Pre-service mathematics teachers' use of the mathematics register

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This paper describes a small-scale study examining one facet of pre-service mathematics teachers' knowledge for teaching. While mathematical knowledge for teaching has received extensive attention from researchers with regards to pre-service teachers, there has been little research on the mathematics register proficiency of this cohort. In this study, we examine a group of pre-service mathematics teachers' mathematics register during a peer-teaching lesson. The authors adapt Rowland's Knowledge Quartet as a framework for the study, by conceptually aligning mathematics register proficiency to each of the four dimensions: foundation, transformation, connection and contingency. Findings indicate pre-service mathematics teachers lack understanding of the significance of the mathematics register and its role in mathematics teaching for eliciting mathematical understanding for students. There was evidence of over-reliance on the everyday register and a lack of fluency with regards to the mathematics register in practice. There is an exigent need for greater emphasis to be placed on developing pre-service mathematics teachers' mathematics register proficiency during initial teacher education, not only to improve pre-service teachers' own knowledge and understanding, but also their ability to facilitate mathematics register proficiency in their future students.

Introduction

Mathematics is both a written and spoken language, the knowledge of which is essential for mathematical understanding to occur (Usiskin, 2015). Kenney (2005) compared learning mathematics to learning a foreign language consisting of specialised nouns and verbs, with the challenge that "it is learned almost entirely at school and is not spoken at home" (p. 3). In teaching secondary school mathematics, Rowland (2012) suggested that the challenge for teachers is not only the complexity of mathematical concepts being taught and the range of prerequisite concepts required, but also the "sophistication of the semiotic systems with which they are represented" (p.21).

In this paper, we consider the mathematics register of a group of pre-service mathematics teachers in an Irish university. We are interested in their ability to exemplify and facilitate mathematics register fluency in a peer-teaching setting, and how their use (or lack of use) of the mathematics register can provide an insight into these pre-service teachers' preparedness for teaching mathematics. Recent studies have focused on the pre-service teachers' knowledge and understanding of specific mathematical topics and the development of this knowledge in a variety of learning settings (see Da Ponte & Chapman, 2008). Research has also examined pre-service teachers' pedagogical approaches, professional identity and beliefs, for example Goos (2005) and Lofstrom and Pursiainen (2015). There is less evidence of pre-service teachers' mathematics register proficiency, although the development of pre-service mathematics teachers' use of appropriate mathematical language is identified as an important aspect of teacher education (Cramer, 2004; Ball, Thames & Phelps, 2008).

Mathematics register

Learning the language of mathematics is an integral aspect of learning mathematics (Chapman, 1993; Schleppegrell, 2007). This has been widely recognised by researchers, stemming from the work of Halliday (1978) who discussed the function of the 'mathematics register'. Halliday defined the mathematics register as:

> ... a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings. We can refer to a 'mathematics register', in the sense of the meanings that belong to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself), and that a language must express if it is being used for mathematical purposes. (p. 195, 1978)

Thus, the mathematics register is much more than the technical language or symbols we use; it also includes the meaning behind these words and structures, the way we communicate them, and the context in which they are used (Chapman, 1993). Ferrari (2004) suggested that mathematical language is designed "not to promote interpersonal communication, but rather to provide an effective, well-organized picture of mathematical knowledge and to support the application of algorithms" (p. 386). It is a language that is discipline-specific and necessary for the accurate representation and application of mathematical knowledge, but by itself is not necessarily conducive to classroom interactions. When learning, teaching, or using mathematics we tend to switch from the regular language we use – the 'everyday register' – to the mathematics register, to read, write, verbalise, represent and importantly, to understand and convey mathematical meaning. Alternating between the everyday and mathematics registers can lead to a loss of mathematical meaning and misunderstanding of mathematical concepts (Moschkovich, 2003; Morgan & Alshwaikh, 2012). For example, everyday language may not be sufficiently precise or explicit to describe a mathematical object or concept.

