



**A comparative study of the mathematical curricula of France and the state of Georgia.**

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## **Abstract**

The purpose of our study is to make a comparison between the high school math content of the State of Georgia and the high school math content of France, whose students perform better at the international level than U.S high school students. A report by the Pew Research Center (2015) found that U.S students are scoring higher on national math assessments than they did two decades ago. However, U.S students' performance is still behind all the other major industrial countries (Desilver, 2015). We use a cross-sectional analysis to investigate the causes of this low performance of U.S students in math. We aim to read the entire math curricula of France and the State of Georgia to identify similarities or any major differences.

## **1- Introduction**

Our current project investigates the differences between the French math curriculum and the State of Georgia math curriculum. Many studies have shown that average U.S students score lower than students in most other industrial nations (Desilver, 2015). The percentages of all U.S students who score at the advanced level is 6%, compared to France 10.8% (Hanushet, Peterson, & Woessmann, 2010)

The French high school system offers many tracks for students to choose that narrow their field of study in the last two to three years of their secondary schooling. The courses offered under the general track in France correspond the most with those offered in Georgia's high schools. Of the general track, a student can choose to focus on literature, economics and social studies, or sciences. We have chosen to explore the science track in hopes of understanding and relating the specific curriculum to that of Georgia's.

In the French high school science field, students are exposed to physics, chemistry, biology, earth science, and various fields of math including but not limited to algebra, geometry, and probability and statistics. These courses are also offered in Georgia high schools but the order in which they are presented or required may differ. The Georgia high school curriculum is designed to give a broader education in preparation for college. It is in college when a student begins to concentrate their studies towards a future career goal.

## **2- How to become a high school teacher in France**

There are essentially five steps to complete to become a teacher in France. This process as explained below is for French citizens and does not wholly apply to expatriates who wish to teach in France, for whom the process is slightly different. The steps below are specifically for someone to become qualified to teach at the high school level in France. Any person who wishes to become a teacher must first obtain a bachelor's degree, which the French call *la licence*.

Most future teachers will earn a degree in the subject they wish to teach. After successfully earning a bachelor's degree, a student may enroll in an ESPE, which stands for *Écoles Supérieures du Professorat et de l'Éducation* (Graduate School for Teaching and Education).

During these two years, they will earn their Master's degree in teaching, education, and training. The French acronym for this degree is MEEF, *Métiers de l'Enseignement, de l'Éducation et de la Formation*.

At the beginning of the first year of the Master's program, or M1, around September and October, students register for a competitive exam that corresponds with their teaching goals. If you wish to teach in a middle or high school, you take the CAPES, *Certificat d'Aptitude au Professorat de l'Enseignement du Second degré*. These exams are administered from March to early July. The CAPES is divided into different sections including, but not limited to, history and geography, mathematics, chemistry, life and earth science, music education, and modern foreign languages. Students take the exam for the subject they intend to teach. A list is published each year that gives the number of teaching positions available for each subject. For example, the list of available teaching positions for students taking the CAPES in 2016 states that 1,440 positions are open for mathematics teachers. A main component of M1 is preparing for these exams. If a student does not pass the CAPES, they must take one more year to prepare for the exam through adapted courses at their ESPE (Cornu, 2015).

Students also complete a four to six week internship during M1 to observe classrooms and practice their teaching. If students succeed on their exams, they continue on to their second year of the MEEF program, or M2. During this year, students complete a work-study program. They spend about 250-300 hours on schoolwork as compared to the 450-550 hours during M1, but they are also part-time teachers at a school or educational establishment. To complete the MEEF program, students write their thesis during M2. They begin to form an outline and reflect on their experience during M1, but they use their first-hand experience during M2 to write a comprehensive thesis. At the end of M2, students will obtain their MEEF. Before these students can become certified teachers, they must gain the approval of an academic jury's evaluation of their internship. Once these students are certified teachers, they become employees of the state and are guaranteed a job for life.

### **3- How to become a High school teacher in the State of Georgia**

There are many options for those who wish to teach in Georgia. If you already teach in another state or are going back to teaching in Georgia, there is information online at the following website on how to earn or renew a teaching certificate in Georgia:

<http://www.gapsc.com/MoveToGeorgia/MoveToGeorgia.aspx>. The information below is for those who have not completed an educator preparation program but wish to teach in the state of Georgia.

Some decisions must be made before enrolling in an educator preparation program. The grade level and subject area you want to teach should be considered. There are four tiers of grade levels: early childhood, middle grades, secondary, and P-12. Early childhood teachers usually teach all subject areas from Pre-K to fifth grade. Middle grades are considered grades four through eight, but most middle schools only have grades six through eight. A middle grades teacher can choose from language arts, math, science, reading, and social science as subject areas. Secondary certification covers grades six through twelve, but most high schools cover grades nine through twelve. The subject areas for high school teachers are generally more specific than those of middle grades. For example, a high school teacher could teach American Literature as opposed to just Reading. Certification in the subject areas of art, music, drama, dance, health, PE, and Special Education is for all grade levels from elementary to high school.

Teaching certification in Georgia requires at least a bachelor's degree. All future teachers must complete an educator preparation program. Many of these programs lead to a bachelor's degree. If you have already earned a bachelor's degree, then there are a couple of options. You may enroll in a master's program that includes an educator preparation program, or you could pursue a certification only program that does not lead to an additional degree. The educator preparation program should include a student teaching internship, which is required for one of the assessments for certification. Before being accepted into an educator preparation program, you must complete the GACE Program Admission Assessment with a passing score. This assessment can be exempted if sufficient scores are obtained on the SAT, ACT, or GRE.

Towards the end of the educator preparation program, future teachers must also complete the GACE (Georgia Assessments for the Certification of Educators) content assessment for their subject area and the edTPA assessment, the Georgia Professional Standards Commission (GaPSC) approved content pedagogy assessment. The GACE is designed for each subject area and depending on the subject may consist of one or two tests. EdTPA is designed to assess planning, instruction, and assessment. Those who are completing the edTPA must submit a portfolio that they have put together during their time student teaching that must include an unedited video of themselves working in a classroom.

Other required components are a passing score on the Georgia Educator Ethics Assessment-Program Exit and the completion of a course in identifying and educating children with special needs. If you are not employed by a Georgia local unit of administration when you apply for certification then, given that all the above requirements are satisfied, you will be issued a Certificate of Eligibility that is converted to an educator certificate when you have become employed.

#### 4- Group age comparison of students

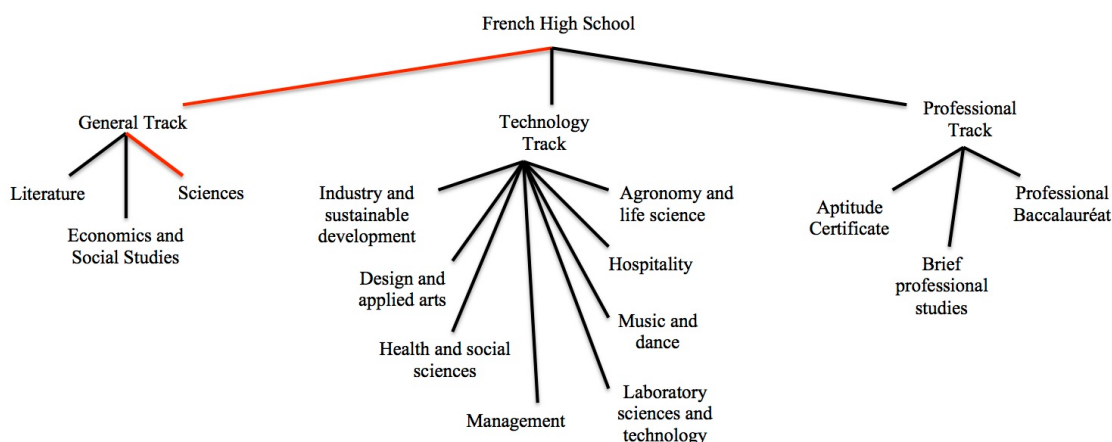
The table below compares age groups of students in the United States to the ages of their counterpart in France. It shows that in both countries, students are expected to enter Kindergarten at the same age and finish high school at the same age.

| Age Upon Entering | Grade Level- United States |            | Grade Level- France                     |                |
|-------------------|----------------------------|------------|-----------------------------------------|----------------|
| 5                 | Kindergarten               | Elementary | Section d initiation a la lecture (SIL) | École Primaire |
| 6                 | 1st grade                  |            | Cours préparatoire (CP)                 |                |
| 7                 | 2nd grade                  |            | Cours élémentaire 1ère année (CE1)      |                |
| 8                 | 3rd grade                  |            | Cours élémentaire 2e année (CE2)        |                |
| 9                 | 4th grade                  |            | Cours moyen 1ère année (CM1)            |                |
| 10                | 5th grade                  |            | Cours moyen 2e année (CM2)              |                |

|    |            |        |                |         |
|----|------------|--------|----------------|---------|
| 11 | 6th grade  | Middle | Sixième (6e)   | Collège |
| 12 | 7th grade  |        | Cinquième (5e) |         |
| 13 | 8th grade  |        | Quatrième (4e) |         |
| 14 | 9th grade  | High   | Troisième (3e) | Lycée   |
| 15 | 10th grade |        | Seconde        |         |
| 16 | 11th grade |        | Première       |         |
| 17 | 12th grade |        | Terminale      |         |

## 5- Comparative study of the curriculum of the state of France and the state of Georgia

### 5.1- Different tracks proposed to high schoolers in France



The French high school system offers many tracks for students to choose that narrow their field of study in the last two to three years of their secondary schooling. The courses offered under the general track in France correspond the most with those offered in Georgia's high schools. Of the general track, a student can choose to focus on literature, economics and social studies, or sciences. We have chosen to explore the science track in hopes of understanding and relating the specific curriculum to that of Georgia's.

