



Linneuniversitetet

Kalmar Västervik

Självständigt arbete II, 15 hp

Mathematics education in Colombia

*How education in mathematics is conducted in a
development country.*



Författare: Rebecka Rundquist
Handledare: Peter Markkanen
Examinator: Torsten Lindström
Termin: HT 2015
Ämne: Didactics of Mathematics
Nivå: Avancerad nivå
Kurskod: 4MAÄ2E

Matematikutbildning i Colombia

Hur matematikundervisning utförs i ett utvecklingsland.

Mathematics education in Colombia

How education in mathematics is conducted in a development country.

Abstract

This study aims to examine the education in mathematics in Colombia and by examining a few cases aspires to describe how education in mathematics in Colombia can operate and which patterns that are common in those cases. This was actualized by using methodological triangulation at three schools in Colombia. The data collection methods that were combined were: observations, interviews with teachers, interviews with students and interpretation of national standards, as well as other essential documents used in mathematics education in Colombia. An analytic framework was created from prior studies that were conducted in Latin America and also from well known pedagogical research across the world. The results of the study were many and they indicated, inter alia, that the students, teachers and other employees had different views of the lessons and classes in mathematics. Furthermore, common concept within education – in mathematics and in general – appeared to be completely non-existent to every party.

Keywords

Colombia, mathematics education, PISA, students' perception, didactics.

Thanks

I would like to thank my two contact persons in Colombia, Marta Osorio and Luis F Maldonado who helped me a lot with my study in Colombia. I would also like to thank the schools, teachers, principals, coordinators and students who participated in this study. I would also like to thank my mentor Peter Markkanen. Finally I would like to thank Linnaeus university who supported me with a MFS scholarship.

Rebecka Rundquist
Number of pages: 50.

Content

Introduction	1
Background of Colombia	1
The purpose of the study	3
Research questions	3
Theoretical background	4
Importance of mathematics	4
Cultural aspects – social norms	4
Sociomathematical norms	5
Teacher’s responsibility	5
<i>Subject matter knowledge</i>	6
<i>Pedagogical content knowledge</i>	7
Student’s perception of mathematics	9
Possible factors for success in childrens’ mathematics education	9
<i>Socioeconomic</i>	9
<i>Homework</i>	10
<i>Formative and summative assessments</i>	11
<i>Gender</i>	12
<i>School</i>	12
<i>Motivation</i>	12
<i>Attention</i>	12
<i>Stress</i>	13
Connections inbetween the variables	13
Method	14
Sample/Participants	14
Measuring instruments	15
<i>Interview students</i>	15
<i>Interview teachers</i>	15
<i>Observations</i>	15
Procedure	15
<i>Analysis</i>	16
Ethics	17
Quality requirement	18
Results	19
The view of mathematics	19
<i>Mathematics in their daily life</i>	19
<i>Mathematics is abstract</i>	20
<i>Mathematics consists of topics</i>	20
<i>The reason of learning mathematics</i>	21
Learning/Learners	23
<i>What motivates students</i>	23

<i>Variables of success</i>	24
<i>Importance of having a good teacher</i>	25
Didactics	26
<i>The structure of the lessons</i>	26
<i>Tools for teaching</i>	27
<i>Exercises and assignments</i>	29
<i>Development in mathematics and in other aspects of life</i>	31
Culture	32
<i>Social and cultural problems</i>	32
<i>Politics</i>	33
<i>The school</i>	34
<i>A group perspective</i>	34
<i>Interaction</i>	34
<i>Time</i>	35
Summary	36
<i>How do the teachers plan and conduct mathematics education?</i>	36
<i>What kind of didactic methods dominates the education in mathematics?</i>	36
<i>How do the students describe their perception of their education in mathematics?</i>	36
Discussion	37
Methodological discussion	37
<i>The interviews</i>	37
<i>The observations</i>	38
<i>Examining documents</i>	39
<i>Time and place of the study</i>	39
Result discussion	39
<i>The view of mathematics</i>	39
<i>Learners/Learning</i>	41
<i>Didactics</i>	43
<i>Culture</i>	45
<i>Summary</i>	48
Further research	49
References	51
Appendix	I
Appendix A Interview students.	I
Appendix B Interview teachers	II
Appendix C Unstructured interviews	III
Appendix D Observation guide	IV
Appendix E Accompanying letter	V
Appendix F National standards	VI

Introduction

This study presents Colombian society from the point of view of education. This aspect is essential in the development of the society, because education is believed to be a tool to get out of poverty (UNICEF, 2016). To create a better society and a better world it is beneficent to have goals to strive for. In the year 2000 all members in the United Nations (UN) approved eight goals, called “the millennium goals”. The millennium goals were developed to create a better world. Out of these eight goals one concerns education, stating that they would “Ensure that, by 2015, children everywhere, boys and girls alike, will be able to complete a full course of primary schooling” (UN, 2000). Unfortunately this goal is not yet achieved in Colombia and in 2015 new goals will be created; the working progress of creating these new goals is called “Post-2015”. The discussion in Post-2015 is leaning towards focusing the goals in education on a more qualitative approach (Millenniemaålen, 2015). This will demand more of each country that will partake in these goals. The 25:th of September in 2015 new goals were created and as in Post-2015 the goal concerning education is more qualitative than before. The new goals are called “Sustainable development goals” and consist of 17 goals. The goal concerning education states: “Ensure inclusive and quality education for all and promote lifelong learning” (UN, 2015).

The Organization for Economic Co-operation and Development (OECD) consist of 34 countries with the mutual goal of developing better policies for better lives (OECD, 2014). One factor of their work is analyzing results from students all over the world. The Programme for International Student Assessment (PISA) is a test designed to measure the knowledge and competence in reading, science and mathematics of fifteen-year old students. During 2012 PISA was administered among sixty-five countries around the world, in both industrialized countries and developing countries. Colombia, located in South America, is one of the development countries that participated in PISA. Colombia was placed sixty-second in mathematics on the PISA test due to the students’ results. They did however perform better in both reading and science on the PISA test (OECD, 2014). Colombia is also a member in the United Nations and will have to reach the Sustainable development goals. Today Colombia does barely meet the criteria’s of the millennium goals and by examining the PISA from 2012 one might suspect that the country could have some difficulties to achieve greater goals. It is therefore necessary to examine the conditions in Colombian society and education, based on prior and recent knowledge.

Background of Colombia

Colombia is a country that has been known for violence, corruption and illegal export and import of drugs and guns, but that is all about to change. Colombias’ government is on the verge of signing a peace agreement with FARC (Fuerzas Armadas Revolucionarias de Colombia – Ejército del Pueblo, translated: Revolutionary Armed Forces of Colombia—People’s Army) and thereby stopping an armed conflict that has been going on for over 40 years (Landguiden, 2012; Peaceworks, 2015).