In addition, the mathematics register often 'borrows' vocabulary from the everyday register, which take on a special meaning when used in a mathematical context, e.g. prime, improper, origin. According to Ferrari (2004), linguistic misunderstanding or misconceptions can hinder students' mathematical progress, particularly when the relationship between mathematical communication and mathematical thinking is considered. This is not to say that the everyday register has no place in the mathematics classroom. The relationship between the everyday and mathematical registers is complex, and teachers often use everyday language to support students in making sense of mathematical concepts. The mathematics teacher is required to contextualise and personalise the mathematics in a manner appropriate to the experience of their students (Ball & Bass, 2002). Pre-service mathematics teachers need to be adequately prepared to negotiate these semiotic systems and to develop their students' mathematics register as part of their classroom context to facilitate mathematical understanding.

Theoretical framework

Subject matter knowledge and pedagogical content knowledge have remained key elements of various models for teacher knowledge since first proposed by Shulman (1986), with subsequent connotations for mathematics teachers' required knowledge of mathematics register and their effective facilitation of mathematics register fluency in learners. For example, Shulman's (1986) definition of pedagogical content knowledge as "the most useful ways of representing and formulating the subject that makes it comprehensible to others" (p. 9) has implications for teachers' mathematics register proficiency. As we are interested in framing the mathematics register within pre-service mathematics teachers' preparation for teaching mathematics in this research study, the authors employed Rowland's *Knowledge Quartet* (2007) in our conceptual framework, due to its suitability as a tool for analysing the mathematical knowledge of pre-service teachers in a classroom or 'teaching' situation.

Although the Knowledge Quartet was originally developed as a tool for analysing primary teachers' mathematical knowledge, it has also been employed in the secondary context (Rowland et al., 2011; Rowland, 2012). Rowland's framework is classified by four dimensions:

- *Foundation:* This includes teachers' mathematical knowledge, and their beliefs and understanding of this knowledge. The foundation dimension is seen as key in informing teachers' pedagogical decisions and strategies employed.

- *Transformation:* This dimension refers to the "knowledge-in-action", both in relation to teachers' planning and their actual teaching. It incorporates the pedagogical strategies employed by teachers in the classroom and especially the representations teachers use to impart mathematical knowledge, be they analogies, examples, explanations or demonstrations.

- *Connection:* The focus of this dimension is on the teacher's coherency between topics, between lessons and within lessons. An important aspect of this dimension is the teacher's awareness of relative connections within and across topics and the relative difficulty of various topics and tasks.

- *Contingency:* The contingency dimension refers specifically to teachers' ability to respond to classroom situations that arise but are not planned for. Thus, it is essentially a measure of teachers' ability to 'think on their feet'. (Rowland, 2007; Rowland et al., 2011).

It is evident that the mathematics register is integral to each of the Knowledge Quartet dimensions, though perhaps not specifically referenced. Thus, we conceptually map the mathematics register to the four dimensions as illustrated in Table 1. Our understanding of mathematics register is adopted from Halliday (1978), with the mathematics register considered to comprise mathematical terminology, vocabulary, symbols and structures together with the expression of these words, symbols and structures in a manner that is mathematically meaningful.

Table 1: Mathematics register conceptualised within Rowland's Knowledge Quartet

Dimension	Foundation	Transformation	Connection	Contingency
Mathematics register component	Knowledge and understanding of the mathematics register, especially mathematical terminology and vocabulary.	Evidence of planning for mathematical language in a classroom setting.	Consistency in mathematics register, especially terminology and vocabulary within lessons and between lessons and across different mathematics topics.	Ability to interpret students' register in line with the mathematics register.
Example in practice	An asymptote is a term specific to mathematics, used to describe a line or curve that approaches a given curve arbitrarily closely.	The wording of an explanation of a given mathematical concept is clearly planned prior to the lesson in order to elicit mathematical meaning.	When referring to a negative number, using the term 'negative' consistently, rather than alternating between 'negative' and 'minus'.	If a student refers to the 'average' of a sample, being able to interpret 'average' as referring to the 'mean' or 'median' as appropriate, within the specific context.
Mathematics register component	Awareness of differences between the everyday register and the mathematics register.	Use of representations and analogies that elicit mathematical meaning for students.	Awareness of difficulties students may experience with the mathematics register.	Ability to facilitate an adherence to the mathematics register during classroom interactions.
Example in practice	The word 'prime' as used in the everyday register may refer to 'the most important' or 'the best quality'. In mathematics, when referring to numbers, the word 'prime' has its own meaning as 'a number only divisible by itself and one'.	Representation and analogies of multiplication should be in the context of objects that can be multiplied. For example, multiplying length by width of an object yields an area, which has mathematical meaning, but multiplying apples by oranges, has no mathematical meaning.	Awareness of the introduction of any new terminology in a mathematical topic being taught and allowing sufficient explanation and association with relevant prior knowledge, e.g. equivalent fractions would be introduced as the appropriate terminology for two fractions of equal value by building on students' prior knowledge of the concept of equality.	If a student refers to one number being 'bigger than' another number, the teacher reinforces the terminology 'greater than' and the appropriate symbol '>'.

