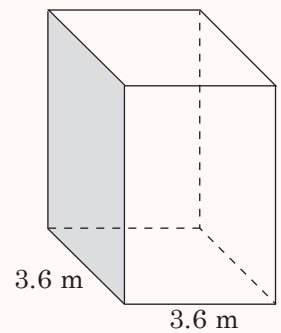
(i) Calculate the area of the shaded part.

(ii) Discuss and find the missing dimension of each figure.

(iii) Explain how you calculate the missing dimension.



$$V = 41.472 \text{ m}^3$$



3) A carpenter made a rectangular wooden box with a volume of 1.44 m^3 . Its length was 1.5 m and width 0.8 m.

(a) What is the area of its base?

(b) Find its height.

(c) How did you get its height? Explain

13.10 Project: Building real-life objects using nets of solids



1. Rationale: This project will help you connect abstract geometry to tangible objects, fostering creativity, teamwork, and critical thinking. By constructing real-life items from nets, you gain a deeper appreciation for mathematics in everyday life.

2. Location and Duration: To be done in the Mathematics corner or outside the class during the periods of co-curriculum activities. It will last 1 week.

3. Project Overview:

In this project, you will explore the concept of **nets of 3D solids** (such as cubes, cuboids, pyramids, and cylinders) and apply your understanding to construct real-life objects. You will design, create, and present models of everyday items using nets, enhancing your spatial reasoning and creativity.

4. Learning Objectives:

By the end of this project, you will be able to:

Identify and describe the nets of common 3D solids.

Construct 3D shapes from their nets.

Relate geometric concepts to real-world objects.

Work collaboratively to design and build a model.

Present and explain your work confidently.

5. Materials Needed: Cardstock or thick paper, Scissors, Rulers, Glue/Tape, Colored pencils/markers, Real-life examples (e.g., cereal boxes, tissue boxes, party hats)

6. Project Phases:

Phase 1: Introduction to Nets (It is done in the ordinary class)

Activity: Teacher introduced nets using real-life examples (e.g., opening a cereal box to show its 2D net).

Group Discussion: You identified objects around you that can be unfolded into nets (e.g., dice, cans, pyramids).

Hands-on Exploration: You practiced drawing and cutting out simple nets (cube, cuboid, pyramid).

Phase 2: Research & Planning

You choose **one real-life object** to model (e.g., a house, a gift box, a tent, a soda can).

You sketch the net needed to construct your chosen object.

Teacher checks designs for accuracy before construction begins.

Phase 3: Construction

You create your 3D models using nets.

You decorate your models to resemble real-life objects (e.g., drawing windows on a house, labeling a tissue box).

Teacher provides guidance on folding and assembling.

Phase 4: Presentation & Reflection (1 Lesson)

You will present your models to the class, explaining the shape you chose, How the net forms the 3D object, Where this object is used in real life.

Class Reflection: The author will explain challenges faced and lessons learned.



End unit assessment

Assessing knowledge

Circle the correct answer

1. The formula for the circumference of a circle is:

- a) $C = \pi \times \text{radius}$
- b) $C = \pi \times \text{diameter}$
- c) $C = 2 \times \text{radius}$

2. A cube has:

- a) 6 faces, 12 edges, 8 vertices
- b) 4 faces, 8 edges, 6 vertices
- c) 6 faces, 8 edges, 12 vertices

3. If L is the length, W the width and H the height of a cuboid, its volume V is calculated using the formula:

a) $V=L+W+H$

b) $V=L \times W \times H$

c) $V=L \times W$

Assessing skills

4. What is the circumference of a wheel whose diameter is 28 cm? Take $\pi = \frac{22}{7}$

5. A cube has a side length of 4 cm. Find its:

a) Surface area

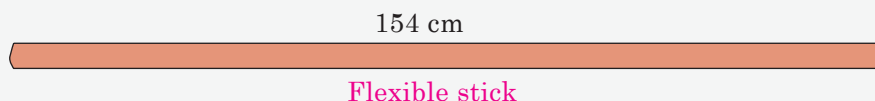
b) Volume

6. A cuboid has a length of 8 m, width of 5 m, and height of 3 m. Calculate:

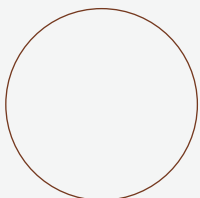
a) Total surface area

b) Volume

7. Find a flexible stick that is 154 cm long.



Fold the stick into a circle to form a wheel. Use tape to attach the ends.

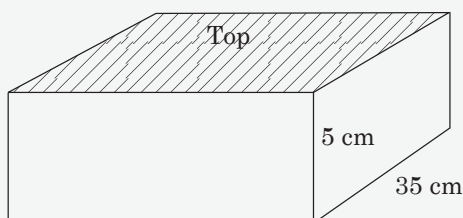


(a) What is the circumference of the wheel made from the flexible stick?

(b) What is the diameter of the wheel. (Use a ruler to measure and calculate to verify).

8. To make a roundabout, the contractor needed a circumference of 88 m. What diameter does he use to get the circumference? Explain your answer.

9. The diagram below represents a box made up of cardboards:

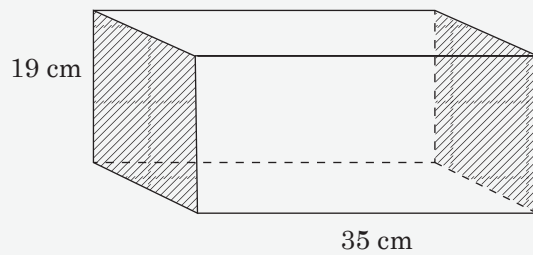


Its volume is 4 530 cubic centimeters. What is the area of its top? Explain how you get the answer .

10. A rectangular container had two faces made from metal sheets. The shaded part represents the metal sheets. Its volume is 19 285 cubic centimeters.

a) calculate the width of the container

b) Calculate the area of the metal sheet used to make the shaded parts. Explain your answer.

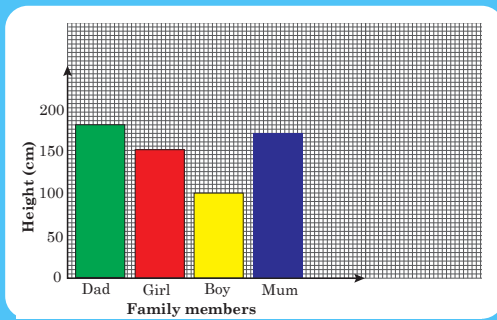


Assessing attitudes and values

11. Why is it important to learn about circumference, surface area, and volume in real life? Give two examples.

UNIT 14

DATA HANDLING



Key Unit Competence: You will be able to collect data, represent and interpret it to answer a question to solve a problem.

Learning activities:

By the end of this unit, you should be able to:

Knowledge and understanding

Define continuous and discrete data.

Explain how to construct bar charts.

Explain line graphs.

Skills:

Distinguish continuous from discrete data.

Collect data and record it in a table.

Represent data in a bar chart.

Represent data on a line graph

Interpret data represented on a bar chart or line graph to answer to a question.

Attitudes and values:

Appreciate that data can be represented on a bar graph or a line graph.

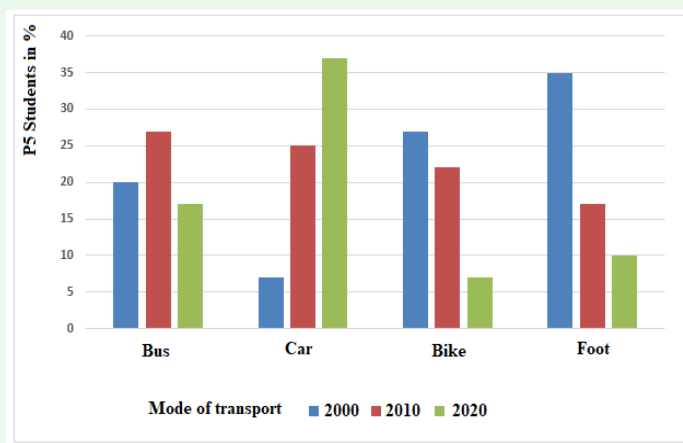
14.0 Introduction

Data are helpful in knowing how things are taking place. They are also helpful in good planning. With records, one can remember what took place in the past. People can know things that happened due to stored data or information.