Furthermore, there are many other things that are evolving in Colombia as well. The industry is blossoming and has shifted focus from coffee and agriculture to coal and oil. According to Colombian law, there has to be at least 30 percent of woman in parliament, 10 percent of Colombia’s budget should be directed to the school system and persecution based on religion or sexual preferences is forbidden. However, today there is only 19,9 percent women in parliament, the school system is underfunded and

the resistance from catholic church and other conservative organizations against homosexual, bisexual and transgender persons is still an issue (Landguiden, 2014).

To sum up, Colombia is developing and doing well but they still have a long way to go. The development that is needed also includes Colombia's schools system. In 1991 only 70.7 percent completed comprehensive school which in 2010 had increased to 91.1 percent. Since the Colombian school system is underfunded it is not uncommon that teachers do not receive their paychecks. Teachers' education is often below standard in rural areas in Colombia (Landguiden, 2014). In spite of Colombia's struggle, mathematic achievements is increasing at the same time costs for education is relatively stable but is still perceived as expensive (Gapminder, 2008). Since the year 2012 comprehensive school is free, there are however many other costs like books and uniforms etcetera (Landguiden, 2014).

In Colombia comprehensive school is divided into elementary school and secondary school, which consists of five years and four years respectively. These nine years of comprehensive school are compulsory. After comprehensive school students can attend high school/ mid secondary school which is non-compulsory and usually consist of two years (UNESCO-UNEVOC, 2014).

According to Post (2011) a lot of students in Colombia work during comprehensive school. In his study twenty-six percent of the children that participated worked after school with family and nine percent worked after school outside home. Post examined the impact of after school employment on academic achievements in mathematics, on children that were approximately twelve years old. His results showed a negative correlation with significant at the .05 level between work and mathematical achievements. The signs of the correlations and significance levels did not depend on whether such work was done at home with their families or outside home (Post, 2011).

Posts (2011) research indicated that many variables that could affect why children choose to work, or rather why their parents thought they should work. Post found that the relationship between school and community affects whether children are more motivated for work or education. Other variables that affects childrens' motivation are how they perceive themselves, costs concerning education and the quality of the school. Possible positive influences on children's motivation for their studies are beneficent rewards for attendants, free food at school and programs that helps with costs for uniforms and books. The teachers are also an important variable; they can have an impact on whether students succeed with their education. The author detected many variables that can have an impact on why children are employed while they are in comprehensive school, however, the main conclusion of Posts (2011) study was that academic achievements decreases when quality of the school decreases. Hence Post emphasizes that when schools improve their quality, the health of community and children will improve as well. Post also recommends that families of children with high attendants and good results should be rewarded (Post, 2011).

There are a lot of variables that could explain why children succeed in their education. Deutsh, Dumas and Silber (2013) attempts to analyze some chosen variables from data collected in the PISA test of 2006 to see how much of the variance that could be explained with individual efficiency as the outcome variable. Individual efficiency refers to the ability to set goals and follow through with speed and precision and what effects that process. They examined variables concerned children's home, school and

personality. Through multiple analysis three variables survived, which were: self-rated ability of students, gender and human capital of parents (level of parents education, what their main job were, how old they were and what language that was mainly spoken at home). These variables could explain the variances of individual efficiency in students in Colombia (Deutsh et al., 2013).

According to Deutsh et al. (2013) these variables say something about the conditions in Colombia. One of their first conclusions is that individual efficiency is pretty “fragile” and that “individual efficiency probably depends on the strength of the intergenerational link” (Deutsh et al., 2013, p. 256-257). They also suggest that the importance of the variable “gender” shows certain signs of discrimination toward girls. The entailment of Deutsh et al. (2013) study is that children’s education would benefit from programs that invests in human capital and fight against child labor. These programs can be designed to give poor families additional income and in exchange they send their children to school, this amount could be higher for the attendants of girl, to fight the gender discrimination (Deutsh et al., 2013). The author of this study think that it is alarming that the results for Colombia showed that the variable “Importance of learning efforts in the eyes of the student” only explained 8.5 percent of variance in individual efficiency within students in Colombia.

With these two studies in mind it does not seem strange that only 74 percent of children in Colombia start upper secondary school (Globalis, 2010). If future goals should be reachable there has to be a change in Colombian schools system and in education in mathematics. To change a system it is crucial that there are awareness of what needs to be changed and therefore knowledge about how the system operates is fundamental if changes should be introduced. As mentioned above, there are many variables that impacts whether a student succeed or fail with their education in mathematics, therefore a holistic view of the mathematics education is needed to implement any possible changes.

The purpose of the study

This study aims to examine the education in mathematics in Colombia and by examining a few cases aspires to describe how education in mathematics in Colombia can operate and which patterns that are common in those cases.

Research questions

How do the teachers plan and conduct mathematics education?

What kind of didactic methods dominates the education in mathematics?

How do the students describe their perception of their education in mathematics?

Theoretical background

This chapter describe different variables that research have shown has an impact on students' achievements in mathematics as well as students' perception of mathematics. This chapter also tries to connect findings in reaserch and se how they impact each other.

Importance of mathematics

Mathematical literacy is defined by OECD (1999) as: "The capacity to identify, to understand and to engage in mathematics and make well-founded judgements about the role that mathematics plays, as needed for an individual's current and future private life, occupational life, social life with peers and relatives, and life as a constructive, concerned, and reflective citizen" (p. 50). Therefore education in mathematics should aim to give students opportunity to develop mathematical literacy. According to this definition students can benefit in their everyday lives by using mathematical techniques (OECD, 1999). These techniques is what PISA focuses on in their tests (OECD, 2014).

According to Skott, Jess, Hansen and Lundin (2010) one can perceive mathematics as situated. This means that mathematics occurs with a context. Therefore when the content during mathematics lessons do not apply in real life situations the knowledge that the students have obtained can not be translated to use in such situations. Hence the teacher should search for existing elements of mathematic within their culture praxis, which should be used within the education of mathematics (Skott et al., 2010). This study will regard mathematics as situated. This implies that depending on what environment and culture teachers and students are located in, these elements mentioned above will vary according to the culture.

Cultural aspects – social norms

Different cultures and groups contain different social norms. This study defines social norms as the following.

"A system of norms specifies the normal pattern that individual actions should correspond to. Norms are in general closely connected with the social aspects of a person's life. Norms determine the margins within which a individual can acquire positions, that can be viewed as valuable for a group of society. The laws of a country express a part of the society's system of norms and other norms can be found in traditions and customs" (Nationalencyklopedin, 1990) (translated by author).

By this definition there is going to be a system of norms for each level (i.e. country, city, school, class and classroom). These norms may be very similar, but they can also vary in many aspects.