In the French high school science field, students are exposed to physics, chemistry, biology, earth science, and various fields of math including but not limited to algebra, geometry, and probability and statistics. These courses are also offered in Georgia high schools but the order in which they are presented or required may differ. The Georgia high school curriculum is designed to give a broader education in preparation for college. It is in college when a student decides to hone their educational paths to their future career goals. We must assess where a French student is in terms of the Georgia high school requirements to place them in the proper grade level.

## 5.2- Grade level comparison of curricula

These tables show where in the Georgia math curriculum the French math curriculum subjects can be found. If a row does not have a Georgia course equivalent, then there was not a standard in the Georgia curriculum that matched with the French curriculum.

| Translation                          | Classe de Seconde                           | GA Course        |                           |                 |                          |                                     |
|--------------------------------------|---------------------------------------------|------------------|---------------------------|-----------------|--------------------------|-------------------------------------|
|                                      |                                             | Unit Number      |                           |                 |                          |                                     |
| <b>1. Functions</b>                  | <b>1. Fonctions</b>                         | <b>Algebra I</b> | <b>Coordinate Algebra</b> | <b>Geometry</b> | <b>Analytic Geometry</b> |                                     |
| Functions                            | Fonctions                                   | Unit 2 and 3     | Unit 2 and 3              |                 |                          |                                     |
| Qualities of functions               | Étude qualitative de fonctions              | Unit 2 and 3     | Unit 3                    |                 |                          |                                     |
| Algebraic expressions                | Expressions algébriques                     | Unit 2           |                           |                 |                          |                                     |
| Equations                            | Équations                                   | Unit 2           | Unit 2                    |                 |                          |                                     |
| Reference functions                  | Fonctions de référence                      | Unit 2           | Unit 3                    |                 |                          |                                     |
| Second degree polynomials            | Études de fonctions                         | Unit 3           |                           |                 |                          |                                     |
| Inequalities                         | Études de fonctions                         | Unit 2           | Unit 2                    |                 |                          |                                     |
| Trigonometry                         | Trigonométrie                               |                  |                           | Unit 3          | Unit 2                   |                                     |
|                                      |                                             |                  |                           |                 |                          |                                     |
|                                      |                                             |                  |                           |                 |                          |                                     |
| <b>2. Geometry</b>                   | <b>2. Géométrie</b>                         | <b>Geometry</b>  | <b>Pre-Calculus</b>       |                 |                          |                                     |
| Coordinate plane and points          | Coordonnées d'un point du plan              | Unit 1           |                           |                 |                          |                                     |
| Shapes on the plane                  | Configurations du plan                      | Unit 5           |                           |                 |                          |                                     |
| Lines                                | Droites                                     | Unit 1           |                           |                 |                          |                                     |
| Vectors                              | Vecteurs                                    |                  | Unit 7                    |                 |                          |                                     |
| Spatial geometry                     | Géométrie dans l'espace                     |                  |                           |                 |                          |                                     |
|                                      |                                             |                  |                           |                 |                          |                                     |
|                                      |                                             |                  |                           |                 |                          |                                     |
| <b>3. Statistics and Probability</b> | <b>3. Statistiques et Probabilités</b>      | <b>Algebra 1</b> | <b>Coordinate Algebra</b> | <b>Geometry</b> | <b>Analytic Geometry</b> | <b>Algebra II/ Advanced Algebra</b> |
| Descriptive statistics, analyse data | Statistique descriptive, analyse de données | Unit 6           | Unit 4                    |                 |                          |                                     |
| Sampling                             | Échantillonnage                             |                  |                           |                 |                          | Unit 7                              |
| Probability of a set                 | Probabilité sur un ensemble fini            |                  |                           | Unit 6          | Unit 7                   |                                     |

| Translation                           | Classe de Première                          | GA course<br>Unit<br>Number |                                              |                                         |                     |
|---------------------------------------|---------------------------------------------|-----------------------------|----------------------------------------------|-----------------------------------------|---------------------|
| <b>1. Analyze</b>                     | <b>1. Analyse</b>                           | <b>Algebra I</b>            | <b>Coordinate Algebra</b>                    | <b>Algebra II/<br/>Advanced Algebra</b> | <b>Calculus</b>     |
| Second degree                         | Second degré                                | Unit 3                      |                                              |                                         |                     |
| Study of Functions                    | Étude de fonctions                          |                             |                                              | Unit 4                                  |                     |
| Differentiation                       | Dérivation                                  |                             |                                              |                                         | Unit 2              |
| Sequences                             | Suites                                      |                             | Unit 3                                       |                                         |                     |
|                                       |                                             |                             |                                              |                                         |                     |
| <b>2. Geometry</b>                    | <b>2. Géométrie</b>                         | <b>Geometry</b>             | <b>Pre-Calculus</b>                          |                                         |                     |
| Plane geometry (vectors)              | Géométrie plane (vecteurs)                  |                             | Unit 7                                       |                                         |                     |
| Trigonometry                          | Trigonométrie                               | Unit 3                      |                                              |                                         |                     |
| Scalar products (vectors)             | Produit scalaire dans le plan (vecteurs)    |                             | Unit 7                                       |                                         |                     |
|                                       |                                             |                             |                                              |                                         |                     |
| <b>3. Statistics and Probability</b>  | <b>3. Statistiques et Probabilités</b>      | <b>Algebra I</b>            | <b>Accelerated Algebra I/<br/>Geometry A</b> | <b>Coordinate Algebra</b>               | <b>Pre-Calculus</b> |
| Descriptive statistics, data analysis | Statistique descriptive, analyse de données | Unit 6                      | Unit 6                                       | Unit 4                                  |                     |
| Probability                           | Probabilités                                |                             |                                              |                                         | Unit 8              |
| Sampling                              | Échantillonnage                             |                             |                                              |                                         |                     |

| Translation                                           | Classe Terminale                                                  | GA Course<br>Unit Number                     |                           |                                                                |                 |
|-------------------------------------------------------|-------------------------------------------------------------------|----------------------------------------------|---------------------------|----------------------------------------------------------------|-----------------|
| <b>1. Analysis</b>                                    | <b>1. Analyse</b>                                                 | <b>Accelerated Algebra I/<br/>Geometry A</b> | <b>Coordinate Algebra</b> | <b>Accelerated Coordinate Algebra/<br/>Analytic Geometry A</b> | <b>Calculus</b> |
| Sequences                                             | Suites                                                            | Unit 2 and 4                                 | Unit 3                    | Unit 3                                                         |                 |
| Limits of functions                                   | Limites de fonctions                                              |                                              |                           |                                                                | Unit 1          |
| Continuity on an interval, Intermediate Value Theorem | Continuité sur un intervalle, théorème des valeurs intermédiaires |                                              |                           |                                                                | Unit 1          |
| Calculating derivatives: complements                  | Calculs de dérivées: compléments                                  |                                              |                           |                                                                | Unit 2          |
| Sine and cosine functions                             | Fonctions sinus et cosinus                                        |                                              |                           |                                                                | Unit 2          |
| Exponential functions                                 | Fonction exponentielle                                            |                                              |                           |                                                                | Unit 1 and 2    |
| Natural logarithmic functions                         | Fonction logarithme népérien                                      |                                              |                           |                                                                | Unit 1 and 2    |
| Integration                                           | Intégration                                                       |                                              |                           |                                                                | Unit 3          |
|                                                       |                                                                   |                                              |                           |                                                                |                 |
| <b>2. Geometry, Complex Numbers</b>                   | <b>2. Géométrie, Nombres Complexes</b>                            | <b>Pre-Calculus</b>                          |                           |                                                                |                 |
| Lines and planes                                      | Droites et plans                                                  |                                              |                           |                                                                |                 |
| Vector geometry                                       | Géométrie vectorielle                                             |                                              |                           |                                                                |                 |
| Scalar product                                        | Produit scalaire                                                  | Unit 7                                       |                           |                                                                |                 |
|                                                       |                                                                   |                                              |                           |                                                                |                 |
| <b>3. Probability and Statistics</b>                  | <b>3. Probabilités et Statistique</b>                             | <b>Geometry</b>                              |                           |                                                                |                 |
| Conditional, independence                             | Conditionnement, indépendance                                     | Unit 6                                       |                           |                                                                |                 |
| Law of Density                                        | Notion de loi à densité à partir d'exemples                       |                                              |                           |                                                                |                 |
| Interval of fluctuation                               | Intervalle de fluctuation                                         |                                              |                           |                                                                |                 |
| Estimation                                            | Estimation                                                        |                                              |                           |                                                                |                 |

## 6- Sample French High school exit exam (Baccalaureat)

### 6.1- Sample Test 1 (France – Mainland, June 22, 2015)

#### Exercise 1 (6 points)

For all candidates

*Round all probabilities to the thousandths place.*

#### Part 1

1. Let  $X$  be a random variable that follows the exponential distribution parameter  $\lambda$ , where  $\lambda$  is a real strictly positive number.  
Recall that the density of probability of this distribution is the function  $f$  defined on  $[0, +\infty)$  by
$$f(x) = \lambda e^{-\lambda x}.$$
  - a. Let  $c$  and  $d$  be two real numbers such that  $0 \leq c \leq d$ .  
Show that the probability  $P(c \leq X \leq d)$  verifies that  $P(c \leq X \leq d) = e^{-\lambda c} - e^{-\lambda d}$ .
  - b. Determine the value of  $\lambda$  to the thousandths place so that the probability of  $P(X > 20)$  is equal to 0.05.
  - c. Find the expected value of the random variable  $X$ .  
  