Introductory Activity

For example, the P5 students at a certain school were surveyed to find out how they usually get to school and return home in three different years 2000, 2010 and 2020. The diagram provided illustrates the results of the survey.



Refer to the diagram and answer to the following questions:

- Is the diagram very helpful to quickly recognize different mode of transport for P5 students in different 3 years 2000, 2010 and 2020?
- Which mode of transport is used by a big number of P5 students in 2020? Which means of transport is used by a small number of P5 students in 2020?
- Do you think that representing data on a diagram like the one given here above can help you to interpret data? Can you explain how to do it?
- Do you think collecting and presenting data is important to you? Give 2 reasons.

14.1 Definition and examples of quantitative data



Activity 14.1

- In the class of P5, members of a group of 10 pupils were asked to tell the number of siblings in their family.

The table below provide the number of siblings by:

Pupil	Eric	Moses	Gakire	Emma	Dora	Ada	Alice	Muvunyi	Kamana	Amanda
Number of siblings	1	2	3	2	3	4	2	1	2	4

Can you count the number of siblings for each pupil? Who has more siblings in their family?

- The teacher asked 10 pupils to go to measure the height of 10 different plants outside the classroom and record them. The following table shows the data in centimeters they brought:

Plant number	Plant 1	Plant 2	Plant 3	Plant 4	Plant 5	Plant 6	Plant 7	Plant 8	Plant 9	Plant 10
Height	1.20	2.25	1.28	1.29	3.01	1.21	1.22	2.27	2.26	1.30

Can you express the height using whole numbers? If they measure many different plants, is it easy to write the height for each plant observed?

- Compare the data used in (1) and data used in (2) are they the same?



Summary

Quantitative data is numerical information that can be measured or counted. It uses numbers and answers questions like “how many?” or “how much?” and can be analyzed using statistics.

Examples

The number of students in a class (e.g., 25 pupils).

The height of a plant (e.g., 15 cm).

The temperature outside (e.g., 30°C).

Qualitative data describes qualities, characteristics, or categories that **cannot be measured using numbers**. It answers questions like “what kind?” or “how does it feel?” and is often based on observations, opinions, or labels.

Examples of qualitative Data

Colors (e.g., The flower is *red*.)

Feelings/Opinions (e.g., The soup tastes *spicy*).

Types/Categories (e.g., The animal is a *dog*).

Text responses (e.g., My favorite hobby is *drawing*).



Application Activity 14.1

1) Dice Rolling Frequency Chart

Work in group and roll a dice 20 times and record each outcome.

Are the outcomes expressed by numbers, do they make quantitative data?

2) Circle **all** the *quantitative* data in the list below.

A cat has *4 legs*.

The soup is *salty*.

The tree is *tall and green*.

The race lasted *10 minutes*.

3) For the following scenario, identify **one** piece of *quantitative* and **one** piece of *qualitative* data.

A scientist studies a plant: It grew 15 cm tall, and its leaves feel fuzzy.

Quantitative: _____

Qualitative: _____

14.2 Type of quantitative data

14.2.1 Definition and examples of Discrete data



Activity 14.2.1

1) **A teacher asked the question to pupils:** How many notebooks do you have?

In groups, make a chart below.

Pupil	Eric	Moses	Gakire	Emma	Dora	Ada	Alice	Muvunyi	Kamana	Amanda
Number of notebooks	10	12	13	12	13	14	12	11	12	14

Observe the values recorded. Do they take various values? Do your values include decimals? Explain your answers.

2) Now, follow the same steps and form questions to collect data on:

Shoe sizes worn by adults.

Shoe sizes worn by children.



Summary

Discrete data is a type of **quantitative data** that consists of:

Whole numbers (no decimals or fractions).

Countable, separate items with clear gaps between values.

Examples of Discrete Data

Number of pupils in a class (e.g., 25 pupils).

Cars in a parking lot (e.g., 10 cars — not 10.5!).

Goals scored in a match (e.g., 4 goals).

Books on a shelf (e.g., 12 books).

Pets in a house (e.g., 2 dogs).

Example:

There are cards with examples: 7 apples, 2.5 kg rice, and 8 children.

Which ones have discrete data?

Answer: Discrete data: “7 apples,” “8 children”

14.2.2 Definition and examples of continuous data



Activity 14.2.2

The following data was collected by a group of pupils.

Pupil	Jean Paul	Anitha	Bernard	Fike
Height	1.40 m	1.38 m	1.43 m	1.42 m

- (i) What type of data was collected?
- (ii) Basing on the definition of discrete data, is the data discrete or not? Explain your answer.



Summary

Continuous data is a type of quantitative data that:

Can take any value within a range (including decimals and fractions).

Is measurable with all possible values between units.

Often involves tools like rulers, scales, or thermometers.

Examples of Continuous Data

Height of a tree (e.g., 2.75 meters).

Weight of a watermelon (e.g., 3.6 kg).

Time taken to run a race (e.g., 12.3 seconds).

Temperature outside (e.g., 23.7°C).

Volume of juice (e.g., 0.5 liters).



Application Activity 14.2.2

- 1) A class collected numerical data on the following.
 - (a) Shoe sizes worn by pupils in school.
 - (b) Distance from home to school for group members.
 - (c) Number of parents for different pupils in a class.

Draw a table and group the data accordingly. Discuss your answers

Discrete	Continuous
-	-
-	-

2) Write “**Discrete**” for discrete or “**Continuous**” for continuous next to each example.

- a) Number of legs on a chair (e.g., 4 legs): _____
- b) Time taken to eat lunch (e.g., 12.5 minutes): _____
- c) Pages in a book (e.g., 120 pages): _____
- d) Length of a jump rope (e.g., 3.2 meters): _____

3) Here are 4 cards with data. Sort these real-world examples into Discrete data and continuous data.

A: 5 puppies

B: 2.3 kg of flour

C: 10 traffic cones

D: 1.5 hours of sleep

4) Answer by true or false.

- a) “The baby’s weight (3.7 kg) is discrete data.”
- b) “The number of eggs in a carton (12) is continuous data.”

14.3 Record data using tally chart



Activity 14.3

A tally chart is a simple way of recording data and counting the amount of something (frequency). To do this we collect the data, sort it into categories and tally the marks, to find the frequencies.

During the election of a class monitor, the class of 40 pupils used the tally table to count votes. Observe the table in which data are completed with tally and complete the missing data:

Candidate	Tally	Number of votes (Frequency)
Musoni		2
Erasme		8
Anitha		11
Kabera		10
Moses		9
Total		40

- a) Who is the winner of the vote?
- b) How do you complete the tally?
- c) How do you count the number?



Summary

To complete a tally chart,

Write all possible answers (categories) in the first column.

Record Data with Tally Marks

For each response, add a tally mark (|) next to its category.

Group tallies in sets of 5: 4 lines + a diagonal fifth (||||).

Count the Tallies: Add up the marks for each category and write the total number in a "Frequency" column.

Check for Accuracy: Ensure the sum of all frequencies matches the total of responses collected

The advantage of using a tally chart is that it can be immediately clear to identify which item(s) occurs the most/least frequently and can be helpful when analysing data.

Example

Let's say we wanted to collect some data on the ways of transport used by students to get to school. We would start collecting data and writing the answers in a list.

The best way to sort this data set is to use tally charts.

The data collection looks like this:

walk, bus, bike, walk, bike, bus, walk, car, walk, bike, bike, bus, walk, walk, walk, car, bus, walk, bus, bus, walk, car, car, walk, walk, train, bike, bus, walk, walk.