In a class there can be different social norms in play depending on the subject. This study will try to examine the ones in the education of mathematics, at different levels. This study will also make the same assumption as Yackel and Cobb (1996) which is that cultural and social processes are integrated in mathematical activity. In a classroom where the subject in hand is mathematics, there are more norms than social norms, besides those there are sociomathematical norms.

Sociomathematical norms

Sociomathematical norms consist of the patterns that are acceptable when one is conducting themselves within the subject of mathematics. Yackel and Cobb (1996) give a few examples of what counts as a sociomathematical norm. “For example, normative understandings of what counts as mathematically different, mathematically sophisticated, mathematically efficient, and mathematically elegant in a classroom are sociomathematical norms. Similarly, what counts as an acceptable mathematical explanation and justification is a sociomathematical norm” (Yackel & Cobb, 1996, p. 461). Sociomathematical norms are created and sustained in the same way as social norms are created and sustained, by social interactions in a group. Sociomathematical norms contains “current goals, beliefs, suppositions and assumptions of the classroom participants”(Yackel & Cobb, 1996, p. 460).

Sociomathematical norms do not depend on any tradition in teaching mathematics; they will be established by the teacher and the students anyway. Hence, it is a variable that is worth mentioning since it can occur regardless of which classroom one is located in. This imply that if a group/class is presented with a new teacher then there can be new sociomathematical norms (Yackel & Cobb, 1996). This can contribute to problems for students that are not used to different kind of sociomathematical norms.

As learners, students develop their understanding of mathematics. Yackel and Cobb (1996) suggests that they do so in three different steps. In the first step one might receive an explanation from a student that could be of social basis instead of mathematical. In the second step students can separate different types of mathematical reasons. In the final step students achieve the ability to use explanations as objects of reflection. Intellectual autonomy can be achieved by going through these steps. Students who are intellectually autonomous can evaluate their own intellectual capabilities in mathematics and interpret their own solutions, calculations and assumptions and therefore evaluate their own work. This is an ability that is perceived as a goal in the education of mathematics (Yackel & Cobb, 1996).

The main conclusion of Yackel’s and Cobb’s (1996) study is that teachers play a big part in establishing social norms as well as sociomathematical norms and they also play a big part in the mathematical quality of the classroom. Furthermore the teachers also have to guide their students through the process of reaching intellectual autonomy (Yackel & Cobb, 1996).

Teacher’s responsibility

Right now the education in mathematics has its major focus on mathematics as a process in which the student should have the opportunity to develop a set of competencies. There are many factors included when students are developing these competencies. One factor is the teacher, and according to Standards (National Council of Teachers of Mathematics (NCTM) (2000) students’ understanding, ability and confidence are shaped by the teaching students encounter in their education. This inflicts certain requirements of the teacher. Teachers need to have a profound knowledge about the mathematics they are teaching and must be able to use their knowledge with flexibility. Teachers should also understand their students as learners in mathematics, and they should be given support in developing their own knowledge (NCTM, 2000). Standards is created by the NCTM, which is the world’s largest

mathematics education organization. The main purpose of standards is to express NCTM's vision for appropriate mathematical goals for all students (NCTM, 2015).

Loewenberg Ball, Thames and Phelps (2008) developed a model named Mathematical Knowledge for Teaching (MKT) that specifies what knowledge the teachers should possess in mathematics and in general, to be able to be a good teacher. This can be divided into two categories, subject matter knowledge and pedagogical content knowledge. Some of these categories will be chosen as guidelines when the data in this study is analyzed. There will be more focus on the pedagogical part of the MKT model since it is more appropriate for the design of this study.

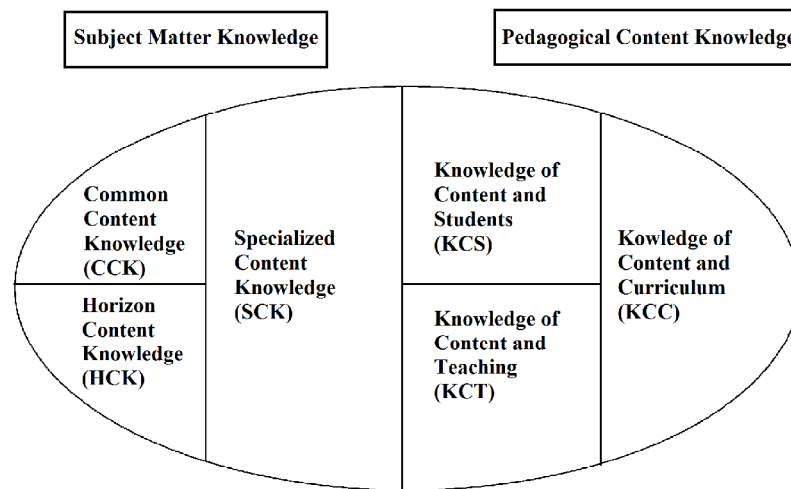


Figure 1 The model for Mathematics Knowledge for Teachers. Adapted from Loewenberg Ball et al. (2008).

Subject matter knowledge

Subject matter knowledge consists of common content knowledge, horizon content knowledge and specialized content knowledge. Common content knowledge is knowledge about the content that is taught to the students, horizon content knowledge focus on a broad knowledge that is needed to connect the different areas in mathematics and specialized content knowledge is a deeper knowledge in the different areas of mathematics (Loewenberg Ball et al., 2008). As mentioned earlier teachers knowledge should be profound and these three categorize can be viewed as guidelines in how to acquire that knowledge.

The knowledge that teachers have provides them with an image of what mathematics is. Yackel and Cobb (1996) describe the teacher's role as a representative of the mathematical community. This implies that the values, beliefs, knowledge and understanding that the teacher possesses is what he or she can transfer to his or hers students. There is a big possibility that students internalizes the view of mathematics that is presented to them through the teacher (Yackel & Cobb, 1996). Therefore it lies within teacher responsibility to have an accurate understanding of mathematics as a subject. It is also their responsibility to try to transfer this knowledge upon their students, so that they in turn acquire an accurate understanding of mathematics. Furthermore, teachers must evaluate student answers directly. This means listening to

their answers, analyze them and give appropriate responses. Without a profound knowledge about mathematics this becomes a process that can easily go wrong. As mentioned earlier teachers have to guide their students through the process of reaching intellectual autonomy (Yackel & Cobb, 1996). This is another reason why it is important for teachers to fully understand mathematics and the process within which individuals build their knowledge.