**In the following exercise, let  $\lambda=0.15$ .**
  - d. Calculate  $P(10 \leq X \leq 20)$ .
  - e. Calculate the probability of the event  $(X > 18)$ .
2. Let  $Y$  be a normally distributed random variable with center 16 and standard deviation 1.95.
  - a. Calculate the probability of the event  $(20 \leq Y \leq 21)$ .
  - b. Calculate the probability of the event  $(Y < 11) \cup (Y > 21)$ .

#### Part 2

A chain of stores hopes to win the loyalty of their customers by offering vouchers to privileged customers. Each of them receives a green or red voucher on which the total offer is inscribed. The vouchers are distributed so as to have in each store a quarter red vouchers and three-quarters green vouchers.

The green vouchers have the value of 30 euros with a probability of 0.067 or the value falls between 0 and 15 euros with unspecified probabilities.

Analogously, the red vouchers have the value of 30 or 100 euros with the respective probabilities of 0.015 and 0.010 or the value falls between 10 and 20 euros with unspecified probabilities.

1. Calculate the probability of having a voucher with a value greater than or equal to 30 euros given that it is red.
2. Show that the value rounded to the thousandths place of the probability of having a voucher of a value greater than or equal to 30 euros is 0.057.

**For the following question, use the value above.**

3. In one of the stores of this chain, of the 200 privileged customers, 6 received a voucher with a value greater than or equal to 30 euros.  
The manager of the store estimates that this number is insufficient and doubts the random distribution of the vouchers in the different stores of the chain.  
Are his doubts justified?

### Exercise 2 (3 points)

#### For all candidates

In an orthonormal coordinate system  $(O, I, J, K)$  of unit 1 cm, consider the points  $A(0, -1, 5)$ ,  $B(2, -1, 5)$ ,  $C(11, 0, 1)$ , and  $D(11, 4, 4)$ .

A point  $M$  travels on the line  $AB$  in the direction of  $A$  to  $B$  with speed 1 cm per second.

A point  $N$  travels on the line  $CD$  in the direction of  $C$  to  $D$  with speed 1 cm per second.

At the time  $t = 0$ , the point  $M$  is on  $A$  and the point  $N$  is on  $C$ .

Denote  $M_t$  and  $N_t$  as the positions of the points  $M$  and  $N$  after  $t$  seconds, where  $t$  is a positive real number.

Let  $M_t$  and  $N_t$  have the following coordinates:  $M_t(t, -1, 5)$  and  $N_t(11, 0.8t, 1 + 0.6t)$ .

*Questions 1 and 2 are independent.*

1.
  - a. The line  $AB$  is parallel to one of the axes  $(OI)$ ,  $(OJ)$ , or  $(OK)$ . Which one?
  - b. The line  $CD$  is located in a plane  $\mathcal{P}$  parallel to one of the planes  $(OIJ)$ ,  $(OIK)$ , or  $(OJK)$ . Which one? Give the equation of the plane  $\mathcal{P}$ .
  - c. Verify that the line  $AB$ , orthogonal to the plane  $\mathcal{P}$ , cuts the plane at the point  $E(11, -1, 5)$ .
  - d. Are the lines  $AB$  and  $CD$  secant lines?
2.
  - a. Show that  $M_t N_t^2 = 2t^2 - 25.2t + 138$ .
  - b. At what time  $t$  is the length  $M_t N_t$  minimized?

### Exercise 3 (5 points)

#### Candidates who have not followed the teaching specialty

1. Solve the equation for  $z$  in the set of complex numbers:  

$$z^2 - 8z + 64 = 0$$
 Consider the two-dimensional complex plane oriented positively  $(O, \vec{u}, \vec{v})$ .
2. Consider the points  $A$ ,  $B$ , and  $C$  with respected affixes  $a = 4 + 4i\sqrt{3}$ ,  $b = 4 - 4i\sqrt{3}$ , and  $c = 8i$ .
  - a. Calculate the modulus and the argument of the number  $a$ .
  - b. Give the exponential form of the numbers  $a$  and  $b$ .
  - c. Show that the points  $A$ ,  $B$ , and  $C$  are on the same circle with center  $O$ , which determines the radius.
  - d. Place the points  $A$ ,  $B$ , and  $C$  on the system  $(O, \vec{u}, \vec{v})$ .

For the following exercise, utilize the figure from question 2d for help. Complete the missing parts of the graph as you answer the question.

3. Consider the points  $A'$ ,  $B'$ , and  $C'$  of respected affixes  $a' = ae^{i\frac{\pi}{3}}$ ,  $b' = be^{i\frac{\pi}{3}}$ , and  $c' = ce^{i\frac{\pi}{3}}$ .
  - a. Show that  $b' = 8$ .
  - b. Calculate the modulus and the argument of the number  $a'$ .

For the following, let  $a' = -4 + 4i\sqrt{3}$  and  $c' = -4\sqrt{3} + 4i$ .

4. Assume that if  $M$  and  $N$  are two points of the plane of respected affixes  $m$  and  $n$ , then the middle  $I$  of the segment  $MN$  has an affix  $\frac{m+n}{2}$  and the length  $MN$  is equal to  $|n-m|$ .
  - a. Let  $r$ ,  $s$ , and  $t$  be the respective affixes of  $R$ ,  $S$ , and  $T$  of the segments  $A'B$ ,  $B'C$ , and  $C'A$ . Calculate  $r$  and  $s$ . Note that  $t = 2 - 2\sqrt{3} + i(2 + 2\sqrt{3})$ .
  - b. What conjecture may be made as to the nature of the triangle  $RST$ ? Justify the result.

### Exercise 3 (5 points)

#### Candidates who followed the teaching specialty

1. Consider the equation E to solve in the set of integers:
 
$$7x - 5y = 1$$
  - a. Verify the pair  $(3, 4)$  is a solution to E.
  - b. Show that the whole number pair  $(x, y)$  is a solution to E if and only if  $7(x - 3) = 5(y - 4)$ .
  - c. Show that the whole number solutions of the equation E are exactly the pairs of whole numbers  $(x, y)$  relatively:

$$\begin{cases} x = 5k + 3 \\ y = 7k + 4 \end{cases} \text{ where } k \in \mathbb{Z}.$$

2. A box contains 25 chips that are red, green, and white. Of the 25 chips there are  $x$  red chips and  $y$  green chips. Given that  $7x - 5y = 1$ , what may be the number of red, green, and white chips?

In the following, suppose that there are 3 red chips and 4 green chips.

3. Consider a random walk of a pawn along the triangle ABC. At each step, you choose at random one of the 25 chips then put it back in the box.
  - When you are on A:
 

If the chip pulled is red, then the pawn moves to B. If the chip pulled is green, the pawn moves to C. If the chip pulled is white, the pawn stays on A
  - When you are on B:
 

If the chip pulled is red, the pawn moves to A. If the chip pulled is green, the pawn moves to C. If the chip pulled is white, the pawn stays on B.

- When you are on C:  
If the chip pulled is red, then the pawn moves to A. If the chip pulled is green, the pawn moves to B. If the chip pulled is white, the pawn stays on C.

At first, the pawn is on A.

For all natural numbers  $n$ , let  $a_n$ ,  $b_n$ , and  $c_n$  be probabilities that the pawn is respectively on the vertices A, B, and C at step  $n$ .

Let  $X_n$  be the line matrix  $(a_n \quad b_n \quad c_n)$  and  $T$  be the matrix  $\begin{pmatrix} 0.72 & 0.12 & 0.16 \\ 0.12 & 0.72 & 0.16 \\ 0.12 & 0.16 & 0.72 \end{pmatrix}$ .

Give the line matrix  $X_0$  and show that for all natural numbers  $n$ ,  $X_{n+1} = X_n T$ .

4. Assume that  $T = P D P^{-1}$ , where  $P^{-1} = \begin{pmatrix} \frac{3}{10} & \frac{37}{110} & \frac{4}{11} \\ \frac{1}{10} & -\frac{1}{10} & 0 \\ 0 & \frac{1}{11} & -\frac{1}{11} \end{pmatrix}$  and  $D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.6 & 0 \\ 0 & 0 & 0.56 \end{pmatrix}$ .