Transport	Tally	Frequency
Walk		13
Bus		7
Car		4
Motorbike		5
Bicycle		1
Total		30

From the table, we see that more students (13) arrive to school through walk.

Only 1 pupil uses a bicycle to arrive at school.

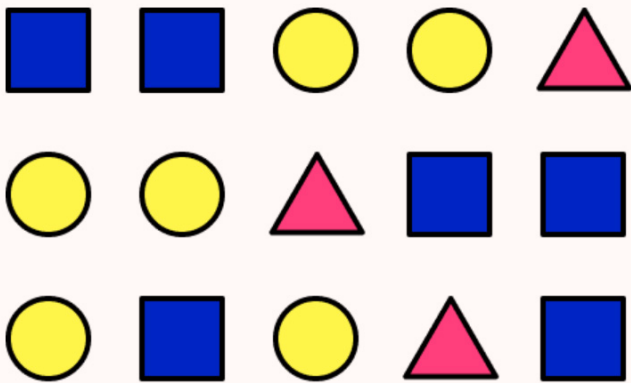


Application Activity 14.3

1) Complete the tally chart that shows the favourite subjects of 40 students:

Subject	Tally	Frequency
Mathematics		11
English		9
SET		7
Kinyarwanda		
Total		40

2) Pupils collected shapes from a mathematics corner. The shapes collected are presented here below.



- Draw a tally chart to show the frequency of these shapes. It includes squares, circles and triangles.
 - Which shape is more presented? In how many times?
 - Which shape is less presented? In how many times?
- 3) Twenty playing cards were picked at random from a deck. By grouping them into their suit, draw a tally chart of the results:



a) Complete the tally chart of the data

Suit	Tally	Frequency
♥ (Heart)		
♣ (Clubs)		
♦ (Diamonds)		
♠ (Spades)		

b) Which type of cards was picked by more pupils? How many?

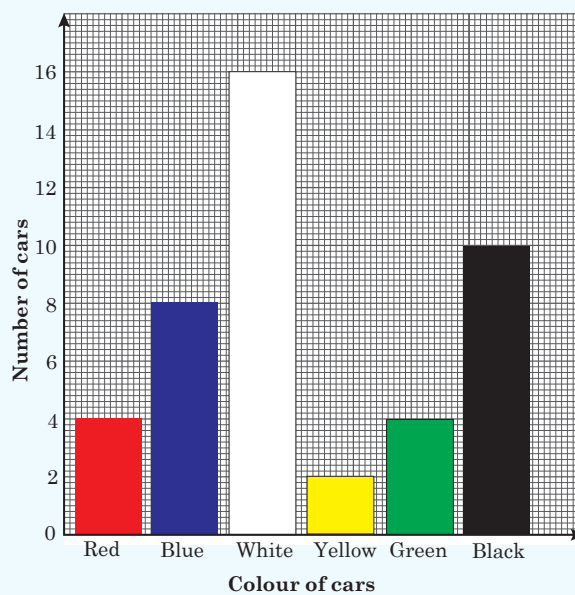
c) Which type of cards was picked by few pupils. How many?

14.4 Representing data using bar charts



Activity 14.4

Observe the following diagram; It is a bar chart representing data about the number of cars in a car park.



The table below contains data read on the chart. Complete the missing numbers.

Red	Blue	White	Yellow	Green	Black
4		16	2		

Summary

A **bar chart (or bar graph)** is a visual tool that uses rectangular bars to represent and compare data.

The length/height of each bar corresponds to the quantity of the data it represents.

Components of a bar chart:

- a) X-axis (Horizontal): Shows the categories (e.g., color of the cars, types of fruit, months).
- b) Y-axis (Vertical): Shows the values/numbers (e.g., number of cars, students, etc).
- c) Bars: Colored rectangles whose height/length matches the number for each.

To read the bars: The **taller/longer the bar, the larger the quantity**.

Example: If the “white cars” bar reaches **16 on the Y-axis**, there are 16 white cars.

To compare data: Ask: *Which bar is tallest? Shortest?*

Example, white cars are more (16) because their bar is the tallest.

Examples

The table below contains data for shoe sizes for different people.

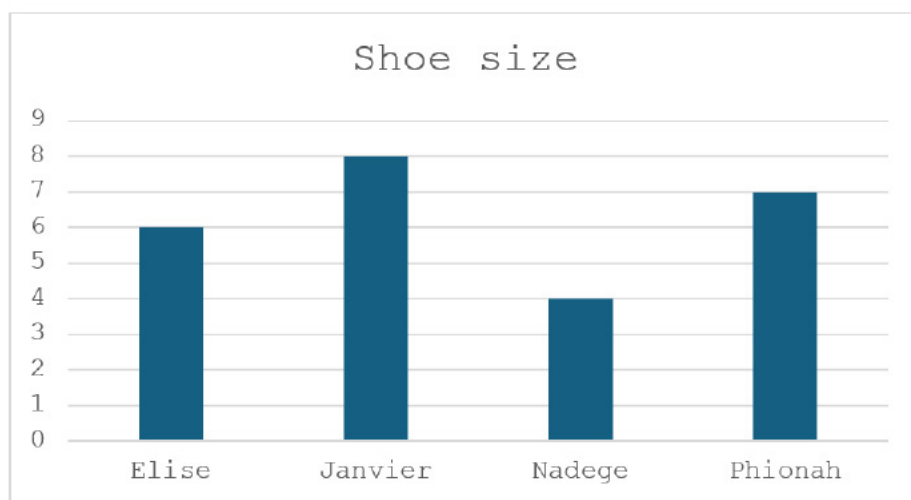
Person	Elise	Janvier	Nadege	Phionah
Shoe size	6	8	4	7

Represent the data using a bar chart.

Solution

On Horizontal axis there are names of people

On the vertical axis there are the numbers showing the shoe size.





Application Activity 14.4

1) The table below contains data for heights of family members.

Family member	Father	Mother	1st child	2nd child	3rd child
Height	1.9 m	1.7 m	1.5 m	1.4 m	1.2 m

Represent the data using a bar chart.

2) The table below contains data about the number of books each pupil has.

Name	Olivier	Thierry	Elie	Christine	Carene
Number of books	10	12	8	15	13

Represent the data using bar chart.

3) Itetero Primary school conducted a survey to know the favourite sports of the students. The table below shows the results of this survey

Name of the sport	Total number of students
Football	14
Basketball	12
Valley ball	10
Table Tennis	5
Hand ball	6
Total	47

From this data,

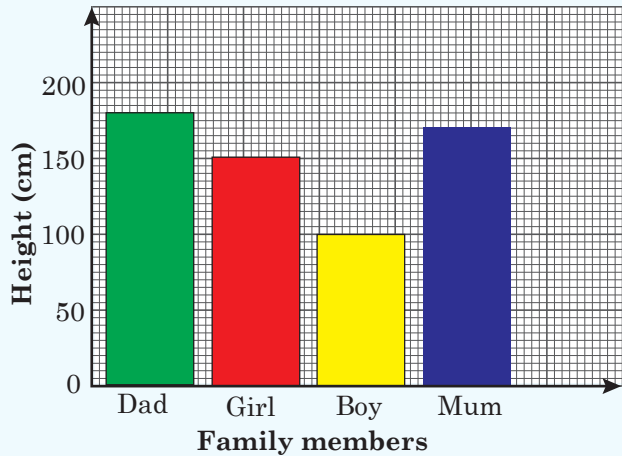
1. Draw a bar graph representing the sports and the total number of students.
2. Which sport is the most preferred one?
3. Which two sports are almost equally preferred?

14.5 Interpreting bar charts to draw a conclusion



Activity 14.5

Study the bar chart below



What is the bar chart about? Explain.