Pedagogical content knowledge

The other category in the MKT-model consists of knowledge of content and students, knowledge of content and teaching and knowledge of content and curriculum (Loewenberg Ball et al., 2008). These categories are about the relationship between knowledge of the mathematical content and different aspects of the teaching situation. The category; knowledge of content and students advise that the teacher has to have knowledge about the students in relation to the content. For examples which exercises that will motivate the students and which ones are easy or hard for the students. Knowledge of content and teaching combines knowing about teaching in relations to the content at hand, for example how to structure the lesson and which method to use when one is teaching. Knowledge about content and curriculum is needed to understand what should be in the course and how to reach the predetermined goals (Loewenberg Ball et al., 2008). This knowledge is needed to understand students as learners, to teach the correct content according to the curriculum and be able to use the flexibility to create terms in which students can learn and develop as learners. Skott et al. (2010) say that: "If teachers should be able to make it easier for students to learn he/she must try to understand the students' mathematical thinking" (p. 211) (translated by author). It is necessary to understand the students' mathematical thinking if the teacher wants to make the education individualized. Individualizing the education means that each student get diagnosed on his/hers prior knowledge and then the education is adapted to each and every students need. This is usually extremely difficult since there are often a lot of students in each class. Therefore teachers need to organize to make individualization possible, and even then it is not always possible (Löwing & Kilborn, 2002). This study argue that it is possible in many cases, since students might have similar experiences, if the teachers have the knowledge and the material to do it. Löwing and Kilborn (2002) argue that individualized mathematics education is important since different students have different problems and different ways of understanding mathematics. Therefore different students need different strategies to fully comprehend mathematics (Löwing & Kilborn, 2002). To be able to understand their mathematical thinking communication must occur during classes on a daily basis and to understand students as learners the teacher also needs to have a social relationship with their students (Skott et al., 2010).

Social relations between teacher and students

Svenningsson and Alvesson (2010) argue that different groups require different types of leadership. A teacher is often thought about as a leader and to be a good leader, the leader/teacher should have different kinds of leadership for different kinds of groups. Depending on what competencies and what level of commitment that students possesses as a group, different kinds of leadership will be required. If teachers should be able to meet this requirement they must be adaptable. In some classes teachers have to be more controlling and in other classes more supporting, sometimes it is appropriate for teachers to make all the decisions and in other cases it might be appropriate to include students in this process. Since teachers should act differently depending on what class he or she is teaching it is important that the teacher get to know his or hers students as a

class, to do so social bonds and social relations between teacher and student are required (Svenningsson & Alvesson, 2010).

Communication is one of the most fundamental founding-stone when a social bond is created and these bonds are continuously tested and recreated (Aspelin, 2010). According to Aspelin (2010) creating social bonds/relationships is a part of teachers responsibilities. Teachers are also responsible for students' cognitive, social and general development and for providing a student with a response that is based on a good communication. To be able to help students in their development communication is vital. Since the teacher need to know where students are in their development to further help them develop (Aspelin, 2010).

Communication

Communication occurs between teachers and students, but there are also other kinds of interactions where communication in mathematics is involved, inbetween students, between the student and teaching materials, within the student and also between the student and his/her parents (Löwing & Kilborn, 2002). Communication in mathematics include both words and symbols. The term is wide and incorporates many dimensions of communication. In short, it is being able to express own ideas and understand, interpret and analyze others ideas (Skott et al., 2010). Therefore it is important that student have opportunities to develop their ability to communicate in mathematics and it is one of the competences that the Standards mention (NCTM, 2000). When students develop their communication, not only do they acquire a skill in mathematics, they can also learn other aspects of mathematics in the process. Communication in mathematics can be perceived as a method which is anticipated to result in a deeper understanding of mathematic terms and procedures (Skott et al., 2010). It is also a opportunity for the teacher to accuire understanding of a students' mathematical thinking and an opportunity for students to understand how their peers think (Yackel & Cobb, 1996). In addition to that it is important that teachers communicate with clarity. According to Hattie's (2009) study teacher clarity had an effect size on students academic achievements at 0.75 and was ranked as eighth of all 138 possible variables, which indicates that it is essential for teachers to communicate with clarity. Clarity in speech is a prerequisite but clarity also lies in the organization, explanation, example and guided practice and assessment of student learning (Hattie, 2009). Hattie's (2009) study was purely quantitative and can therefore be used to create certain guiding principle, but it can not insure that this is the case everywhere. Hattie analyzed over 50 000 studies on different variables, but that does not insure that the different variables will have the same impact if one would to do the study again in a selected country. Therefore this study will have chosen parts of Hattie's study in mind but still be aware of the cutrual influences of Colombia and its people.

There are different kinds of communication that can occur in education in mathematics and in education in general. One of them is the initiation-reply-evaluation model (IRE) that has been, and still is, common in traditional teaching. In this model the communication takes form in three steps. Initially the teacher usually asks a question, receives a reply from the student and evaluate if the answer is correct. If not, the teacher repeats or simplifies the question until he or she have received the correct response (Skott et al., 2010). Skott et al. (2010) say that certain sorts of initiations enable certain types of responses. Many times when this model is used teachers tend to ask questions that merely call for students to finish the teacher's sentence. Some researchers believe that these kinds of questions and communication diminish the learning potential in the

activities and rather than asking these kinds of questions teachers should ask questions that aim towards commencing meta-processes. A meta-process is about more than just facts, it involves reflection concerning the student's thoughts and helps answer questions like how, why, what if, etcetera (Skott et al., 2010).

Depending on what kind of communication that exists in teacher-student conversations, in books and in other teaching materials, different messages will be sent to the students. These messages create different expectations of mathematics and mathematics education, which in turn creates sociomathematical norms, as mentioned earlier, and social norms that applies in mathematics education. These social norms include a lot of aspects, for example: what student's responsibilities are, how students work, how students learn and much more (Skott et al., 2010).

Student's perception of mathematics

How students perceive mathematics is essential. In general, prior education in mathematics was often focused on receiving the right answer from student regardless of how they achieved it. This is known as product and content standards. Nowadays the main focus has switched and now centers around the process standard that focus on how the student figured out the answer, instead of the answer itself. Students who are under the impression that the answer is what matters will tend to focus on that. However, if they believe that the process is more important, that is what their focus will be on when learning mathematics. This is important since the development of a lot of abilities occur in the process (Skott et al., 2010). As mentioned earlier these beliefs are many times transferred from the teacher (Yackel & Cobb, 1996).

The student's perception of his/hers own ability is also a factor in mathematical education as it can be a predictor of their future achievements. This phenomenon is called a self-fulfilling prophecy and this "occurs when people's erroneous expectations lead them to act towards others in a way that brings about the expected behavior, thereby confirming their original impression" (Holt, 2012, p.515). For example if a student believe that he/she is bad at mathematics. Then he/she might avoid more difficult assignment and will thereby not make any substantial progress, which will confirm the students perception of being bad at mathematics. Moreover, Post (2011) argues that how students perceive themselves could affect their motivation. This study agree with Post's research and conclusion, since Post's research is based on relevant data from well known organisations that are active in the selected countries. Furthermore, when one considers the phenomenon self-fulfilling prophecy which confirms one's perception of themselves, it might also be a rather logical conclusion that this would eventually affect one's motivation.