- Using a calculator, give the coefficients of the matrix  $P$ . It may be noted that they are whole numbers.
- Show  $T^n = P D^n P^{-1}$ .
- Give without justification the coefficients of the matrix  $D^n$ .

Note that  $\alpha_n$ ,  $\beta_n$ , and  $\gamma_n$  coefficients of the first line of the matrix  $T^n$  are as follows:

$$T^n = \begin{pmatrix} \alpha_n & \beta_n & \gamma_n \\ \dots & \dots & \dots \\ \dots & \dots & \dots \end{pmatrix}.$$

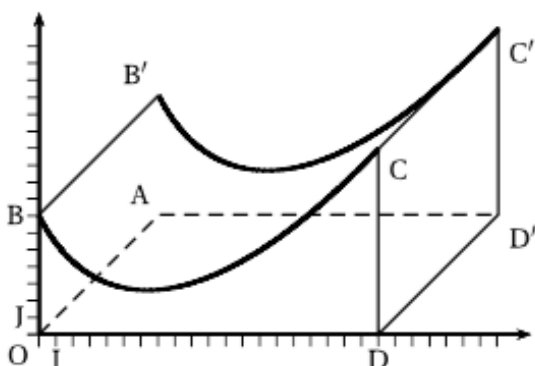
Assume that  $\alpha_n = \frac{3}{10} + \frac{7}{10} \times 0.6^n$  and  $\beta_n = \frac{37 - 77 \times 0.6^n + 40 \times 0.56^n}{110}$ .

Do not try to find the coefficients of the second or third lines.

- Recall that for all natural numbers  $n$ ,  $X_n = X_0 T^n$ .
  - Determine the numbers  $a_n$  and  $b_n$  using the coefficients  $\alpha_n$  and  $\beta_n$ . Deduce  $c_n$ .
  - Determine the limits of the sequences  $\{a_n\}$ ,  $\{b_n\}$ , and  $\{c_n\}$ .
  - On what vertex is the pawn most likely to be on after a large number of iterations of the random walk?

#### Exercise 4 (6 points)

For all candidates



A city council decided to install a skateboard ramp in a city park.

The drawing opposite provides a parallel perspective.

The quadrilaterals  $OAD'D$ ,  $DD'C'C$ , and  $OAB'B$  are rectangles.

Consider the face plane  $OBD$  with orthonormal system  $(O, I, J)$ .

The unit is the meter. The width of the ramp is 10 meters, that is  $DD' = 10$ , the length  $OD$  is 20 meters.

**The goal of the problem is to determine the area of the different surfaces to be painted.**

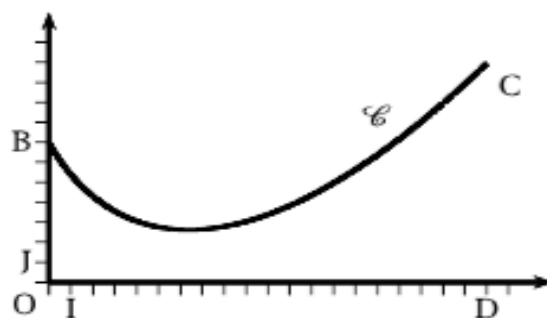
The profile of the skateboard ramp is modeled by an image of the function  $f$  defined on the interval  $[0, 20]$  as

$$f(x) = (x + 1) \ln(x + 1) - 3x + 7.$$

Note that  $f'$  is the derivative function of  $f$  and  $\mathcal{C}$  is the curve representative of  $f$  in the plane  $(O, I, J)$ .

### Part 1

1. Show that for all real numbers  $x$  in the interval  $[0, 20]$ , we have  $f'(x) = \ln(x + 1) - 2$ .
2. Deduce the changes of  $f$  on the interval  $[0, 20]$  and draw up a table of changes.
3. Calculate the slope of the tangent line of the curve  $\mathcal{C}$  at the abscissa 0.



The absolute value of the coefficient is called the slope of the skateboard ramp at the point B.

4. Assume that the function  $g$  defined on the interval  $[0, 20]$  by

$$g(x) = \frac{1}{2}(x + 1)^2 \ln(x + 1) - \frac{1}{4}x^2 - \frac{1}{2}x$$

has the derivative function  $g'$  defined on the interval  $[0, 20]$  by  $(x + 1) \ln(x + 1)$ .

$$g'(x) =$$

Determine an antiderivative of the function  $f$  on the interval  $[0, 20]$ .

### Part 2

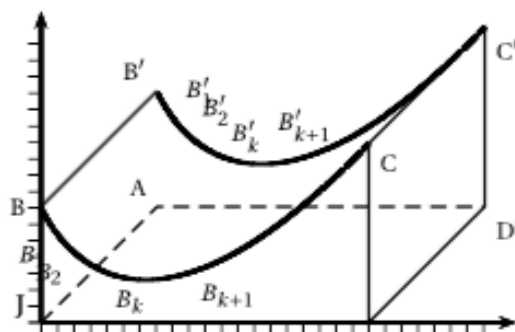
The three questions of this part are independent.

1. Are the following propositions exact? Justify your response.

$P_1$ : The height difference between the highest point and the lowest point of the ramp is at least equal to 8 meters.

$P_2$ : The slope of the track is close to two times greater at B than C.

2. We want to cover the four side faces of the ramp in a layer of red paint. The paint used covers a surface of  $5 \text{ m}^2$  per liter. Determine, within 1 liter, the number of liters of paint necessary.



3. We want to paint the curved face black, in other words the top surface of the ramp. In order to determine the approximate value of the area to paint, consider in the system (O, I, J) of the face plane the points  $B_k(k, f(k))$  for the variable  $k$  from 0 to 20. Thus,  $B_0=0$ .

Assume that you move along the curve  $C$  from  $B_k$  to  $B_{k+1}$  by the segment  $[B_k B_{k+1}]$ . Thus the area of the surface to paint will approach the sum of the areas of the rectangles of type  $B_k B_{k+1} B'_{k+1} B'_k$  (see figure).

- Show that for all whole numbers  $k$  from 0 to 19,  

$$B_k B_{k+1} = \sqrt{1 + [f(k+1) - f(k)]^2}.$$
- Complete the following algorithm in order to make an estimation of the area of the curved portion.

|           |                                                                                |
|-----------|--------------------------------------------------------------------------------|
| Variables | S: real<br>K: natural                                                          |
| Function  | f: defined by $f(x) = (x+1)\ln(x+1) - 3x+7$                                    |
| Treatment | Let S equal 0<br>For K between ... and ...<br>S takes the value ...<br>End For |
| Output    | Display ...                                                                    |

## 6.2- Analysis of Sample Test 1

The Georgia Department of Education has created the Georgia Standards of Excellence as the basis for the Georgia high school curriculum. Georgia high school students are required to fulfill four units of credit in mathematics that include Coordinate Algebra or Algebra I or the equivalent, Analytic Geometry or Geometry or the equivalent, Advanced Algebra or Algebra II or the equivalent, and one additional unit to be selected from the list of GSE/AP/IP/dual enrollment designated courses. Course options from the list of GSE/AP/IP/dual enrollment designated courses include but are not limited to Pre-Calculus, Calculus, AP Statistics, AP Calculus AB, AP Calculus BC, Multivariable Calculus, and Advanced Mathematical Topics (<https://www.gadoe.org/Curriculum-Instruction-and-Assessment/Curriculum-and-Instruction/Documents/Mathematics/Georgia-High-School-Graduation-Mathematics-Requirements-2015-2016.pdf>).

The Georgia Department of Education has identified six mathematics sequence options for grades six through twelve that layout the routes students may take as they progress through their mathematics courses through middle and high school. Of these six sequences, only four of them would allow students to take a calculus course and only one of them would allow students to take anything higher course such as Multivariable Calculus or Advanced Mathematical Topics during their high school education.

The test shown above was given to students taking the mandatory high school exit exam for Mainland France. There are different tests depending on the high school track followed by the students. The tests used in our analysis are for student registered for the track S. Students who graduate with a high school degree from the track S are eligible to go to college in majors such as mathematics, physics, chemistry, any engineering field, or accounting.

This exam has four exercises. Exercise 1 has two parts. Question 1 of Part 1 covers exponential probability distributions, which are not covered in the Georgia high school standards. This means that a student graduating from a high school in the state of Georgia will not be able to solve Question 1 of Part 1. Question 2 of Part 1 asks students to calculate probabilities on a normal distribution, which is covered in Georgia high schools under the standard MGSE9-12.S.ID.4. Georgia high school students would be able to complete this section of Part 1. Part 2 of Exercise 1 covers conditional probability. Georgia high school students would be able to complete Questions 1 and 2 under the standards MGSE9-12.S.CP.3, MGSE9-12.S.CP.6, MGSE9-12.S.CP.7, and MGSE9-12.S.CP.8, but would not be able to construct the confidence interval that is required to complete Question 3 of Part 2 unless they chose AP Statistics as their fourth mathematics course.