Read the height of:

- (i) Dad (ii) Mum (iii) Boy (iv) Girl



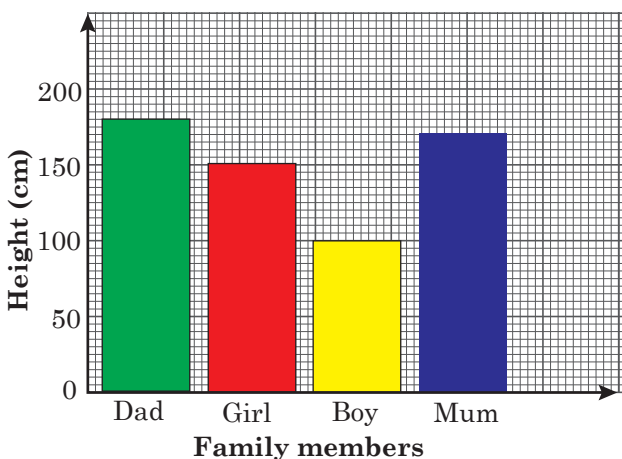
Summary

In interpreting bar charts,

- (i) State the information represented by each bar. Check the horizontal axis
- (ii) Read the lengths of bars and the information they represent. Check the vertical axis.

Examples:

Observe the data on the bar chart and answer the following questions.



- (a) What information is shown by the bar chart?
- (b) Who is the tallest?
- (c) Who is 150 cm tall?
- (d) How is bar chart important to you?

Solution

- (a) On the vertical axis, we have height (in cm). On the horizontal axis, we have family members. The information shown is the heights of family members.
- (b) Dad is the tallest. His height is 180 cm.
- (c) Starting from vertical axis, at 150 cm put a ruler. Draw a dotted line to see which bar is at 150 cm. The bar representing the Girl is at 150 cm. The girl is 150 cm tall.
- d) Bar chart are important to me because:

They make numbers fun: Instead of just looking at boring numbers in a table I can see the sizes with colorful bars!

Easy to compare: I can tell in **one second** who has the biggest shoes, or the smallest. No counting needed!

Great for projects: When I survey my friends about their favorite snacks or sports, I can draw a bar chart to show the results. My teacher says it's "very scientific"!

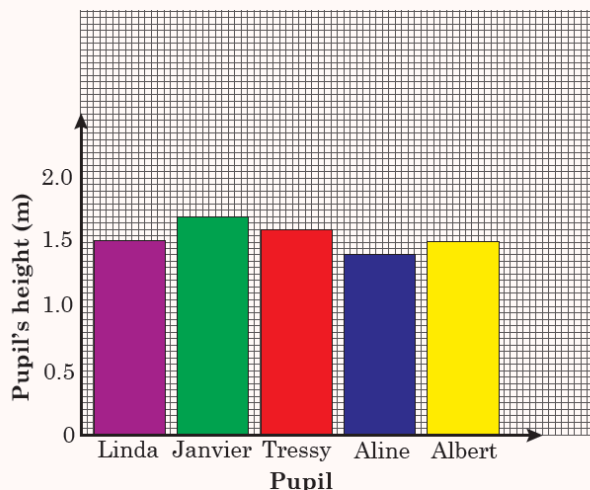
Helps me understand real life: Weather reports use bar charts for rainfall. Even game scores use them.

Mistakes: If I label the axes and color carefully, my chart never lies. It is like a **math superhero**. It is always telling the truth!



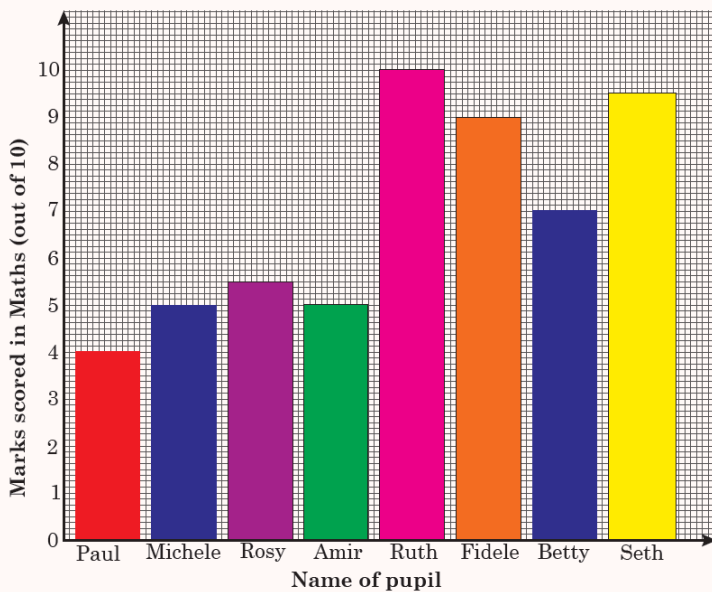
Application Activity 14.5

1. Look at the bar graph below



- (a) How many pupils measured their heights?
- (b) What information is shown on the graph?
- (c) Who is the shortest?
- (d) How many meters tall is the tallest pupil?
- (e) Which pupils were the same height?

2. Study the bar chart below.



- (a) What is the bar chart about?
- (b) How many pupils did the mathematics activity?
- (c) What is the highest score?
- (d) Who scored the lowest mark?
- (e) Which pupils scored the same mark?
- (f) Who scored the highest mark?
- (g) How many more marks did Ruth score than Rosy?
- (h) What score did Seth get?

14.6 Representing data using line graphs

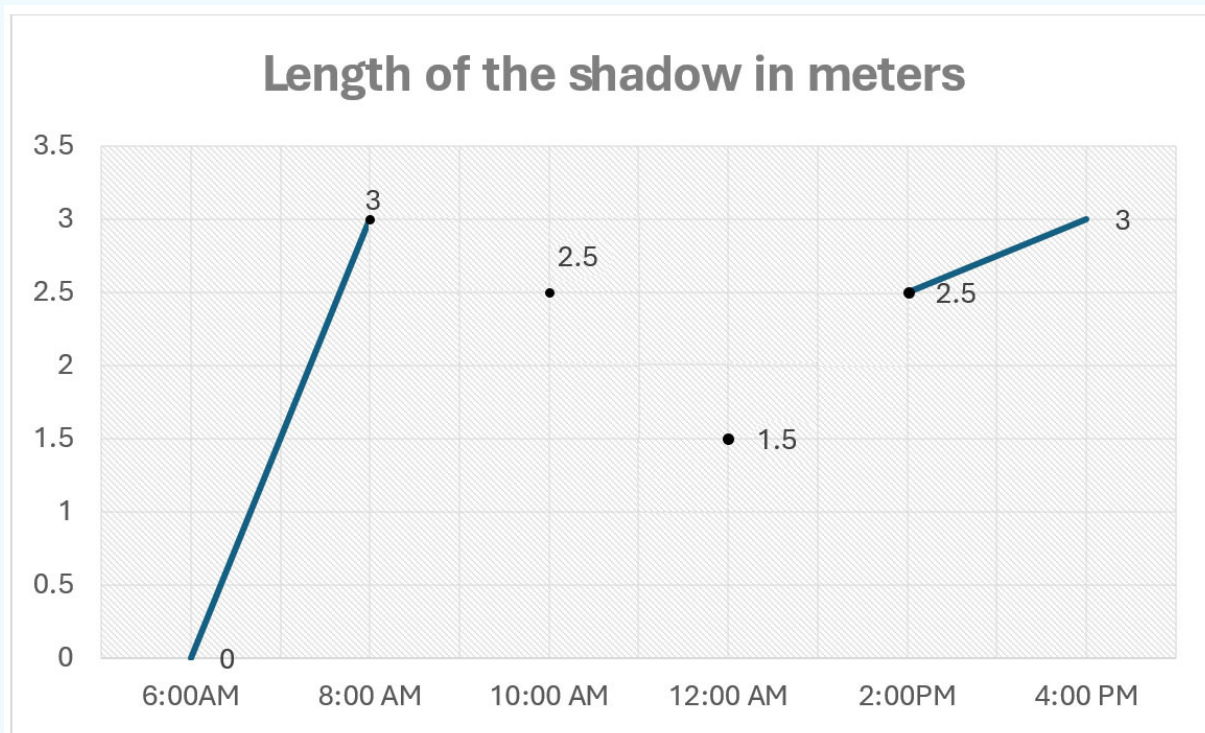


Activity 14.6

Use metre rule or tape measure to measure the length of a tree shadow at the following times.