Table 1 illustrates with examples how the mathematics register is key in each of the Knowledge Quartet dimensions, and thus constitutes an integral aspect of a teacher's knowledge for teaching mathematics. We employed this conceptual mapping as a framework in our study to identify possible gaps in a group of pre-service mathematics teachers' mathematics register proficiency and to determine the source of any deficiencies in terms of their preparation for teaching mathematics.

Method

This research is part of a study examining a group of pre-service mathematics teachers' peer-teaching lessons. The participants (N=13) comprised an entire class group in their third year of a four-year degree program in an Irish University, at the end of which they will be qualified to teach Mathematics at post-primary level (ages 12 to 18) in Ireland. The participants were aged 19 to 22 years, with 6 males and 7 females. The group can be said to be high achievers as they were in the top 10% of the country in terms of academic achievement when they finished post-primary education, based on national data (CAO, 2018). Prior to this study, the pre-service teachers had completed one other mathematics pedagogy module, six mathematics content modules (two on algebra, two calculus, one science mathematics and one linear algebra), and an eight-week post-primary school placement teaching mainly Junior Cycle students (the first three years of post-primary education in Ireland, ages 12-15 years).

Table 2: List of assigned topics from the number strand

Pre-service teacher (coded)	Mathematical concept
PST8	Multiplying a negative number by a positive number
PST11	Multiplying a negative number by a negative number
PST12	Addition of fractions
PST1	Multiplication of fractions
PST2	Division of fractions
PST4	Adding two negative numbers
PST3	The associative property for multiplication
PST5	The commutative property for addition
PST10	Finding a fraction of a number
PST9	$(x^m)(x^n) = x^{m+n}$
PST13	Multiplying decimals
PST7	Inverse proportion
PST6	Converting fractions to percentages

As part of their second mathematics pedagogy module, each pre-service teacher was assigned by the module leader (also one of the researchers) a mathematical concept from the Junior Cycle Number strand of the Mathematics syllabus (DES, 2018), to teach for ten minutes to their peers. The Junior Cycle Mathematics syllabus is divided into five strands: statistics and probability; geometry and trigonometry; number; algebra; and functions. Table 2 lists the assigned topics from the number strand. Each pre-service teacher had three weeks to prepare his or her short lesson. The lesson was to be prepared as for a Junior Cycle class. These teaching tasks were observed by the three researchers and video-recorded. The pre-service teachers watched the video of their own teaching and wrote a 1,500-word reflection on their teaching, based on this observation.

Ethical approval was granted for the study by the university ethics board and all participants gave consent to participate in the study. The researchers watched the videos individually and using a grounded-theory approach (Glaser & Strauss, 1967), collectively compared their observations to identify key episodes of interest relating to each pre-service teacher's mathematical knowledge for teaching (Rowland, 2007). Based on the identified episodes, questions were posed to each pre-service teacher in individual semi-structured interviews conducted by the two researchers not involved in teaching the module. The purpose of the interviews was to clarify meaning behind the identified episodes within each pre-service teacher's teaching task. The interviews were transcribed for analysis. A common occurrence identified was the pre-service teachers' inaccurate use of mathematics language and lack of fluency with the mathematics register, leading to the following research questions:

1. What gaps can be identified in pre-service mathematics teachers' mathematics register proficiency?
2. How do these gaps relate to the pre-service teachers' mathematical knowledge for teaching?