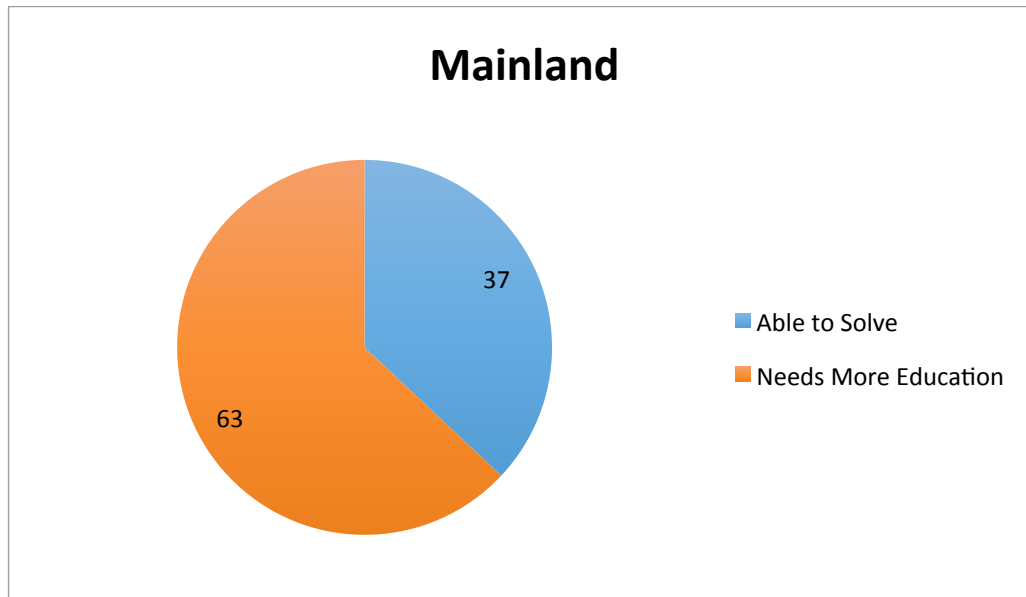
Only students who pursued Multivariable Calculus in high school would be able to complete Exercise 2, which covers lines and planes in a three-dimensional coordinate system. Although not a part of the Georgia Standards of Excellence, students who take Multivariable Calculus in high school would be able to complete Exercise 2 under the standard MMCA1.

There are two Exercise 3s in this exam. The high school degree track S gives the option for students to select an emphasis in math, physics, or chemistry. The first Exercise 3 is reserved for students who did not choose mathematics as their focus of study. The material covered in the first Exercise 3 about complex numbers is covered in the Georgia Standards of Excellence MSGE9-12.N.CN.3, MSGE9-12.N.CN.4, MSGE9-12.N.CN.6, and MSGE9-12.N.CN.7. A Georgia high school student would be able to complete this exercise.

Exercise 3 is for students who have selected track S with emphasis in mathematics. This exercise requires students to apply concepts about number theory, matrices, and limits of sequences. There are no standards or courses in the Georgia high school curriculum that cover number theory or limits of sequences. While the Georgia Standards of Excellence include some work with matrices, they do not require as rigorous work as needed for this portion of the exam. A Georgia high school student would not be able to complete this exercise.

Exercise 4 covers derivatives and integration, which is not covered in the Georgia Standards of Excellence. A Georgia high school student would not be able to complete this exercise without taking Calculus or AP Calculus as their fourth mathematics course option during their high school career.

After analyzing this test, I found that a high school student graduating in the state of Georgia should be able to solve 37% of this test. This percentage includes Exercise 1, Exercise 2, Exercise 3 for track S who did not have an emphasis in mathematics, and Exercise 4. The other 63% will be achieved if they take Calculus I, Calculus II, Calculus III, Number Theory, and an advanced Probability course in college.



### 6.3- Sample Test 2 (Pondicherry, April 17, 2015)

#### Exercise 1 (4 points)

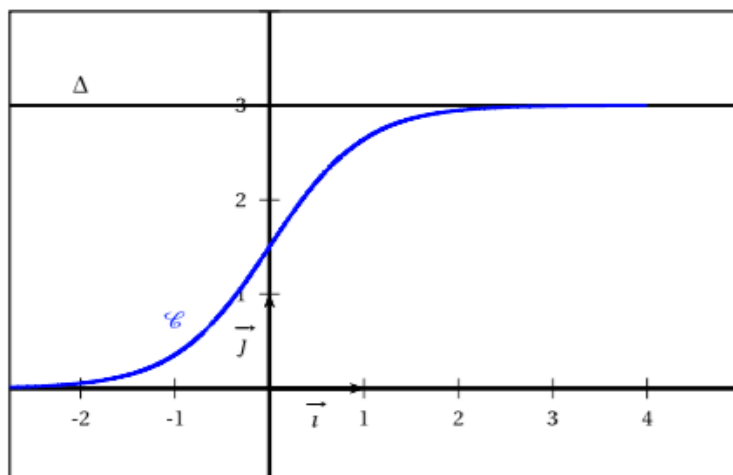
For all candidates

#### Part A

Let  $f$  be a function on  $\mathbb{R}$  by

$$f(x) = \frac{3}{1+e^{-2x}}.$$

On the figure below, in an orthogonal system  $(O, \vec{i}, \vec{j})$  is the curve  $\mathcal{C}$  of the function  $f$  and the line  $\Delta$  of the equation  $y = 3$ .



1. Show that the function  $f$  is strictly increasing on  $\mathbb{R}$ .
2. Justify that the line  $\Delta$  is the asymptote of the curve  $\mathcal{C}$ .
3. Show that the equation  $f(x) = 2.999$  gives a unique solution  $\alpha$  on  $\mathbb{R}$ . Determine an approximation of  $\alpha$  with an error of 0.001.

### Part B

Let  $h$  be the function defined on  $\mathbb{R}$  by  $h(x) = 3 - f(x)$ .

1. Justify that the function  $h$  is positive on  $\mathbb{R}$ .
2. Let  $H$  be the function defined on  $\mathbb{R}$  by  $H(x) = -\frac{3}{2} \ln(1 + e^{-2x})$ . Show that  $H$  is an antiderivative of  $h$  on  $\mathbb{R}$ .
3. Let  $a$  be a strictly positive real number.
  - a. Give a graphic interpretation of the integral  $\int_0^a h(x) dx$ .
  - b. Show that  $\int_0^a h(x) dx = \frac{3}{2} \ln \left( \frac{2}{1 + e^{-2a}} \right)$ .
  - c. Let  $D$  be the set of points  $M(x, y)$  of the defined plane by
 
$$\begin{cases} x \geq 0 \\ f(x) \leq y \leq 3 \end{cases}$$
 Determine the area, in unit area, of the domain  $D$ .

### Exercise 2

5 points

For all candidates

### Part A

Let  $\{u_n\}$  be the sequence defined by its first term  $u_0$  and, for all natural numbers  $n$ , by the relation

$$u_{n+1} = au_n + b \text{ (where } a \text{ and } b \text{ are nonnegative real numbers and } a \neq 1).$$

For all natural numbers  $n$ ,  $v_n = u_n - \frac{b}{1-a}$ .

1. Show that the sequence  $\{v_n\}$  is geometric with common ratio  $a$ .
2. Deduce that if  $a$  belongs in the interval  $[-1, 1]$ , then the sequence  $\{u_n\}$  has the limit  $\frac{b}{1-a}$ .

### Part B

In March 2015, Max buys a foliage plant measuring 80 cm. Someone informs him to cut it every year in the month of March by a quarter of its height. The plant grows 30 cm over the course of the following 12 months. As soon as he returns home, Max cuts his plant.

1. What will be the height of the plant in March 2016 before Max cuts it?
2. For all natural numbers  $n$ , denote  $h_n$  as the height of the plant before it's cut in March of the year  $(2015 + n)$ .
  - a. Justify that for all natural numbers  $n$ ,  $h_{n+1} = 0.75h_n + 30$ .
  - b. Use your calculator to guess the variations of the sequence  $\{h_n\}$ .  
Prove this conjecture (you may use reasoning by recurrence)
  - c. Is the sequence  $\{h_n\}$  convergent? Justify your response.

### Exercise 3

6 points

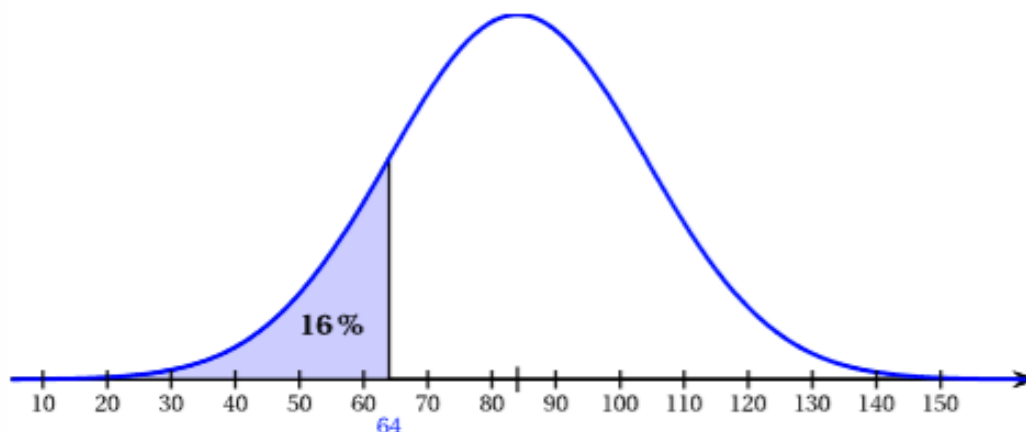
For all candidates

Parts A and B may be treated independently.