Time	6:00	8:00 AM	10:00 AM	12:00 AM	2:00PM	4:00 pm
Length of the shadow	0	3m	2.5m	1.5m	2.5m	3m

In the diagram below, some points are identified. Join them. What type of graph do you get?



Collect your own data, represent them using a line graph.

Where did you find line graphs that represent data in your life experience?



Summary

To draw a line graph;

Make sure you have a set of data (set of values or categories and set of affected values)

(i) Choose a suitable scale for all values.

(ii) Draw axes and label them:

Write categories on Horizontal axis

Decide on equal intervals for the Y-axis

Write the values on Vertical axis.

(iii) Plot points on the graph. Each point is described by a pair of values. Point P(category, affected value).

(iv) Join the points using straight line: draw straight lines between the points in order.

(v) **Add a Title:** Write a clear title at the top.

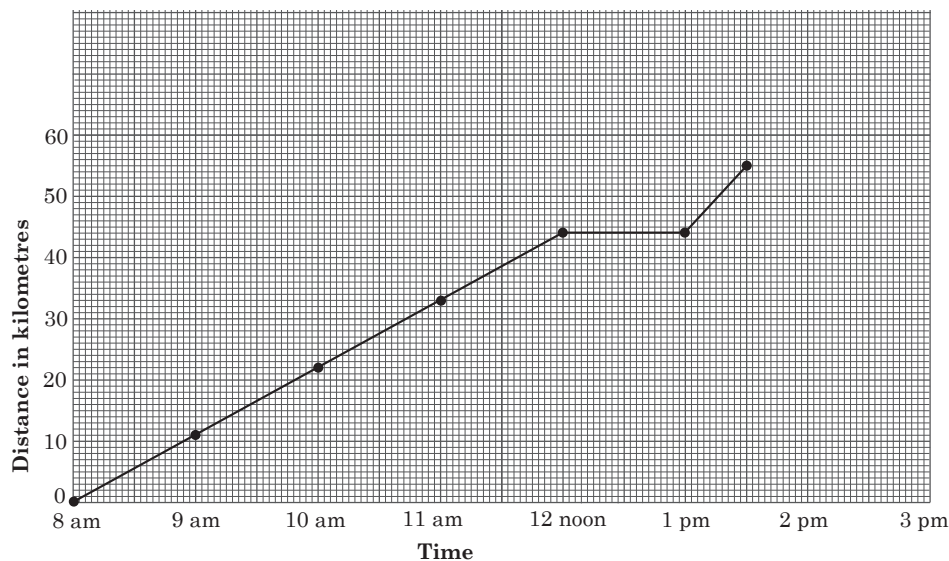
Example:

Below is data for the distance covered by a cyclist at different times

Time	8:00 a.m	9:00 a.m	10:00 a.m	11:00 a.m	12:00 p.m	1:00 p.m	2:00 p.m
Distance (km)	0	11 km	22 km	33 km	44 km	44 km	55 km

Represent the data using a line graph.

Solution



Application Activity 14.6

1) Study the data in the table below. It shows the distance travelled by a motorist

Time	10:00 a.m	11:00 a.m	12:00 noon	1:00 p.m	2:00 p.m	3:00 p.m	4:00 p.m
Distance covered (km)	0	50	100	150	150	250	350

Represent the data using a line graph.

2) The data below is the amount of water used by a family after every two hours

Time	7:00 a.m	9:00 a.m	11:00 a.m	1:00 p.m	3:00 p.m	5:00 p.m
Water used (litres)	0	30	60	60	90	120

(a) Represent the data using a line graph.

(b) Why was the amount of water the same from 11:00 a.m to 1:00 p.m? Explain your answer.

3) A farmer planted a crop. She measured its height after every two months. She recorded the data below

Time	0	2 months	4 months	6 months	8 months	10 months
Height (cm)	0	50	100	150	180	200

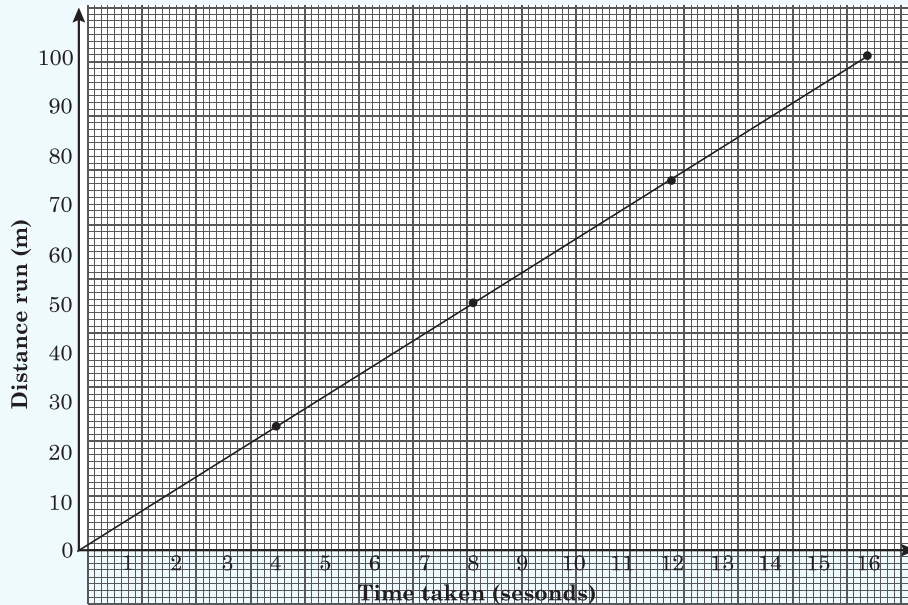
Represent the data using a line graph. Discuss your steps to draw an accurate line.

14.7 Interpreting line graphs to draw a conclusion



Activity 14.7

Look at the line graph below. It represents data for distance covered by an athlete.



- What is the line graph about?
- How many seconds does the athlete take to cover 100 m? Explain the steps you used to find the answer.



Summary

Interpreting a line graph means understanding what the graph shows.

To do it:

Read the title: the title tells you what the graph is about.

Check the axes labels. Horizontal-axis (bottom): shows categories (e.g., Days, months).

The vertical axis (side): shows measurements or values (e.g., temperature, height).

Look at the scale: see how numbers increase on the y-axis (e.g., 0, 5, 10...).

Follow the points and lines: each point represents a value. The lines show changes over the change of category.

Observe trends:

Rising line = increase (e.g., temperature going up).

Falling line = decrease (e.g., temperature dropping).

Straight line = no change.

Compare values: find the highest and lowest points.

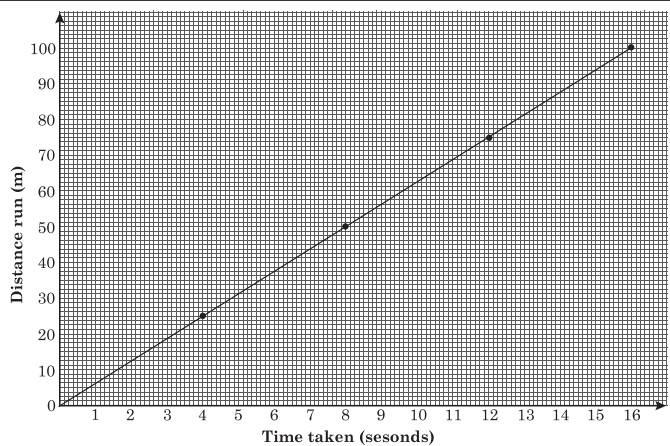
Answer questions: use for example a ruler and follow the line graph to find answers.