Possible factors for success in childrens' mathematics education

Socioeconomic

The variable of the student's socioeconomic status have been a predictor to many outcome variables within mathematics education. It has been argued that children that are from a lower socioeconomic background get stuck in a spiral if the school does not intervene (Deutsh et al., 2013). In Deutsh's et al. (2013) study it seems that the socioeconomic variable mostly relies on the mother's education, since it is more common that the woman spends a lot of time with the children. Therefore women will transmit their knowledge and beliefs, much like the teacher does in the classroom, and children will acquire their mother's mathematical thinking. According to Deutsh et al.

(2013) this creates a factor of a poverty trap. Deutsch's et al. (2013) study is limited to a quantitative approach but the data is processed in a proper manner where they include different aspects of childrens' learning environment. Therefore Deutsch's et al. (2013) study appear to have high validity and it is relevant to this study since it is limited to five Latin-American countries, including Colombia.

In a major study where over a hundred variables were analyzed socioeconomic status was ranked as 32:nd out of 138 with an effect size of 0.57. On the other hand this variable was also analyzed with achievements in different subjects as an outcome variables. This analyze showed that in mathematics the effect size was 0.70, which is obviously more than in general (Hattie, 2009). So if the outcome variable would had been achievements in mathematics socioeconomic status would have been ranked around 13:th place instead of 32:nd. This enable one to see that the socioeconomic status is a significant predictor of the education of mathematics in Hattie's study as well.

The level of socioeconomics of a students may be a reason for students to work after school time. As mentioned earlier this can affect the mathematical achievements of students negatively (Post ,2011). Students from lower socioeconomics classes are not always able to attain a better level of education, since better education costs more money (Landguiden, 2014).

Homework

Murillo and Martinez-Garrido (2014) conducted a study on homework and its impact on students' achivements in language and mathematics in sixteen countries in Latin America. Their results yielded to a great amount of information about habits of setting homework, the amount of time that teachers estimate for the homework, how often teachers correct the homework and how many teachers that build on homework in the classroom (Murillo, Martinez-Garrido, 2014).

Murillo and Martinez-Garrido (2014) highlight some of their results from thier research on homework. These results show that older students are more inclined to benefit from homework than younger students. They also present earlier results that indicate that setting homeworks "is a "powerful tool" for children's educational advancement and development"(Murillo & Martinez-Garrido, 2014, p. 664).

Although the use of homework has been indicated to be a great tool in mathematics education and have a positive impact on mathematical achievements, this is not always the case. If homework should have a positive effect they should be designed according to each student (Murillo & Martinez-Garrido, 2014). Once again this partly lies within the teacher's responsibilities. If homework is poorly-designed students' achievements in mathematics can decrease (Murillo & Martinez-Garrido, 2014). And even though homework is well-designed this does not benefit every student. Students with a lower socioeconomic status benefit more from doing homework that requires less time than students from a higher socioeconomic status (Murillo, Martinez-Garrido, 2014). One might speculate that this could have a connection with the "poverty trap" mentioned above. Thusly Murillo's and Martinez-Garridos' (2014) results indicate that homework in mathematics is very important. However they have to be designed to fit each student and the homeworks should also have a strong connection to the classroom.

Murillo's and Martinez-Garridos' (2014) study is based on relevant data but it very narrow, which can be both a benefit and a disadvantage. The hypotheses are narrow, which is beneficial since it becomes easier to answer them. The disadvantage of a narrow study like this is that it eliminates possible factors that could be of importance to the context. The conclusions they have are clearly based on their collected data, but it is not clear what the different variables contain which lowers the validity within the study. The data analysis is briefly mentioned as "multilevel model" with the accompanying statistics (Murillo & Martinez-Garrido, 2014, p. 667-668). To increase the study's reliability, a better description of the data analysis is needed. However, since the data is collected from well known sources and the study has a clear purpose, the study's results still becomes reliable.

Homework can yield into a lot of information about the students knowledge. This information could be processed and assessed in different ways. Two distinguished assessments are formative and summative assessments.

Formative and summative assessments

Summative assessments

Summative assessments are when information is gathered at the end of a unit of instruction, for example at the end of a semester, with the purpose of categorizing the performance of the students. For instance through a test such as midterm or final exams through which students get a grade based on their score. It could also be used to categorize students into groups like basic, proficient or advanced. Summative assessments are said to be the most visible test that students encounter in education today (Andrade & Cizek, 2010).

Formative assessments

Formative assessments are when information is gathered in the middle of a unit of instructions i.e. before a chapter or a semester is finished. Its purpose should be one of the following: "to identify the student's strengths and weaknesses; to assist educators in the planning of subsequent instruction; to aid students in guiding their own learning, revising their work, and gaining self-evaluation skills; and to foster increased autonomy and responsibility for learning on the part of the student" (Andrade & Cizek, 2010, p. 4). Formative assessments is more of a collaborative process in which both educator and students are involved. These kind of assessments can be gathered through a variety of different methods for example through tests, observations, class discussions, homework and many more (Andrade & Cizek, 2010). In Hattie's (2009) study providing formative evaluation was ranked as the 3:rd most powerful impact on academic achievements, with an effect size of 0.90. This shows that formative assessments could be a powerful tool in education.

To summarize summative assessments and formative assessments:

"Whereas the focus of summative assessments is on coarse-grained information for evaluation purposes with little direct application to instructional interventions for individual students, the focus of formative assessment is nearly opposite...the focus of formative assessment is on obtaining fine-grained information about student strengths and weaknesses in a nonevaluative context in which both the teacher and student see the information as valuable and useful for determining the subsequent activities that would be most beneficial for reaching predetermined educational goals" (Andrade & Cizek, 2010, p.15).

Gender

In Colombia there are inequalities between men and women, which affects the country and the people to a major extent. In the city things are more equal between the genders but the man is still in a position of power in relation to the woman (Landguiden, 2014). Deutsh et al. (2013) presented data which shows certain signs of discrimination toward girls. In their study individual efficiency was the outcome variable and the independent variable gender was accounted for 51 percent of the variance (Deutsh et al., 2013).