#### Part A Study of the life expectancy of household appliances

Statistical studies modeled the life expectancy, in months, of a type of dishwasher by the random variable  $X$  following the normal distribution  $\mathcal{N}(\mu, \sigma^2)$  of  $\mu = 84$  and standard deviation  $\sigma$ . In addition,  $P(X \leq 64) = 0.16$ .

The graph of the density function of  $X$  is given below.



1. a. Using the graph, determine  $P(64 \leq X \leq 104)$ .  
b. What might be an approximation of  $\sigma$ ?

2. Note that  $Z$  is a random variable defined by  $Z = \frac{X-84}{\sigma}$ .
  - a. What is the law of probability followed by  $Z$ ?
  - b. Justify that  $P(X \leq 64) = P(Z \leq \frac{-20}{\sigma})$ .
  - c. Deduce the value of  $\sigma$ , round to the thousandths place.
3. In this question, consider that  $\sigma = 20.1$ .  
Round the probabilities to the thousandths place.
  - a. Calculate the probability that the life expectancy of the dishwasher is between 2 and 5 years.
  - b. Calculate the probability that the dishwasher has a life expectancy of more than 10 years.

### Part B Study of the extended warranty of El'Ectro

The dishwasher has a free warranty for the first two years. The company El'Ectro offers their customers an extended warranty of 3 additional years.

The statistical studies conducted on **the customers who chose the extended warranty** showed that 11.5% invoked the warranty.

1. We chose at random 12 customers among those who chose the extended warranty (since the number of customers is high we can assume it is a random choice with replacement).
  - a. What is the probability that exactly 3 of the customers invoked the extended warranty? Explain the precise process the law of probability used. Round to the thousandths place.
  - b. What is the probability that at least 6 of the customers invoked the extended warranty? Round to  $10^{-3}$ .
2. The offer for the extended warranty is as follows: for 65 additional euros, El'Ectro will reimburse the customer the initial value of the dishwasher, its 399 euros, **if an irreplaceable failure occurs between the beginning of the third year and the end of the fifth year**. The customer may not invoke this extended warranty if the failure is repairable.  
We chose at random a customer among the customers who have subscribed for the extended warranty and denote  $Y$  as the random variable that represents the algebraic gain in euros fulfilled for the customer by the company El'Ectro, thanks to the extended warranty.
  - a. Justify that  $Y$  takes the values 65 and -334, then give the probability distribution of  $Y$ .
  - b. Is this extended warranty offer financially advantageous for the company? Justify.

### Exercise 4

5 points

**For candidates who did not follow the teaching specialty**

Let ABCDEFGH be a cube with side 1.

In the system  $(A, \overrightarrow{AB}, \overrightarrow{AD}, \overrightarrow{AE})$ , consider the points M, N, and P of respective coordinates  $M(1, 1, \frac{3}{4})$ ,  $N(0, \frac{1}{2}, 1)$ ,  $P(1, 0, -\frac{5}{4})$ .

1. Place M, N, and P on the figure given in the appendix.
2. Determine the coordinates of the vectors  $\overrightarrow{MN}$  and  $\overrightarrow{MP}$ .  
Deduce that the points M, N, and P are not aligned.
3. Consider algorithm 1 given in the appendix.

- a. Complete by hand the algorithm with the coordinates M, N, and P given above.
  - b. What is the displayed result by the algorithm? What can you deduce about the triangle MNP?
4. Consider algorithm 2 given in the appendix. Complete it such that you can test if a triangle is right or isosceles.
5. Consider the normal vector  $\vec{n}(5, -8, 4)$  of the system MNP.
  - a. Determine a Cartesian equation of the plane MNP.
  - b. Consider the line  $\Delta$  passing through F and of vector direction  $\vec{n}$ . Determine a parametric representation of the line  $\Delta$ .
6. Let K be a point of intersection on the system MNP and the line  $\Delta$ .
  - a. Show that the coordinates of the point K are  $\left(\frac{4}{7}, \frac{24}{35}, \frac{23}{35}\right)$ .
  - b. We are given  $FK = \sqrt{27/35}$ .  
Calculate the volume of the tetrahedron MNPF.

#### Exercise 4

5 points

#### For candidates following the teaching specialty

The numbers of the form  $2^n - 1$  where  $n$  is a natural nonnegative number are called **Mersenne numbers**.

1. Let  $a$ ,  $b$ , and  $c$  be three nonnegative natural numbers such that  $\text{PGCD}(b, c) = 1$ .  
Prove, with the help of the Gaussian theorem, that:  
if  $b$  divides  $a$  and  $c$  divides  $a$  then the product  $bc$  divides  $a$ .
2. Consider the Mersenne number  $2^{33} - 1$ .  
A student uses their calculator and obtains the results below.

|                        |             |
|------------------------|-------------|
| $(2^{33} - 1) \div 3$  | 2863311530  |
| $(2^{33} - 1) \div 4$  | 2147483648  |
| $(2^{33} - 1) \div 12$ | 715827882.6 |

It confirms that 3 divides  $2^{33} - 1$  and 4 divides  $2^{33} - 1$  but 12 does not divide  $2^{33} - 1$ .

- a. How does this assertion contradict the result demonstrated in question 1?
  - b. Justify that in reality 4 does not divide  $2^{33} - 1$ .
  - c. Noticing that  $2 \equiv -1 \pmod{3}$ , show that in reality 3 does not divide  $2^{33} - 1$ .
  - d. Calculate the sum  $S = 1 + 2^3 + (2^3)^2 + (2^3)^3 + \dots + (2^3)^{10}$ .
  - e. Deduce that 7 divides  $2^{33} - 1$ .
3. Consider the Mersenne number  $2^7 - 1$ . Is it prime? Justify.
4. Given the following algorithm where  $\text{MOD}(N, k)$  represents the remainder of the Euclidean division of  $N$  by  $k$ .

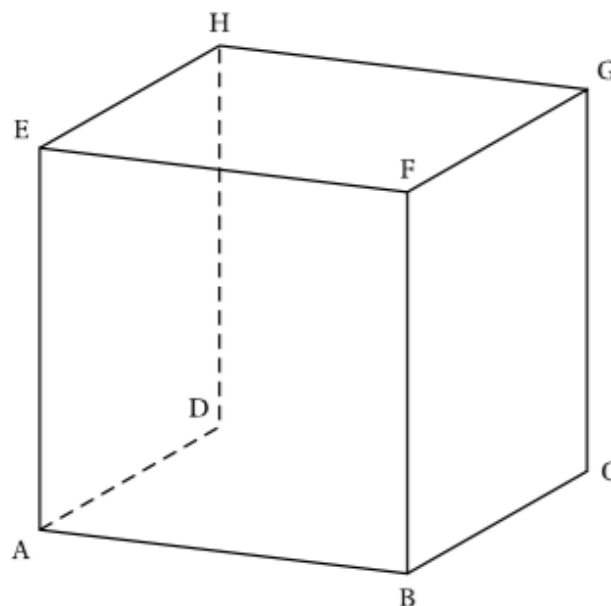
|                 |                                                                                                |
|-----------------|------------------------------------------------------------------------------------------------|
| Variables:      | $n$ natural number greater than or equal to 3<br>$k$ natural number greater than or equal to 2 |
| Initialization: | Ask the user for the value of $n$<br>Assign to $k$ the value 2                                 |

|            |                                                                                                                          |
|------------|--------------------------------------------------------------------------------------------------------------------------|
| Treatment: | As long as $\text{MOD}(2^n - 1, k) \neq 0$ and $k \leq \sqrt{2^n - 1}$<br>Assign to k the value k+1<br>End of As long as |
| Output:    | Display k.<br>If $k > \sqrt{2^n - 1}$<br>Use "Case 1"<br>If not<br>Use "Case 2"<br>End of If                             |

- What is the output if we enter  $n=33$ ? And if we enter  $n=7$ ?
- What represents case 2 for the Mersenne number studied?  
What does the number  $k$  display for the Mersenne number studied?
- What represents case 1 for the Mersenne number studied?