Example:

Study the graph in Activity 14.7.

Answer these questions:

- (a) What distance does the athlete cover in 4 seconds?
- (b) How long does it take the athlete to cover 75 m?
- (c) Find the distance the athlete covered in 8 seconds.
- (d) How is line graph important to you?



Solution:

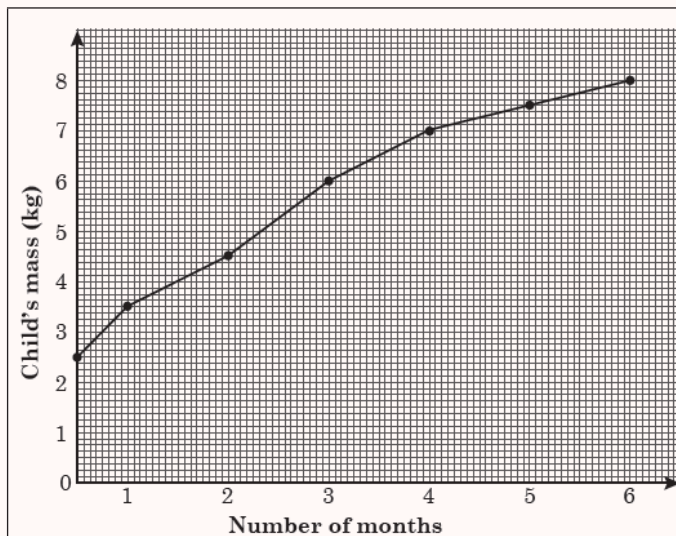
- (a) Put the ruler vertically at 4 seconds. It cuts the line at 25 m. The athlete covers the distance of 25 m in 4 seconds.
- (b) Put the ruler horizontally at 75 m. It cuts the line at 12 seconds. The athlete covers the distance of 75m in 12 seconds.
- (c) In 8 seconds, the athlete covered 50 m.
- d) Line graphs are important because they help visualize how data changes over time or categories, making trends easy to spot.

Additionally, line graphs are simple to read and widely used in real-world situations. Students, scientists, and professionals rely on them to analyze data efficiently.



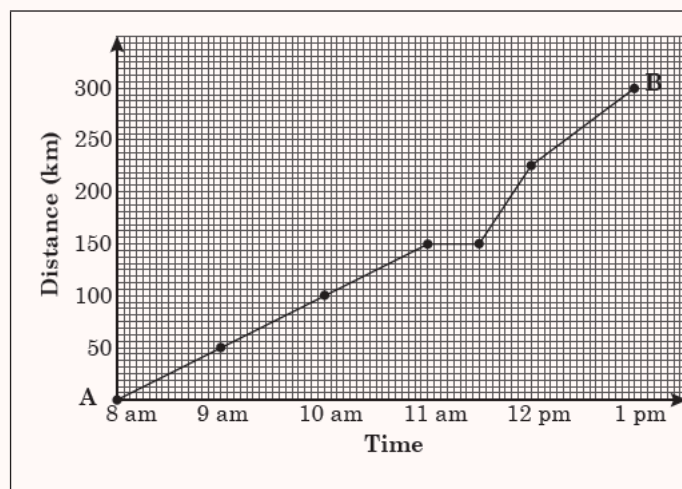
Application Activity 14.7

1) Look at the graph below:



- (a) What is the graph about?
- (b) What is the mass of the child at birth?
- (c) Read the mass of the child at 3rd month.
- (d) At what month is the mass of the child 7 kg?
- (e) What is the change in the child's mass from the 3rd to the 4th month?

2) A motorist started a journey from town A to B. The data is representing the graph below:



- (i) How far is town A from B?
- (ii) What time did the motorist start the journey?
- (iii) What time did the motorist start to rest?
- (iv) What distance did the motorist cover from 11:00 a.m to 11:30 a.m?
- (v) Find the time taken to reach town B after resting.



14.8 Project Title: Data Detectives: Collecting and graphing real-life events

1. Rationale: This project makes data handling **fun and meaningful** by connecting math to real-world observations. Learners develop critical thinking, teamwork, and communication skills while mastering graphs and tables.

2. Location and duration: It will be done outside the classroom. It can take 1 week.

3. Project Overview:

In this project, learners will become “**Data Detectives**” by collecting real-life data, organizing it using **tally tables and frequency tables**, and representing it visually through **bar graphs** or **line graphs**. They will apply their maths skills to answer questions about their findings.

4. Learning Objectives:

By the end of this project, learners will be able to:

Collect and record data from a real-life event or situation.

Organize data using **tally marks and frequency tables**.

Represent data on a **bar graph** or **line graph** (depending on the type of data).

Interpret graphs to draw conclusions.

Present findings clearly to peers.

5. Materials Needed: Clipboards, paper, pencils, Rulers, graph paper, or digital tools (if available), Colored pencils/markers, Sticky notes (for brainstorming)

6. Project Phases:

Phase 1: choose a real-life event (done in the previous lessons)

Brainstorming: Learners suggest events they can observe and record (e.g., weather for a week, favorite fruits in class, favorite sport or subject, etc).

Selection: Each group (or individual) picks **one event** to study.

Planning: Define what data to collect.

Phase 2: Data Collection

Learners collect data over time (e.g., daily temperature, lunch choices, favorite subject in a school area).

Example Projects: *Weather trackers:* Record daily temperature/rainfall.

=Lunch Survey: Count students' favorite fruits. *Subject survey:* favorite subject learnt in the class.

Phase 3: Organize Data: Create a tally table and then a frequency table.

Phase 4: Graph the Data: Draw graphs manually or use digital tools (e.g. Excel)

Bar Graph: Used for categories (e.g., favorite fruits, car colors).

Line Graph: Used for changes over time (e.g., temperature over a week).

Phase 5: Present Findings:

Each group presents: Their research question (e.g., "What is the most popular fruit in our class?"). How they collected data. Their tally table, frequency table, and graph.

Conclusion: What is the observation of the group? (e.g., "Bananas are the most liked").



End unit assessment

Assessing knowledge and understanding:

1.(a) Define quantitative data and give two examples.

(b) Differentiate between discrete and continuous data with an example of each.

2.a) Study the table below showing favorite fruits of 20 pupils. Complete the tally and frequency columns.

Fruit	Tally	Frequency
Apple		6
Banana		
Orange		5
Mango		4

(b) Which fruit is the most preferred? Which is the least preferred?

Assessing skills

3) In a school the following data was recorded

Class	P1	P2	P3	P4	P5	P6
Number of pupils	50	47	48	45	45	40

a) Name the type of quantitative data above. Explain.

(b) Represent the data using a bar chart. What do you notice?

(c) From your bar graph,

(i) Which class had the highest number of pupils?

(ii) Which two classes had the same number of pupils?

4) The amount of rainfall was recorded for a certain town

Month	January	February	March	April	May	June
Rainfall (mm)	10	2.5	15	30	45	20

(a) What type of quantitative data was collected?

(b) Represent the data using a line graph.

(c) From your graph, which month had the highest rainfall? Explain.

5. A farmer recorded plant growth (cm) over 4 weeks:

Week	1	2	3	4
Height (cm)	5	8	11	14

(a) Represent this data in a line graph.

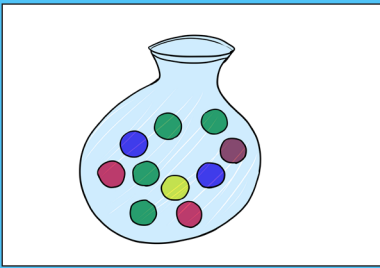
(b) If the trend continues, what will be the height in Week 5?

Assessing attitudes and values

6. Think about the transport survey, what are two reasons why walking is the most common mode used by pupils to arrive at school. How can the school encourage safer walking routes?

7. A shopkeeper says, “Line graphs are useless for my business.” Do you agree? Explain how a line graph could help track daily sales of items like kilograms of rice.