The authors focus on identifying gaps in the pre-service mathematics teachers' mathematics register, as it is beyond the scope of this study to determine their mathematics register proficiency. The researchers qualitatively analysed the video data, interview transcripts and reflections to identify patterns relating to the pre-service teachers' mathematics register, with a focus on identifying inaccuracies in terminology, vocabulary, symbols and structures, as well as any lack of mathematical meaning in the explanation, representation or demonstration of mathematical concepts. Content analysis was employed to categorise gaps in the pre-service teachers' mathematics register in line with the theoretical framework of the study (Krippendorff, 2004).

However, due to the context of the pre-service teachers' lessons (ten-minute peer-teaching), there was less evidence of the pre-service teachers' mathematics register in relation to the connection and contingency dimensions, so we only report findings with regards to the foundation and transformation dimensions. For ease of analysis, examples were allocated to the dimension they were most strongly indicative of. Triangulation using multiple researchers and data sources was employed to add validity and reliability to this analysis (Johnson, 1997). As this was a small-scale study (in terms of participants, duration and context), the authors do not claim generalisability of findings. Rather we aim to gain an insight into an aspect of pre-service mathematics teachers' preparedness for teaching mathematics which has received inadequate attention to date.

Findings

Foundation

All pre-service teachers in our study demonstrated some gaps in their mathematics register in terms of the Foundation dimension. We describe some examples of these identified gaps in the pre-service teachers' mathematic register identified in the video recorded lessons.

Misuse of mathematical vocabulary

Analysis of the videos highlighted the misuse of certain basic mathematics terminology. An example of this is the term 'equation'. The word 'equation' was used by three pre-service teachers to describe a mathematical statement or expression (numbers and/or variables with operations). In teaching the division of fractions, one pre-service teacher stated: "So our equation is eight thirds divided by one third" (PST2). When questioned about her use and understanding of the term 'equation' during the interview, she recognised the error in her terminology, although it is not clear whether she has a complete understanding of the meaning behind 'equation':

Researcher (R):	You also refer to that [example] as the equation eight thirds divided by one third.
PST2:	Yeah.
R:	What's an equation?
PST2:	Okay. Yeah, no it's not an equation. Because an equation is something with an equals sign.
R:	Yeah so what would that [example] be?
PST2:	It would have just been an expression.

The pre-service teacher corrects her terminology – expression rather than equation. However her description of an equation appears to focus on the 'equals sign', with no mention of the need for two expressions that are equivalent. Similarly, another pre-service teacher summarises a word problem as an equation as described in the following extract from his teaching task video:

> So if they owed [Bank] ten thousand euro, you want to figure out how much they're actually going to save, how much they're not actually going to spend. So by the end of the lesson we're going to hopefully actually be able to do out that equation. (PST11).

The pre-service teacher uses the term 'equation' as a synonym for 'problem' or when there is a need to find a solution to a question. The colloquial term "to do out" the equation is also synonymous with carrying out procedural steps suggesting a procedural focus in this pre-service teacher's thinking. When asked about his understanding of the word equation in the interview, the pre-service teacher gives an over-simplified definition and sounds uncertain about his knowledge.

Researcher (R):	What's an equation?
PST11:	An equation … so it's when there's an equ- … I suppose when there's an equals sign and numbers on either side.
R:	Okay and what's an expression?
PST11:	Expression. When there is an equals sign is it?
R:	When there's what?
PST11:	When there isn't an equals sign?

The pre-service teacher is unsure of the difference between an equation and an expression and again focuses on the existence of the equals sign, appearing to guess the required answer. These examples of participants' misuses of the word 'equation' are indicative of a lack of understanding of the significance and meaning of this basic mathematics terminology which contributes to their inaccurate mathematical language use while teaching.