School

The private schools usually have tuition which provides them with a stable income. Public schools does not acquire a tuition and does therefore depend on the government. Since the school system is underfunded this has a big affect on the public schools (Landguiden, 2014). Because of the fact that the schools system is underfunded many schools can not reach an acceptable level of quality. Post (2011) states that the mathematical achievements decreases when the quality of the school decreases. He also states that “As schools improve, so will community and child health, making more visible the connection between the two”(Post, 2011, p. 275). According to him the quality of the school is a very important variable. In his study the quality was measured with twelve yes or no questions to see if “the school has the following items: electricity, running water, indoor plumbing, a telephone, sufficient bathrooms, a kitchen, a lunchroom, a library, nutritional feeding programmes, medical services, transportation, and free textbooks” (Post, 2011, 267). This study will use the same questions to evaluate the quality of the schools in this study’s sample. Additionally Post’s (2011) study indicated that the quality of the school affects students’ motivation.

Motivation

Vallerand, Pelletier, Blais, Brière, Senécal, Vallières (1992) claimed that motivation is one of the most important psycholoical concepts of education. Vallerand et al. (1992) refers to research that shows strong tendencies which indicates that motivation is connected to curiousness, endurance, learning and achievements. Those are all important components within education (Vallerand et al., 1992). Furthermore, Katz, Eilot and Nevo (2014) mention that motvation appears to be a good predictor of students educational experiences, like emotions during academic activities, feeling of competence and concentration. According to the authors mentioned above motivation can be essential if the student should be successful within their education (Vallerand et al., 1992; Katz et al., 2014).

Both Vallerand’s et al. (1992) research and Katz’s et al. (2014) research were built on reliable previous research and both have a well implemented method. However, research on other variables have shown that students’ success in education can rely on other things. Furthermore, there are other variables that have an impact on student’s motivation and in turn on their achievements, therefore the success might not rely on the actual motivation but the factors that are affecting it.

Attention

Attention or concentration depends on a couple of factors. One of those things is the working memory, which consist of among other things the short-term memory. The short-term memory’s maximun capacity is between five to nine units/objects, but if a stimulus is presented while the short-term memory is at work it may interrupt ones attention. This kind of attention is called selective attention, or concentration (Groome, 2010). During a class the student have to concentrate on a lot of things, they have to

take notes, listen to the teacher and keep their visual focus on the board. All of these processes can be interrupted by different stimulus. According to a hypothesis called streaming, a competing audio stimulus may interrupt one's short-term memory's capacity. This depends on how many variations there are in the audio. Audio with many variations interrupts the memory more than audio with few variations (Groome, 2010). If a person should concentrate on two things simultaneously these things should use different parts of the attention process, otherwise it is difficult to recall the information that has been processed. For example it is possible to concentrate on both audio and visual stimulus, but it is difficult to concentrate on two audio stimuli at the same time. Furthermore, the ability to concentrate on more than one thing diminishes when tasks become more difficult. Therefore one is easier interrupted when one is performing tasks where the difficulty level is higher (Groome, 2010). These are just some theories/hypotheses about how one's concentration works, thus there are many things that can have an impact on one's concentration. Another distraction can occur when the mind is troubled. Then it becomes occupied, ergo it becomes harder to concentrate, for example when a person is under a lot of stress (Holt, 2012).

Stress

Holt (2012) defines stress as "a pattern of cognitive appraisals, physiological responses and behavioural tendencies that occur in response to a perceived imbalance between situational demands and the resources needed to cope with them" (p. 609). Stressors are specific stimuli that are demanding or threatening situations. If a person is able to deal with these stressors they can actually be beneficial and enhance one's performance. If a person does not have the resources to deal with some stressors and does not find a way to cope with the stress that could result in a variety of negative consequences, both psychological and physiological (Holt, 2012). There are different kinds of stressors that students can encounter some examples are academic deadlines, academic failure and high demands that require major effort. Stressors outside the school also affect the students, for instance a serious illness, financial worries or the death or loss of a loved one. When stress proceeds during a long period of time a person normally reaches exhaustion. Therefore stress can diminish one's performance (Holt, 2012).

Connections inbetween the variables

The variables above are important both on their own but also in relation to the other variables since many appear to affect one another. That is why it is important to try to analyze the relationship between the different variables as well as analyzing the variables themselves. Therefore the background on the different variables will be used to create an analytic framework.

Method

This chapter aims to stepwise describe how this study has been conducted. It also describes why certain decisions have been made and how this affected the study in hand.

The field of didactics of mathematics is often distinguished by two features. The first is that research often is executed in the actual classroom and tends to analyze everyday situations. The second attribute is the aim to contribute to the praxis of mathematics education (Skott, et al., 2010). To get a more complete view of a situation or phenomenon Denscombe (2009) argues that the researcher should use multiple data collection methods within the study. This study intends to present an accurate view of the education in mathematics in Colombia and therefore three data collection methods were used by methodological triangulation. Methodological triangulation indicates that two (or more) data collection methods are used in one study to view a phenomenon/data with different approaches in order to confirm the results (Denscombe, 2009). The data collection methods that were combined were: observations, interviews with teachers, interviews with students and interpretation of national standards, as well as other essential documents used in mathematics education in Colombia. Thereby this study is a mix between quantitative and qualitative research but have a more qualitative approach. By using triangulation the researcher can increase the validity of the data and the research procedure (Denscombe, 2009).

Sample/Participants

The sample consisted of three schools, at each school one class was selected to be observed and interviewed. Therefore the participants consisted of both teachers in mathematics and students who worked or attended one of the selected schools. During the observations other students and personnel participated as well. The schools were all located in a major city in Colombia.

The three classes that were observed included students who were between 15 to 19 years old. The total amount of students was 84 which were divided over the three classes and the total amount of structured observations was five. The students who were chosen to participate in the interviews were 15 to 17 years old and partook in the mathematic education that their school provided. Choice of age was motivated by the fact that the PISA tests are taken by 15 year olds and since the PISA results were used as guidelines it was appropriate to involve participants who were close to that age. Two students were interviewed at each school, which resulted in a total of six students as respondents through interviews. When the interviews in the public schools were conducted one female and one male student were chosen to participate. The private school was an all boy school and therefore there were only male participants. The private school had catholic traditions which were why it was an all boy school. The three teachers who participated in observations and interviews were 30, 47 and 49 years old and the sample consisted of one woman and two men. Unstructured interviews were conducted with principals, coordinators, other teachers or random students who attended the schools. Unstructured observations were conducted frequently during the course of four weeks. Some of the students who were asked to participate in the study declined because they wanted to do other things during their recess but none of the students that were asked during class declined. Therefore there was an external drop-out of around five students. There was also an internal drop-out in one of the classes that were supposed to be observed. The teacher changed his mind in the last minute and said

that he felt uncomfortable with the researcher's presence because the class would not be an ordinary class and it would be quite chaotic. The class was shortened because of a ceremony that the school arranged and because of that the teacher chose another kind of activity for the class that the teacher thought would be chaotic to observe.