8. Write a short message to a younger pupil explaining why data presentation on a bar graph is a useful skill in everyday life.



UNIT 15

INTRODUCTION TO LIKELIHOOD OF EVENTS

Key Unit Competence: You will be able to conduct experiments and decide how likely something is to happen.

Learning activities:

By the end of this unit, you should be able to:

Knowledge and understanding:

Explain different likelihood of occurring events.

Skills:

Conduct experiment and record outcomes

Use vocabulary of likelihood to compare events.

Attitudes and values:

Appreciate that random events cannot be predicted.

Appreciate the appropriate use of vocabulary of likelihood of events.



15.0 Introduction

This unit introduces you to the basic concepts of probability by helping them understand and describe the likelihood of events happening in everyday situations. You will explore probability through hands-on experiments, games, and real-life scenarios, such as weather predictions, drawing coloured balls, tossing fair coins and rolling fair dice.



Introductory Activity

Many things in nature happen in different ways. Some things occur by chance, while others must happen. Some have an equal chance of happening, while others are almost impossible.

For example, before a football match starts, no one knows which team will win. At the beginning, both teams have an equal chance. But by the end, different results can happen.

Before the match, the referee tosses a coin to decide which team gets which side of the field in the first half.

Questions:

- (a) Is it certain, likely, or impossible for the Team 1 to start the game on a the right side of the field?
- (b) Is it possible or impossible for both teams to have the same score in a football match?

15.1 Representing the outcomes of a die tossed many times



Activity 15.1

This practical activity is conducted to show that there is a chance of winning but it is not possible to predict the winner before a game. In real life, the game of tossing a die opens our mind and we become able to differentiate fair game from unfair game of chance.

a) Required materials: dice, pens, sheets of paper, 30 Counters, a graph paper/squared paper, number cards 1 to 6.

b) Procedure.

Step 1: Put dice on the table and count the number of dots for their faces.



Step 2: On the surface of your table, put number cards on a row marked 1 up to 6 corresponding to the sides of a die.



Step 3: Toss a die and observe the side that is facing up. For every toss observe the side that faces up, observe the number and put a counter along the number card representing the side. For example, if a side of 5 shows up after a toss, put a counter on number card 5.

1	2	3	4	5	6
				●	

If role a die once,

- i) What is the possible outcomes you can observe?
- ii) Considering that you will have one outcome, what is the chance does each case have?

Step 3: Toss the **die other twenty-nine times** to make it thirty times and for each face place a counter on the number card that corresponds to the side that shows up.

Step 4: Count the number of counters on each number card which represents the number of times that sides 1, 2, 3, 4, 5, and 6 of a die have shown up during the thirty tosses. Record the results in a table shown below.

Face of a die (number card)	1	2	3	4	5	6
Outcomes = Number of times a face showed up (number of counters)						

Which Face had the highest frequency?

Step 5: Represent the results from step 4, as a bar chart.



Summary

When you roll a die once, The die can land on any one of the six faces: **1, 2, 3, 4, 5, or 6**.

Each face has an equal chance of $\frac{1}{6}$ to face up.

Before the roll, the result is **uncertain**, you cannot predict which number will appear.

After the roll, the outcome is **fixed (e.g., «4»)**, and **no other result is possible for that single trial. It is impossible to get two faces at the same time.**

If you bet on a single outcome (e.g., “I bet it’s a 6”), you have a $\frac{1}{6}$ chance to win and $\frac{5}{6}$ chance to lose.

When you role a fair die many times, all faces will appear with approximately equal frequency.

When we toss a die many times, we can record the results in different ways to see patterns. Here are simple methods:

Listing the Outcomes: Write down each number that appears.

Example: If you toss a die 6 times, you might get: 3, 5, 1, 2, 4, 6.

Using a tally chart.

Drawing a bar chart (Pictograph).

Example:

For example, when a die was tossed thirty times the following results were gotten:

The Face of **1** showed up seven times (7 counters on number card 1)

The Face of **2** showed up five times. (5 counters on number card 2)

The Face of **3** showed up once. (1 counter on number card 3)

The face of **4** showed up three times. (3 counters on number card 4)

The face of **5** showed up six times (6 counters on number card 5)

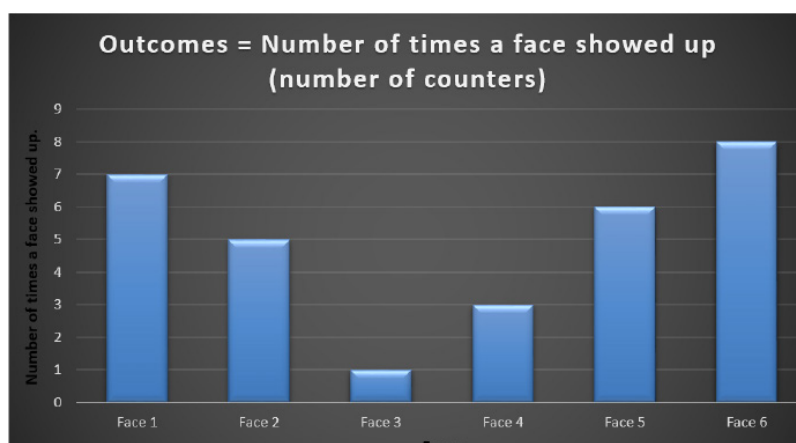
The face of **6** showed up eight times. (8 counters on number card 6)

The recorded table of outcomes looks as below:

Face of a die (number card)	Face 1	Face 2	Face 3	Face 4	Face 5	Face 6
Outcomes = Number of times a face showed up (number of counters)	7	5	1	3	6	8

These data can be represented on a bar graph as follows:

- We find that the face 6 is the one that faced up more times. It has the frequency 8.
- The face 1 is the one that faced up few times. It has the frequency 1.



Application Activity 15.1

- In a group of 6 pupils, play the game of tossing a die 20 times. Each pupil will select his/her face (for example Moses can select the face number 1) and play the game.

Use the table like the following to record the results

Face of a die	1	2	3	4	5	6
Name of the player (Owner)	Moses					
Tally for outcome						
Total	4					

After the game, check who will be the winner.

The winner will be the one who will have a face with a bigger number of facing up (Tally for outcome).

2) In an experiment, two pupils toss a dice 48 times. They recorded their results as below.

Face of dice	1	2	3	4	5	6	Total
Total times it faced up	9	8	8	7	8	8	48

- (i) Represent the data on a bar graph.
- (ii) Which face had the highest chance of showing up? Explain.
- (iii) Which face showed up the least? Explain?
- (iv) What the total times the face 5 has faced up?

15.2 Vocabulary of chance



Activity 15.2

- (a) Inside your class, what is there?
 - (i) Is it certain that there is a pupil?
 - (ii) Is it impossible to find a pupil in class?
 - (iii) Can you find a book in your class? Which vocabulary of chance can you use? Certain or impossible?



Summary

When talking about the chance, the following are the most useful words: experiment, outcomes, equal chance, certain, impossible, equally likely, less likely, likely and more likely.

Experiment: A test or activity you do to see what will happen.

Example: Tossing a coin to see if it lands on heads or tails is an experiment.

Outcomes: The possible results of an experiment.

Example: For a coin toss, the outcomes are heads or tails.

Equal chance: When two or more outcomes are equally likely to happen.

Example: When flipping a fair coin, heads and tails have equal chances.

Certain (sure): An event that will definitely happen.

Example: The sun rises each morning; it is certain to happen.

Impossible: An event that cannot happen.

Example: Getting a tail and a head at the same time when tossing one coin.

Equally likely (even chance): Outcomes that have the same chance of happening.

Example: Rolling a fair six-sided die; each number (1–6) is equally likely.

Less likely: An event that has a smaller chance of happening compared to another.

Example: Drawing a red card from a deck with many black cards is less likely.

Likely: An event that has a good chance of happening.