Simplification of terminology

At times, 11 of the 13 pre-service teachers used terminology that adhered more to the everyday register than the mathematics register. One example of this is a pre-service teacher's labelling of improper fractions as 'top-heavy fractions'. When asked about this terminology in the interview, he explains his reasoning as:

> Well if students are looking at it, especially if younger students are looking at a fraction, the denominator is usually more than the top when you're looking at a normal fraction. More values like say, a half, a third. But if the top of it is heavy … If you think of it as a top-heavy fraction, like the way I would have – say for instance if they were only learning fractions I would be saying to students if you've something top heavy what's wrong with it? It's obviously too heavy on top so you take off what isn't heavy to make it balanced and leave it over to the side. (PST12).

The pre-service teacher's explanation is not entirely lucid, particularly his reference to "normal" fractions and the evidence of procedural emphasis when describing how to convert an improper fraction to a mixed fraction – "take off what isn't heavy and leave it over to the side". Yet, his analogy of top-heavy does have some meaning in the context of younger students. However, rather than using the idea of a fraction being top-heavy as an analogy, he replaces the correct terminology with this analogic phrase. This is evidenced in the fact that he cannot remember the correct terminology for an improper fraction when asked in the interview:

Researcher (R):	Is there any other way that we refer to a top-heavy fraction? Is there any other terminology?
PST12:	There is another term … And there's literally no point in me thinking of it because I won't remember it.

In his attempt to simplify his teaching of this concept to students, the pre-service teacher adopts a register that is more in line with everyday speech and resultantly, forgets the appropriate mathematical terminology. There appears to be an inability to consolidate both the everyday and mathematical registers, both in his own knowledge and in the teaching context.

Understanding of terminology

A third illustration of the pre-service teachers' foundational knowledge with regards to the mathematics register is the use of terminology in their teaching which they reference but are unable to explain. This was the case for three of the pre-service teachers in this study. For example, in her teaching of the multiplication of fractions, a pre-service teacher displayed a formal proof on a *PowerPoint* slide – see Figure 1. She does not explain the proof line by line but states: *"If we work our way down, you can see by the definition of division, by the associative and commutative property, we can work our way down and we're going to get it at the end"* (PST1). Not only is the explanation incoherent, furthermore, she ignores the line with the phrase 'uniqueness of multiplicative inverse'.

Formal proof

Essentially we are looking that for any integers a,b,c,d where $b \neq 0$, $d \neq 0$, $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

$\frac{a}{b} \times \frac{c}{d} = \left(a \times \frac{1}{b}\right) \times \left(c \times \frac{1}{d}\right)$........definition of division

$\frac{a}{b} \times \frac{c}{d} = a \times \left(\frac{1}{b} \times c\right) \times \frac{1}{d}$........associative property

$\frac{a}{b} \times \frac{c}{d} = a \times \left(c \times \frac{1}{b}\right) \times \frac{1}{d}$........commutative property

$\frac{a}{b} \times \frac{c}{d} = (a \times c) \times \left(\frac{1}{b} \times \frac{1}{d}\right)$........associative property

$\frac{a}{b} \times \frac{c}{d} = ac \times \frac{1}{bd}$........uniqueness of multiplicative inverse

$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$........definition of division

Figure 1: Multiplication of fractions

The researcher asked this pre-service teacher to explain the proof in the interview during which she stated she didn't understand the phrase 'uniqueness of multiplicative inverse'. The lack of understanding of the phrase in this instance is a clear signal that the pre-service teacher lacks knowledge of mathematics terminology and does not fully understand the proof she used in her teaching task. Presenting the proof failed to elicit mathematical meaning and understanding, however, due in part to the pre-service teacher's lack of knowledge and understanding of the mathematical terminology and hence her knowledge and understanding of the concept.

Beliefs about the mathematics register

A final point from the analysis relates to the belief aspect of the foundation dimension (Rowland, 2007), specifically, to one pre-service teacher's belief about fractions which is indicative of a lack of understanding of the mathematics register. In discussing the use of the division of fractions in a real-world context with the class, she made the following statement:

> The point I was trying to make is it's kind of hard to come up with [real life] examples because we don't really see fractions in real life anymore. We kind of more focus on using decimals instead. For example, two euro fifty cents, we don't say that's two and a half euro anymore. It's two point five, or like you know. So that's one thing to consider. Fractions aren't really a way of communicating anymore. (PST2)

The researcher followed up on this statement in the interview.