## Appendix

### Exercise 4: Candidates who did not follow the teaching specialty



### Algorithm 1

Take  $x_M, y_M, z_M, x_N, y_N, z_N, x_P, y_P, z_P$   
 $d$  equals  $x_N - x_M$   
 $e$  equals  $y_N - y_M$   
 $f$  equals  $z_N - z_M$   
 $g$  equals  $x_P - x_M$   
 $h$  equals  $y_P - y_M$   
 $i$  equals  $z_P - z_M$   
 $k$  equals  $d \times g + e \times h + f \times i$   
Show  $k$

### Algorithm 2 (to complete)

Take  $x_M, y_M, z_M, x_N, y_N, z_N, x_P, y_P, z_P$   
 $d$  equals  $x_N - x_M$   
 $e$  equals  $y_N - y_M$   
 $f$  equals  $z_N - z_M$   
 $g$  equals  $x_P - x_M$   
 $h$  equals  $y_P - y_M$   
 $i$  equals  $z_P - z_M$   
 $k$  equals  $d \times g + e \times h + f \times i$

## 6.4 Analysis of Sample Test 2

Georgia Department of Education has created the Georgia Standards of Excellence as the basis for the Georgia high school curriculum. Georgia high school students are required to fulfill four units of credit in mathematics that include Coordinate Algebra or Algebra I or the equivalent, Analytic Geometry or Geometry or the equivalent, Advanced Algebra or Algebra II or the equivalent, and one additional unit to be selected from the list of GSE/AP/IP/dual enrollment designated courses. Course options from the list of GSE/AP/IP/dual enrollment designated courses include but are not limited to Pre-Calculus, Calculus, AP Statistics, AP Calculus AB, AP Calculus BC, Multivariable Calculus, and Advanced

Mathematical Topics (<https://www.gadoe.org/Curriculum-Instruction-and-Assessment/Curriculum-and-Instruction/Documents/Mathematics/Georgia-High-School-Graduation-Mathematics-Requirements-2015-2016.pdf>).

The Georgia Department of Education has identified six mathematics sequence options for grades six through twelve that layout the routes students may take as they progress through their mathematics courses through middle and high school. Of these six sequences, only four of them would allow students to take a calculus course and only one of them would allow students to take anything higher course such as Multivariable Calculus or Advanced Mathematical Topics during their high school education.

The test shown below was given to students taking the mandatory high school exit exam in Pondicherry, India. There are different tests depending on the high school track followed by the students. The tests used in our analysis are for student registered for the track S. Students who graduate with a high school degree from the track S are eligible to go to college in majors such as mathematics, physics, chemistry, any engineering field, or accounting.

Exercise 1 of this exam consists of two parts, Part A and Part B. Georgia high school students would be able to complete Question 1 of Part B, which asks students to justify that a function is positive for all real numbers, under the standard MGSE9-12.F.IF.7e. Georgia high school students would need to take either Calculus or AP Calculus to be able to complete all other questions of Exercise 1.

Exercise 2 consists of two parts, Part A and Part B, both of which cover material on geometric sequences and limits of sequences. Students would be able to complete Question 1 of Part A, Question 1 of Part B, and Question 2a of Part B. However, the Georgia Standards of Excellence do not cover the concepts required to complete the questions about sequences and limits of sequences. A Georgia high school student would need to take AP Calculus to successfully complete this exercise.

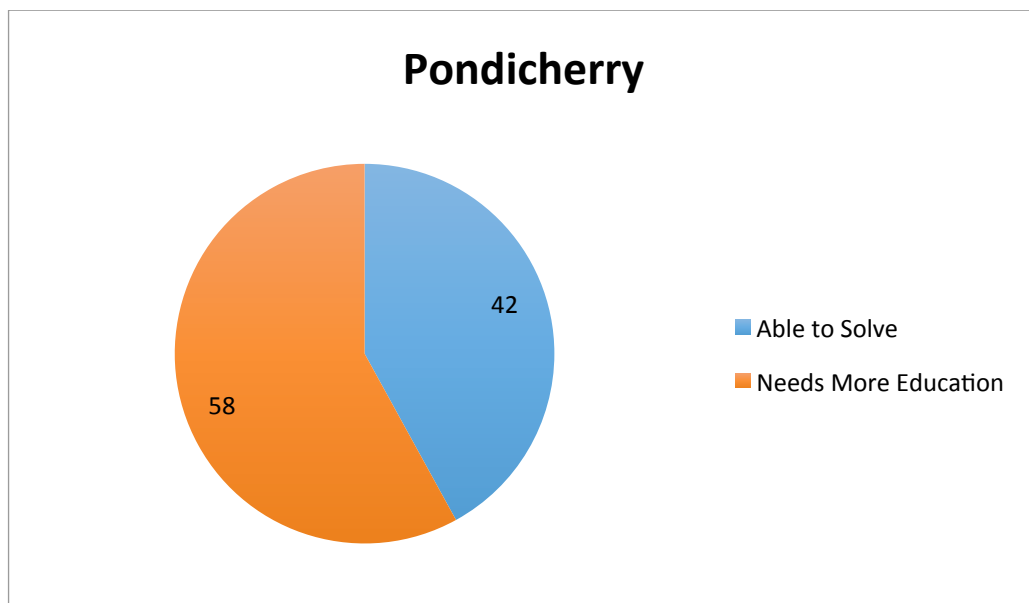
Part A of Exercise 3 requires students to have an understanding of the normal probability distribution. Georgia high school students would be able to complete this section from the Georgia Standards of Excellence MGSE9-12.S.ID.4, MGSE9-12.S.MD.4, and MGSE9-12.S.MD.7. Students would not be able to complete Part B of Exercise 3, which covers binomial distributions. A Georgia high school student would need to take AP Statistics to complete Part B of this exercise.

There are two Exercise 4s in this exam. The high school degree track S gives the option for students to select an emphasis in math, physics, or chemistry. The first Exercise 4 is reserved for students who did not chose mathematics as their emphasis. This exercise covers points, lines, and vectors in a three-dimensional coordinate system. A Georgia high school student could complete Question 1, Question 2, and Question 3a under the standard MGSE9-12.N.VM.2. They would need to take Multivariable Calculus to be able to complete the rest of this exercise.

The second Exercise 4 is for students who chose mathematics as their focus of study. This exercise covers Mersenne primes, modular arithmetic, and the definition of divides, which is not covered in the Georgia Standards of Excellence. There are no courses on the list of GSE/AP/IP/dual enrollment courses as a fourth mathematics course option that covers this material.

After analyzing this test, I found that a high school student graduating in the state of Georgia should be able to solve 42% of this test. This percentage includes Exercise 1, Exercise 2, Exercise 3, and Exercise 4

for track S who did not have an emphasis in mathematics. The other 58% will be achieved if they take Calculus I, Calculus II, Calculus III, Number Theory, and an advanced Probability course in college.



## **7- Sample Georgia High School Exit Exam**

The last Georgia High School Graduation Test (GHS GT) was given in the spring semester of 2015. In March 2015, Georgia governor Nathan Deal signed a House Bill that no longer required high school students to pass the GHS GT to earn a high school diploma. As a result the test was no longer administered. At this time, there is no statewide test that students must pass in order to earn a high school diploma. Below is a sample copy of the exam.

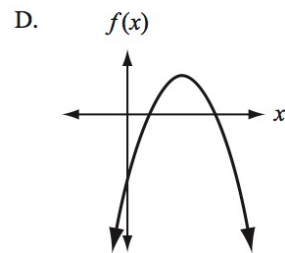
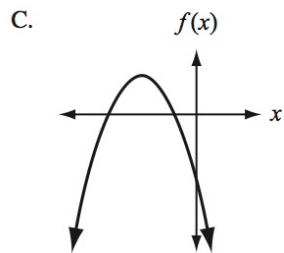
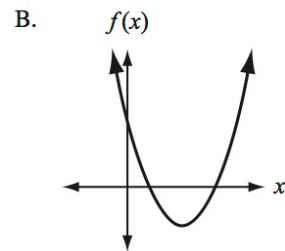
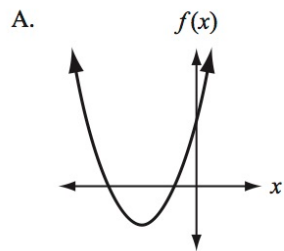
1. The function  $g(x) = |x - 5|$  is the result of a translation of the function  $f(x) = |x|$ . How is the graph of  $g(x)$  different from the graph of  $f(x)$ ?
  - A. The graph of  $g(x)$  is 5 units up.
  - B. The graph of  $g(x)$  is 5 units down.
  - C. The graph of  $g(x)$  is 5 units to the left.
  - D. The graph of  $g(x)$  is 5 units to the right.
2. Which expression is equivalent to  $\sqrt{32b^{16}}$ ?
  - A.  $16b^4$
  - B.  $16b^8$
  - C.  $4b^4\sqrt{2}$
  - D.  $4b^8\sqrt{2}$

## Mathematics

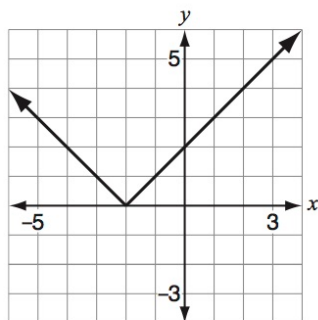
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3. A student is studying the quadratic function  $f$ . The student determined that  $f(0) > 0$ . The student also determined that  $f$  has two real roots,  $a$  and  $b$ , such that  $a < b < 0$ .

Which graph could represent  $f$ ?



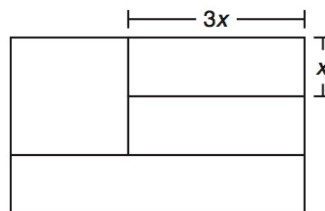
4. A student drew this graph of the function  $f$ .



Which value of  $x$  satisfies  $f(x) = 1$ ?

- A.  $x = -3$
- B.  $x = -2$
- C.  $x = 1$
- D.  $x = 3$

5. The Georgia state flag consists of a square and three rectangles. Each rectangle has the same width,  $x$ . The length of each of the two smaller rectangles is equal to  $3x$ , as shown in this diagram.