Example: It's likely to rain if clouds are very dark.

More likely: When an event has a higher chance of happening than another.

Example: you are more likely to win a game if you practice a lot.



Application Activity 15.2

- 1) Go outside your classroom. What is there in your school? Are you likely or unlikely to find the following? Explain.
i) a tree ii) a lion iii) a bird iv) grass v) a car vi) a cow vii) a motor bike
- (2) Toss a coin. What side is likely to face up? Head or tail?
- 3) Use the correct vocabulary of chance in each of the following sentences. Discuss your answers.
 - a) In a museum, you are ____ to see preserved animals.
 - b) In animal park, you are ____ to see a wild animal in cage.
 - c) Inside your class, it is ____ to see a living lion lying next to you.
 - d) It is ____ to see a bird flying by your school
 - e) It is ____ that you have tails only in several tosses of the coin.

15.3 special events: Certain, uncertain and impossible events



Activity 15.3

- 1) The Sun Always Rises.
Is the event “*the sun will rise tomorrow at noon*” a certain or impossible event?
- 2) Discuss the answer: Will we have the rain tomorrow?
Can you predict if it will rain tomorrow?
Therefore, conclude if it is certain that it will rain tomorrow.
- 3) There is a bag with 5 red marbles and 1 blue marble.
“If you pick a marble without looking, which color is more likely?”
Is blue marble likely or unlikely to be picked?
- 4) Is the following possible or impossible? Explain your answer.
 - i) A dog will meow like a cat.
 - ii) A banana will turn into a car
 - iii) The teacher will come to school tomorrow.



Summary

Certain event = It will **always** happen (100% chance).

Uncertain event = It might happen (more likely/less likely).

Impossible event = It will **never** happen (0% chance).



Application Activity 15.3

Say if the following event is certain, uncertain or impossible.

- 1) *If you drop a ball from your hand, It will fall to the ground. It is _____*
- 2) *If you roll a standard six-sided die, you will get the number 3. It is _____*
- 3) *A cow flies like a bird. It is _____*
- 4) *“Look at these statements. Which one is certain, uncertain, and impossible?”*
 - i) *“The sun will rise tomorrow.” (_____)*
 - ii) *“It will rain next Tuesday.” (_____)*
 - iii) *“A rock will grow into a tree.” (_____)*

15.4 Conducting experiments and deciding how likely something is to happen



Activity 15.4

Toss a coin 20 times. Record the results in the table below. For example, if in 1st throw, head faces up, then tick (✓) head in table.

Throw	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Total
Head	✓																				
Tail																					

Count the total number of heads and write this in the total.

Count the total number of tails and write this in the total. Compare your results with the other groups.

1. What is the number of times of getting a tail in a throw? Explain.
2. What the number of times of getting a head in a throw? Explain.
3. When you toss a coin, which side is likely to face up? Why?

4. Does the outcome depend on what was observed previously? Why?

Fill in the table below by ticking the appropriate box. Discuss your answer.

Possible outcome in tossing a coin	Vocabulary of chance applicable				
	Certain	Impossible	Likely	Unlikely	Equally likely
Getting a head					
Getting a tail					
Getting tail and head at once					
Getting no tail and no head					



Summary

1. Likely (Probable)

Used when an event has a **good chance** of happening (chance > 50%).

Examples:

“It’s **likely** to rain today because the sky is cloudy.»

“You’re **likely** to roll a number less than 5 on a die.» (4 out of 6 chances)

2. Unlikely / Less Likely (Improbable)

Used when an event has a **low chance** of happening (chance < 50%).

Examples:

“It’s **unlikely** to snow in summer.» (Very rare)

“Rolling a 7 on a standard die is **impossible**, but rolling a 1 is just **unlikely** in one try.» *(Same 1/6 chance as other numbers, but we say «unlikely» for emphasis)*

Note:

“**Less likely**” compares two events (e.g., «Rain is **less likely** than sunshine tomorrow»).

“**Unlikely**” stands alone (e.g., «A lion in the classroom is **unlikely**”).

3. More Likely (Higher chance)

Used when one event has a **higher chance** than another.

Examples:

“Flipping **heads** is **more likely** than rolling a 6 on a die.»

“In a school, seeing a bird is **more likely** than seeing a cow.»

Example:

1) If you flip a coin, is it more likely to land on heads or tails? Why?

Answer:

It's **equally likely** to land on heads or tails because a fair coin has two sides, each with the same chance.

2) In a bag with 2 red balls and 8 blue balls, is it more likely or less likely to pick a red ball? Why?

Answer:

It is **less likely** to pick a red ball because there are fewer red balls than blue balls.



Application Activity 15.4

- 1) A bag has 4 red marbles and 1 blue marble. Is it more likely or less likely to pick a red marble?
- 2) There are 3 sunny days and 12 rainy days in a month. Is a rainy day more likely or less likely than a sunny day?
- 4) In a class of 30 students, 27 students prefer pizza, and 3 prefer sweets. Is a student more likely or less likely to choose pizza?
- 5) A dice has 6 faces numbered 1 to 6. Is rolling a 6 more likely or less likely than rolling an odd number (1,3,5)?



End unit assessment

Assessing knowledge

1) Fill in the blanks with the correct vocabulary of chance:

- a) When flipping a fair coin, heads and tails have an _____ chance. (*equal/likely*)
- b) The sun rising tomorrow is a _____ event. (*certain/impossible*)
- c) A cow flying like a bird is an _____ event. (*uncertain/impossible*)

2) A pupil went to bathe by a river. Use vocabulary of chance to fill in the following.

- (i) It is ____ he will wet his body. (certain, unlikely).
- (ii) It is ____ for him to bathe without getting wet. (certain, impossible)
- (iii) It is ____ he will use soap. (unlikely, likely).
- (iv) It is ____ that he will wet and then dry his body. (equally likely, unlikely)
- (v) It is ____ that he will bathe without soap. (likely, unlikely).

3) Match the term to its definition:

Term	Definition
1. Experiment	A. Will never happen (0% chance)
2. Impossible	B. A test to see what happens
3. More likely	C. Higher chance than another outcome

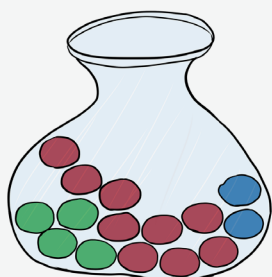
Assessing skills

4) A die was tossed 20 times, and the results were recorded:

Face	1	2	3	4	5	6
Number of times it faced up	3	5	2	4	3	3

- Draw a **bar graph** to represent this data.
- Which face appeared **most frequently**?
- What is the number of times did a **3 rolled up**?

5) A bag contains: 9 maroon marbles, 2 blue marbles and 4 green marbles.



If you pick one marble without looking:

- Is it **more likely** or **less likely** to pick a red marble?
 - Which color is **least likely** to be picked?
- 6) Eric and Olive conducted an experiment. They tossed a coin ten times. They recorded their results below.

Throw	1	2	3	4	5	6	7	8	9	10
Head faced up	✓	✓		✓			✓	✓		✓
Tail faced up			✓		✓	✓			✓	

✓ shows the result that was observed at each throw.

- How many times did heads face up?

(ii) How many times did tails face up?

(iii) Find the total times heads and tails face up. Explain your steps.

(iv) Was it possible to have both head and tail at once in a toss? Is having both head and tail at once a possible or impossible event?

(v) Is head faced up a more likely or unlikely event?

(vi) If a bottle top was used, would the results be the same? Explain your answer.

(vi) Tell the importance of learning probability.

7) A coin was tossed 15 times, landing on: Heads: 7 times and Tails: 8 times

Is the result **equally likely**? Explain.

Assessing attitudes and Values

8) Explain why probability is important in real life.

9) If a game is unfair, how does it affect players?

10) Gambling is when someone risks money or something valuable (like toys or belongings) on a game or event where the outcome is uncertain. Why should we avoid gambling?

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