Researcher (R):	So on watching the video [of her own teaching], do you still agree with [the statement]?
PST2:	I do. Because I don't know, people don't seem to use fractions the way they use decimals. They, well the same thing, but do you know? I don't know, people just prefer to use decimals. I just thought … I don't know … I'd point that out in the presentation. That fractions are kind of lesser used …
R:	Right. So why do you think that we still teach them in schools?
PST2:	Obviously because you need them.
R:	For?
PST2:	For … No I get what you mean now. But … I can't put my finger on it now.

The pre-service teacher appears to believe that fractions are an obsolete aspect of mathematics, particularly in an everyday context. Despite being within one year of being a qualified mathematics teacher, she is unable to give any reason why fractions are taught in secondary schools or provide one instance of where we encounter fractions in everyday life. Her belief appears to be connected to an overly simplistic view that the mathematics register applied to an everyday context should equate to the everyday register. If we do not say the word fraction or use a fraction in our everyday register, then fractions are not applicable to real world contexts. While this did not appear to be a view shared by her peers, the struggle between the everyday and mathematics register is a common occurrence in this study, highlighting the pre-service teachers' need for support in this regard.

Transformation

Planning explanations

One of the most commonly observed incidences from the teaching tasks was a lack of clarity and mathematical meaning in explanations of all the pre-service teachers. From further questioning of the pre-service teachers in the interviews and from reviewing their reflections, the reason for these incidences appeared to fall into two categories: lack of planning of explanations and lack of mathematical knowledge and understanding. We provide an illustrative example of the lack of planning aspect as most relevant to the transformation dimension of our theoretical framework. The pre-service teacher in this example was teaching inverse proportion and had presented the following problem on a *PowerPoint* slide to the class: *A farmer has enough corn to feed 30 hens for 9 days. How long will the corn last 18 hens?* The pre-service teacher began the solution for the problem by stating that 30 multiplied by 9 will equal 270. She used the letter z to represent the number of days it takes 18 hens to eat the corn and stated that z multiplied by 18 will also equal 270. The pre-service teacher asked the class to complete the problem individually. She then asked one of her peers for their answer and to explain how they found the answer, to which her peer stated, *'I did 270 divided by 18'*. The pre-service teacher reinforced this response saying: *'Yes, very good. It's just literally using algebra to manipulate the equation. So you're just bringing over your 18 and dividing it into 270 to get your answer'* (PST7).

This explanation of 'bringing over' to solve equations is one which is commonly taught to students in the Irish context without clarifying why. For students, this can lead to confusion as numbers 'disappear' from one side of an equation and 'appear' on the other side. For example, in the above problem, the pre-service teacher describes the procedure as "bringing" 18 from the left-hand side of the equation to the right-hand side of the equation rather than emphasising that both sides of the equation are being divided by 18. It is a key example of non-adherence to the precision and rigour that the mathematics register should endorse and is not conducive to eliciting mathematical meaning for students. Similarly, the description of 'dividing it into' is an inaccurate expression of the concept of division. The researchers queried this explanation with the pre-service teacher in the interview and the pre-service teacher was able to identify the imprecision of her explanation and to provide the correct explanation, indicating that a lack of knowledge was not the issue in this case. Rather, there appeared to be a lack of planning in terms of the pedagogical aspect of the lesson, particularly with regards to using the appropriate mathematics register in her explanations to develop deep mathematical understanding of the concept being taught. This was supported by the pre-service teacher's written reflection of her teaching task in which she admitted:

> If I were to redo this task I think that I would put a lot more thought and planning into my teaching task. I think I would spend more time in planning the actual words that I would be using when explaining the concept to the students. Looking back I did not actually think about the words I would use when explaining to the students all that much. (PST7)

This pre-service teacher clearly understood the mathematics involved in "bringing over 18" and a lack of planning can be attributed to the deficits in her mathematics register. This underlines the importance of not only teachers' knowledge of the mathematics register, but also their planning of explanations to adhere to the mathematics register in their teaching and to elicit mathematical understanding for students.