The area of this particular Georgia flag is 60 square feet. What is the length of  $x$ ?

- A. 2 feet
  - B. 4 feet
  - C.  $2\sqrt{5}$  feet
  - D.  $2\sqrt{15}$  feet
6. Which of these true statements also has a true inverse?
- A. If the product of integers  $a$  and  $b$  is odd, then both  $a$  and  $b$  are odd.
  - B. If  $x$  is a multiple of 6, then  $x$  is an even number.
  - C. If  $a$  and  $b$  are consecutive integers, then the sum of  $a$  and  $b$  is odd.
  - D. If  $p$  is negative, then  $|-p|$  is positive.

## Mathematics

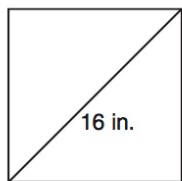
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7. One interior angle of a pentagon has a measure of  $120^\circ$ . The other four interior angles are congruent to each other.

What is the measure of one of the four congruent angles?

- A.  $30^\circ$
- B.  $60^\circ$
- C.  $105^\circ$
- D.  $195^\circ$

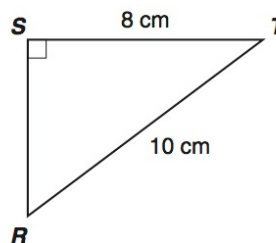
8. This diagram shows a square tile with a diagonal length of 16 inches.



What is the approximate area of the tile?

- A. 64 square inches
- B. 128 square inches
- C. 181 square inches
- D. 256 square inches

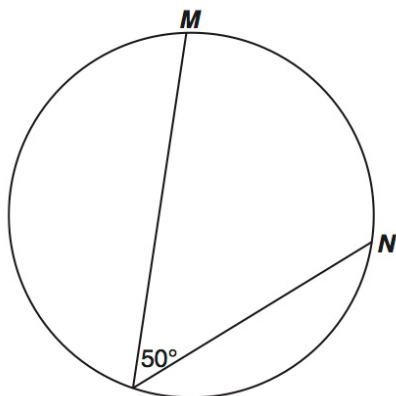
9. A student drew this diagram of a right triangle.



What is the value of the tangent of  $\angle R$ ?

- A.  $\frac{4}{5}$
- B.  $\frac{5}{4}$
- C.  $\frac{3}{4}$
- D.  $\frac{4}{3}$

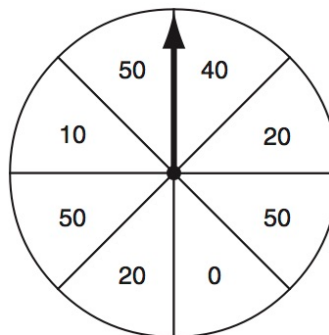
10. This circle has a radius of 9 inches.



What is the approximate length of arc  $MN$ ?

- A. 8 in.
  - B. 16 in.
  - C. 23 in.
  - D. 35 in.
11. There are 10 students who applied for internships. Only 3 positions are available.
- How many different groups of 3 can be selected from the 10 students?
- A. 30
  - B. 120
  - C. 720
  - D. 1000

12. Jerry will spin the arrow on this spinner once.



What is the expected value of Jerry's spin?

- A. 20
- B. 25
- C. 30
- D. 50

## Mathematics

13. A group of 100 people were asked to rate two restaurants on a scale from 0 to 10. The results are represented by this double box-and-whisker plot.



Which statement is correct?

- A. The range of ratings is greater for Restaurant A than for Restaurant B.
- B. The range of ratings is greater for Restaurant B than for Restaurant A.
- C. The interquartile range of ratings is greater for Restaurant A than for Restaurant B.
- D. The interquartile range of ratings is greater for Restaurant B than for Restaurant A.

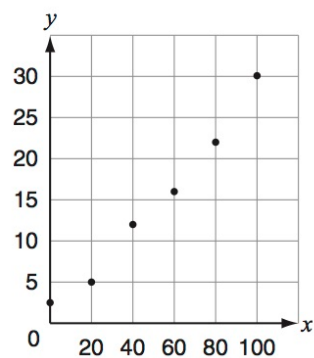
14. A marketing researcher asked a random selection of adults to rate two different brands of toothpaste on a scale from 1 through 10.

- Brand X had a mean rating of 7.5 with a standard deviation of 1.1.
- Brand Y had a mean rating of 6.8 with a standard deviation of 2.0.

Based on the data, which statement **must** be true?

- A. The data is more dispersed for Brand X.
- B. The data is more dispersed for Brand Y.
- C. The range of the data is greater for Brand X.
- D. The range of the data is greater for Brand Y.

15. A student drew this scatter plot.



Which equation best models the data in the scatter plot?

- A.  $y = 0.1x + 3$
- B.  $y = 0.3x + 1$
- C.  $y = x + 0.3$
- D.  $y = 3x + 0.1$

**Georgia High School Graduation Tests  
Mathematics Formula Sheet**

Below are the formulas you may find useful as you work the problems. However, some of the formulas may not be used. You may refer to this page as you take the test.

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|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p style="text-align: center;"><b>Area</b></p> <p>Rectangle/Parallelogram <math>A = bh</math></p> <p>Triangle <math>A = \frac{1}{2}bh</math></p> <p>Circle <math>A = \pi r^2</math></p> <p>Trapezoid <math>A = \frac{1}{2}(h)(b_1 + b_2)</math></p> <p style="text-align: center;"><b>Circumference</b></p> <p><math>C = \pi d \quad \pi \approx 3.14</math></p> <p style="text-align: center;"><b>Volume</b></p> <p>Rectangular Prism/Cylinder <math>V = Bh</math></p> <p>Pyramid/Cone <math>V = \frac{1}{3}Bh</math></p> <p>Sphere <math>V = \frac{4}{3}\pi r^3</math></p> <p style="text-align: center;"><b>Surface Area</b></p> <p>Rectangular Prism <math>SA = 2lw + 2wh + 2lh</math></p> <p>Cylinder <math>SA = 2\pi r^2 + 2\pi rh</math></p> <p>Sphere <math>SA = 4\pi r^2</math></p> <p style="text-align: center;"><b>Trigonometric Relationships</b></p> <p><math>\sin(\theta) = \frac{\text{opp}}{\text{hyp}}; \cos(\theta) = \frac{\text{adj}}{\text{hyp}}; \tan(\theta) = \frac{\text{opp}}{\text{adj}}</math></p> <p style="text-align: center;"><b>Special Right Triangles</b></p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p><b>45°–45°–90° Triangle</b></p> </div> <div style="text-align: center;"> <p><b>30°–60°–90° Triangle</b></p> </div> </div> | <p style="text-align: center;"><b>Pythagorean Theorem</b></p> <p><math>a^2 + b^2 = c^2</math></p> <p style="text-align: center;"><b>Quadratic Formula</b></p> <p><math>x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}</math></p> <p>Standard Form <math>ax^2 + bx + c = y</math></p> <p>Vertex Form <math>a(x - h)^2 + k = y</math></p> <p style="text-align: center;"><b>Expected Value</b></p> <p><math>E(x) = \sum_{i=1}^n x_i p(x_i)</math></p> <p>the sum of each outcome multiplied by its probability of occurrence</p> <p style="text-align: center;"><b>Permutations</b></p> <p><math>{}_n P_r = \frac{n!}{(n-r)!}</math></p> <p style="text-align: center;"><b>Combinations</b></p> <p><math>{}_n C_r = \frac{n!}{r!(n-r)!}</math></p> <p style="text-align: center;"><b>Interquartile Range</b></p> <p>the difference between the first quartile and third quartile of a set of data</p> |
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## 8- Conclusion of Our Work

To become a high school teacher in France, it is required to complete a degree in the subject you wish to teach before going on to a two-year teacher training school, or ESPE (*Écoles Supérieures du Professorat et de l'Éducation*). Students also have to complete a competitive exam after their first year at an ESPE. Mastery of the subject is required before mastery of teaching. Comparatively, while teaching certification in Georgia requires at least a bachelor's degree, it does not have to be subject specific. Many university students major in education with a concentration on the grade level, for example middle grades or early childhood. However, some students may choose to obtain a bachelor's degree in a subject area and complete the education program requirements by obtaining a master's degree in teaching.

Studying the table comparing the age of students in each grade level, students in France spend more time in their middle school and three years on a more focused curriculum in high school that is based on which track they want to pursue. The high school curriculum in Georgia is broader, requiring students to complete courses in math, science, language arts, literature, and often a foreign language. The curriculum is structured for college preparation. It is in college that Georgia students will begin to take more courses in a selected area.

A glaring difference in the mathematical expectations of students in France and Georgia can be seen when comparing the different graduation exams. The French *baccalaureat* is a written exam, which requires students to show their work and write explanations and proofs. Subjects in the exam include Calculus I, Calculus II, Calculus III, Number Theory, and an advanced Probability course. The Georgia High School Graduation Test consists of 15 multiple-choice questions. However, these exams are in line with the structure of schooling in each area. The French are expected to concentrate on a particular field during their high school years, whereas the Georgia curriculum supplies students with a broader high school education.

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