Measuring instruments

Interview students

The interviews were constructed according to what Bryman (2011) chooses to call semi-structured interviews. This meant that the interviewer used an interview guide that was created in advance and the questions were rather broad. Furthermore, the interviewer also included follow-up questions frequently during the interview. The interview guide, that was used when interviewing the students (Appendix A), had questions that were phrased in a way that aimed to be easily accessible to the students. The questions were designed to create a holistic view for some variables. The chosen variables were from prior studies on the Colombian school system which are presented in the prior chapter and the author of this study chose the most salient variables in those studies. During the interview some of the students did not understand the questions. The researcher chose to ask about these things again. If the student had not given a response at the second or third time the question was asked, the researcher moved on to the next question. The main reason for repetition or rephrasing was because sometimes there were difficulties with the different languages and/or the cultures.

Interview teachers

The interview with the teachers was also semi-structured and therefore an interview guide had been created in advance (Appendix B). The questions were designed to create a holistic view for some variables, but also to thoroughly examine a few aspects of the mathematical education. These variables and aspects were derived from a theoretical framework as well as from prior studies. During the interview some of the teachers did not understand the questions. The researcher chose to repeat or rephrase in the same manner as she did in the interviews with the students.

Observations

For the structured observations an analytic framework had been created in advance (Appendix D). This consisted mainly of the variables that have been processed in the theoretical background of this study such as sociomathematical norms, communication and teacher. Each part of this framework eventuated different themes that help the observer to estimate different aspects of the mathematical education that was conducted during the hours of class.

Procedure

The schools that participated in this study were based on a convenience sampling. Each school was addressed with the same letter, which explained the study's purpose and methodology. As the study was approved by the principal, the selected teachers were informed and asked to participate. Each teacher was asked to give the students a presentation of the researcher and information about the study. First part of the field study was based on unstructured observation. This gave the researcher the opportunity to have calm and informal conversations with both students and teachers, to better estimate social environment and working environment. The idea was to get to know the school and get acquainted with both teachers and students to easier blend in when the data collection from observations began. In the structured observation the researcher

took the part of a complete observer and did not interfere with the education. By observing multiple lessons with the same teacher a time-bias was avoided. The observations took place in a natural environment with ordinary lessons and since it was conducted at the end of the semester the observations also included some evaluations of the students' work. After a few observations an interview was conducted with the teacher.

Four criteria were used as student informants were selected: (1) student's approval, (2) age, (3) gender, and (4) teacher's recommendation on which students could provide the most relevant information. During the unstructured observations of the schools a lot of unstructured interviews were conducted (Appendix C). All of the observations and the different interviews were conducted by the author herself and all interviews were recorded for analysis.

Analysis

The analysis started when the author began to observe, this part of the analysis is what Fangen (2005) calls an interpretation of the first level. This interpretation was necessary to develop a sense of theory concerning the context. Since this study has mixed data collection methods, this also gave the researcher an opportunity to ask about different interpretations and their meaning in each moment.

The second level of interpretation can be conducted in different ways. This study is designed as a comparison between a number of schools. Allowing for partitioning and elementwise comparisons, this is an analytic advantage. Both differences and similarities can be detected. These can be emphasized when they are put into the context of other cultural frameworks, too. (Fangen, 2005).

The interviews were analyzed in a similar manner to the observations. When a researcher conducts an interview, the researcher should start to interpret right away. This is because it requires a first level of interpretation to know which follow up questions that should be used (Trost, 2005). The researcher has chosen not to transcribe the interview because of the fact that all the interviews were translated by an interpreter and therefore it would not be the respondents' original words. There were one exception with student 6, who occasionally spoke English during the interview; those parts were transcribed. Other than with student 6 the interviews were not transcribed since it was not the respondents' original words, and therefore it loses some purpose for the transcription. Instead the researcher chose to interpret the entirety of each question and the entirety of the interview and then the questions/interviews were summarized. This procedure gives the data the same structure and is therefore easier to analyze. Another benefit is that uninteresting or nonessentials material can be excluded. However, since all interviews have been recorded, the researcher can go back and listen to the material again. Thus, avoiding the loss of important data (Trost, 2005).

The analyze of the interview proceeded as following. The researcher listened to each interview and summarized what seem relevant to the study. After all of the interviews with the teachers were summarized the researcher attempted to discover keywords. Keyword either indicates that many or every respondent mentioned it or that the respondent had expressed this word with great importance. After keywords were found the keyword were compared to each other to create different themes. Each theme was examined to see if the themes could be divided into categories that would contain a minimum of two keywords. When the interviews with the student were

summarized/transcribed the same procedure was used to analyze those interviews. After this a comparison was made between the themes of the teachers and the students. The results of this comparison worked as a basis when the result was presented. Then findings from observations and documents that was relevant for each theme was presented at the same time. Observations were analyzed in a similar way. By finding keyword or actions and compare the different observations to each other. The mathematical content that was presented during the classes were compared with standards, both national and international, in order to categorize and analyze the mathematical content. Each school gave the researcher access to their manual. This was examined to detect differences and similarity between the schools. Lastly the different analyses were combined. This process was conducted in order to try to create a holistic view of the phenomenon (Denscombe, 2009). The data that was presented in the result was presented according to the standardized tradition, which indicates that quotes and observations are rewritten in order to make it easier for the reader to apprehend the meaning. When quotes or observations are rewritten, unnecessary word or pauses are removed but the text is still consistent with the original purpose/meaning (Fangen, 2005). In this study the researcher needed to edit some quotes a bit more since the interpreters occasionally used language that were grammatically incorrect.

Ethics

This study will comply with the demands drafted in order to protect the participants within a study. To comply with this demand there are four aspects to consider; these aspects/demands are: the demand of information, demand of approval, demand of confidentiality and demand of usage (Vetenskapsrådet, 2011). The demand of information implicates that the participants have knowledge about the study, both method and purpose. To make sure that all students participating in the study understand their rights, the information will be in Spanish. The observer and interviewer will to some degree be able to speak Spanish, as this can be significant if the students have unexpected questions that need to be replied to in Spanish. By informing about both method and purpose in Spanish this demand will be met. The demand of usage implies that the data collected during the study is limited to the purpose of the study, which this study will comply with. No personal data concerning the students or teachers will be documented during the observation and during the interviews both teachers and students can choose what they would like to share. Possible information that is collected during the study will be treated according to the demand of confidentiality, which implies that information about the participants will be carefully handled and encoded. The demand of approval is going to be more difficult to follow among the students. Since all the students were 15 years old or older their parents'/guardians' approval were not needed, it is sufficient with the student's approval. If the parents should object against any part of the study it will be dealt with if it becomes an issue. Teachers must give their approval in order to go through with observation and interviews (Vetenskapsrådet, 2011).