Representations and analogies

Analysis of the videos of the teaching tasks in this study found that while pre-service teachers recognised the importance of using representations and analogies in their teaching, for some (7 out of 13), their focus on 'simplifying' the mathematics procedure led to the mathematical conceptual meaning becoming negligible. One instance that exemplifies this finding was observed in a pre-service teacher's 'analogy' for multiplying two negative numbers. The pre-service teacher presented the following in his lesson:

> If you were say feeding your dog, if you were saying to eat, you're encouraging your dog to eat, so that's positive. So if I said 'do eat', that's a positive thing. Whereas if I said 'do not eat', putting a negative in with the positive, it's a negative as I'm actually telling the dog to not eat. But if I said to the dog, 'do not not eat', what am I actually trying to get him to do? Eat yeah. So that's just a simple way of getting through it. (PST11)

To explain why the product of two negative numbers is positive, the pre-service teacher uses the notion of two negatives making a positive in English grammar. However, he fails to adhere to the mathematical meaning of the concept in that his analogy does not involve multiplication or objects that can be multiplied. His own words "a simple of way of getting through it" are illustrative of his focus on simplifying the mathematical concept for students to be able to remember *what* to do, without abiding by the mathematical meaning of the concept so students understand *why* they need to do this. This is further clarified in the interview with the pre-service teacher when the researcher asked him about his analogy.

Researcher (R):	Your explanation of multiplying two negative numbers, you used the example of telling a dog to not not eat.
PST11:	Oh yeah I thought that really broke it down easy. But then, maybe it was kind of to the wrong group of people. Maybe it was for children or something like that. So I was saying there was two double negatives. So you say to not not do something then you're actually doing it. So that's how I was trying to break it down.

This extract emphasises the pre-service teacher's concentration on simplification of procedure rather than developing conceptual meaning. Furthermore, the pre-service teacher admitted he did not know how to represent the concept of multiplying two negative numbers and this lack of knowledge led to the use of an inappropriate analogy. It shows a lack of depth in terms of pedagogical thinking and similarly to the foundation dimension, a tendency to revert to the everyday register. In doing so, mathematical meaning was lost and the analogy became ineffective.

In analysing the teaching tasks, it was found that the pre-service teachers were more likely to use the mathematics register during introductory elements of the lesson only, in which they briefly recounted the origins of the notation or formulae relevant to their teaching task (something they may have experienced in university lectures). However, they tended to revert to the everyday register during representations, analogies or demonstrations which may indicate a lack of fluency in practical aspects of their teaching.

Discussion

In this paper, the researchers wanted to identify possible gaps in the mathematics register of a group of pre-service mathematics teachers during a peer-teaching lesson (research question 1). We also wanted to connect the gaps to these pre-service teachers' knowledge for teaching mathematics (research question 2), to understand the source of any deficiency with a view to informing future pre-service mathematics teacher education courses. To relate the mathematics register with knowledge for teaching mathematics, we conceptually mapped mathematics register proficiency onto Rowland's *Knowledge Quartet*, using the four dimensions as a theoretical base for analysing gaps in the pre-service teachers' mathematics register. However, due to the context of the study, only the foundation and transformation dimensions are examined.

In relation to the first research question – what gaps can be identified in pre-service mathematics teachers' mathematics register proficiency – all the pre-service teachers demonstrated gaps in their mathematics register to some extent. They demonstrated misuse and lack of understanding of certain basic mathematics terminology in this study and their explanations sometimes lacked mathematical meaning. These shortcomings in the pre-service teachers' mathematics register could lead to misunderstanding or misconceptions if they occur in a mathematics classroom and, as suggested by Schleppegrell (2007), could hinder students' mathematical progress. There was less adherence to the mathematics register particularly during practical demonstrations and representations. This may be indicative of a lack of fluency with the mathematics register in practice, with a tendency to revert to a more everyday register.

Schleppegrell (2007) averred that the construction of mathematical knowledge for students is particularly dependant on the teachers' oral language explanations and classroom interactions. As such, a lack of fluency with the mathematics register during these key aspects of their teaching may impede the pre-service teachers' ability to facilitate students in developing their mathematical knowledge. A common pattern of reliance on the everyday register in lieu of the mathematics register emerged in this study, with resultant loss of mathematical meaning in the pre-service mathematics teachers' teaching. This loss of mathematical meaning due to the use of the everyday register has also been highlighted in the literature (Moschkovich, 2003; Morgan & Alshwaikh, 2012). Schleppegrell (2007) highlighted the role of a mathematics teacher in exemplifying and facilitating students in the shift to the more academic mathematics register, while maintaining mathematical integrity. Our findings suggest that these pre-service teachers are not fully prepared to fulfil this role.

The second research question in this study is: How do these gaps relate to their mathematical knowledge for teaching? Previous research has discussed the importance of understanding, using and applying the mathematics register to develop mathematics proficiency (Riccomini et al., 2015). Therefore, the pre-service teachers' misuse and lack of understanding of some basic mathematical terminology is indicative of inadequate mathematics proficiency in terms of the foundation dimension. Some of the pre-service mathematics teachers in this study tended to equate the everyday register with the mathematics register applied to everyday life; for example, one pre-service teacher equated non-use of the word fraction in the everyday register to mean fractions are not relevant in a real-world context. Stubbs (1986) proposed that a person's language can convey their beliefs and attitudes about the subject, what he termed the 'prepositional attitude'. Our findings reveal a serious misconception in at least one of the pre-service mathematics teachers' prepositional attitudes to, and understanding of, the mathematics register, with possibly detrimental implications for their future teaching and their students' mathematical development, as suggested by previous research on linguistic misconceptions (Ferrari, 2004).

Gaps were also found in the pre-service teachers' mathematics register with regards to their planning of mathematical language and in the representations and analogies. Rowland (2012) suggested that for secondary teaching (the level our pre-service teachers are preparing to teach) the representation of abstract mathematical concepts is critical. All the pre-service teachers in this study were cognisant of the importance of representations and the importance of transforming knowledge to build learners' understanding. Yet, during the pre-service teachers' lessons, explanations sometimes lost mathematical meaning due to a lack of planning with regards to the language that should be used to evoke mathematical understanding of the concept being taught. Previous research by Lofstrom and Pursiainen (2015) identified a weakness in pre-service teachers' pedagogical knowledge and ability to relate pedagogical theory to practice. This could also be seen in this study as, at times, the pre-service teachers' focus in using analogies in their teaching was on simplifying a concept to procedure rather than maintaining the mathematical integrity of the concept. While a mathematics teacher is required to contextualise and personalise the mathematics in a manner appropriate to the experience of their students (Ball & Bass, 2002), this personalisation should not replace mathematical meaning.

Conclusion

This paper examines a group of pre-service mathematics teachers' mathematics register during peer-teaching lessons as part of their initial teacher education. While this paper describes a small-scale study in the Irish context, it highlights the need for greater emphasis to be placed on the development of pre-service teachers' mathematics register proficiency during initial teacher education. The importance of students' and teachers' mathematics register has been highlighted by previous researchers in developing students' mathematics proficiency (Ferrari, 2004; Kenney, 2005; Riccomini et al., 2015), but this study advocates the necessity of facilitating pre-service teachers' mathematics register fluency as an integral aspect of their mathematical knowledge for teaching.

We take a novel approach in developing a conceptual framework for examining pre-service mathematics teachers' facility with the mathematics register as a facet of their mathematical knowledge for teaching, through conceptually mapping mathematics register to the four dimensions of Rowland's *Knowledge Quartet* (2007). Our findings indicate that these pre-service mathematics teachers need to be supported in the development of the mathematics register proficiency required to exemplify and facilitate their future students' mathematics register development. We hypothesise that this would also facilitate the enhancement of the pre-service mathematics teachers' knowledge and understanding of mathematics and in turn, their teaching of mathematics. Further research is required on pre-service mathematics teachers' mathematics register proficiency/deficiency to examine pre-service mathematics teachers' use and facilitation of the mathematics register in a classroom context, and to investigate approaches to develop pre-service mathematics teachers' mathematics register fluency during their initial teacher education.

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