The demands above is drafted by a Swedish institute but is argued to be valid abroad as well (Vetenskapsrådet, 2011). However, since the researcher is in a different environment/culture than usual there might arise disagreement about what is considered to be a sensitive question. If there are such questions, and they are not of great importance for the purpose of this study they will be eliminated. If they are of great importance each question will be considered one by one, and eliminated if researcher seems it to be appropriate.

Quality requirement

When data is measured with different data collection methods it provides an opportunity for the researcher to check the data and see if the different methods present the same image; regardless of what data collection method that was used to collect the data. If these images differ to a great extent the researcher may have an opportunity to discover this early and can try to understand why this data is diverse (Denscombe, 2009). A methodological triangulation tends to give a holistic perspective on a phenomenon, which is the aim of this study. This indicates that the method can increase the validity of the data, since it provides an overall picture of the education in mathematics.

Additionally, this study is conducted in a natural environment which increases the ecological validity. Therefore it is most likely possible to apply the results in other real situations that are similar to the ones observed (Denscombe, 2009). Since this study tends to be more of a qualitative study than a quantitative study the sample is small, but the data collection methods does deliver a profound view over the small sample.

Because of the small sample the researcher has not tried to reach any general conclusion, however this study have tried to reach some transferability. Transferability indicates that the results can be applied or transmitted to another situation than the actual sample (Denscombe, 2004).

To establish a sense of comfort in the participants the researcher started to ask them very open question that made it possible for the respondent to talk freely (Trost, 2005). A common way to begin an interview was "I thought we could start with you telling me a little about yourself". This gave the respondent to start talking about what ever they wanted while the got used to being interviewed, the recorder etcetera. All interviews have been conducted by one researcher which is a benefit both when interviewing and when analyzing. It is a benefit when one is interviewing because the interviewer will have an impact on the respondent and when it is the same person interviewing there will most likely be a similar impact on all respondent. Usually, the collected interviewing material is better understood by the person that actually conducted the interviews (Trost, 2005).

Results

*This chapter presents the results of this study which consist of themes, categories and keywords. Where themes contain the categories and categories contains keywords. The keyword will be written in italic, categories in **italic and bold** and themes in **bold**. The respondent will be represented as student 1, student 2, etcetera, teacher 1, teacher 2 or teacher 3. Teacher 1 was the teacher of student 1 and 2, teaching at school 1. Teacher 2 was the teacher of student 3 and 4, teaching at school 2. Teacher 3 was the teacher of student 5 and 6, teaching at school 3.*

The view of mathematics

Through the interviews in this study there have been many opinions on how the respondents view mathematics and also many opinions about how they think that other people view mathematics.

Mathematics in their daily life

A keyword that was discovered was *help*. All the people that were interviewed were sure that mathematics could help you in life in many ways. The main example was when you would need to buy something and that you would need it for further studies or in future careers. The teachers described scenarios where they explained how mathematics could be used in the students' everyday life and a lot of times they said that they tried to connect it to what the student would do in the *future*. For example, teacher 3 said that

"I always try to connect mathematics the reality of the students, so for them to understand how to apply it in real life... so for example logarithms. I have students who say: what am I going to use this for if I am going to do social sciences. Then I reply: if you are going to do social sciences, logarithms are going to be needed if you are going to study population and populations increases because it increases exponentially or linearly at a logarithmical scale".

Student 6 was asked about applications of mathematics and how mathematics can be used in the student's everyday life. Student 6 said "Sometimes I don't know why mathematics is working in our lives ... They don't explain the functionality of the topic in our lives, unless we ask for it". The student said that the only thing the teachers explained about that was how the curriculum was organized during the year and that "the only situation that I have used it in is during tests and when I spend my own money or administer my own money". On the other hand this student also says that mathematics "...is a structure that makes our society" and "I think that mathematics *helps* a person a lot". The student emphasized that knowing mathematics would be of help in the *future*. Student 2 talks about how "they say" that students will encounter mathematics in their future educations. The student also states that it is important to do well in mathematics because a person can make a lot of money if they are good at mathematics and the student's parents agree on this point, according to the student.

Student 5 said that mathematics fosters logical thinking, both in mathematics and in life. "... there is a logical way to solve the problems, that you have to find the logical way to solve the problem and that is sort of what happens in life ... you have to think logically about how to solve it and approach it from that logical point of view".

Mathematics could also be identified in the *technology* and teachers 2 and 3 said that they wanted to use more technology in their teaching, but unfortunately they did not

have the resources. Teacher 2 does however implement it in some sense into the students' education by giving them homework that are designed to make the students go online and search for information on the internet. Thereby making the students at least interact with technology when they do their homework.

When the teachers are talking about mathematics they often talk about the *applications* of mathematics, the ability of applying mathematics and the ability of solving exercises with applications. As quoted earlier by teacher 3 the teacher tries to make the students understand how to apply mathematics in real life. Teacher 1 says that "The students do not see the application of mathematics... and according to them it is not something that they are going to use in their lives. But as time passes on they realize that mathematical terms are in every context". Teacher 1 often expressed that the teacher wanted the students to show their skills in applied mathematics.

Mathematics is abstract

Even though many people thought that mathematics could help them in their daily life, the respondent expressed an opinion about how in general in Colombia people did not appreciate mathematics and how people in general thought that it was not a useful thing. Teacher 1 said that the students thought that mathematics was useless for their lives. Teacher 3 said that mathematics is very *isolated* in the curriculum in Colombia. That seems to be a good description of what students and teachers think that the general opinion about mathematics is. Teacher 3 also said that mathematics is considered to be a very *abstract* thing here in Colombia.

As teacher 2 said "They got some problems with addition but when they are using something real like money they don't make the same mistake". Student 4 which is a student of teacher 2 was asked; when can you use mathematics? Student 4 responded "When you have to use money. We don't usually develop mathematical thinking but we develop something related with distance... sizes, altitude". I. e. the student did not know when one could use mathematics, except with money and distance.

Mathematics consists of topics

An ordinary lesson in mathematics is described by student 1 as "Then we sit down and the teacher starts a *new topic*, she explains the new topic. Next lesson the teacher gives us feedback on the new topic and at the third lesson the teacher makes an evaluation of the new topic". This student describes the lessons like a repeated procedure. The next topic is presented and then they go through these different steps. After that they have a new topic and do the same thing again with that topic.

In the mathematics education in Colombia each year is divided into four periods. Each period covers a different part of mathematics. During the period that this study was conducted teacher 1 worked with limits, teacher 2 and teacher 3 worked with analytical geometry. The teachers all teach a certain area in mathematic in one course, which indicates that they do not mix different areas of mathematics. Teacher 2 was asked about if the teacher ever mixed different areas in mathematics and he responded "it would be an interesting exercise but it does not happen".

Student 3 talked about the students mothers' opinion about mathematics. "She says that it is important but she doesn't know which area of mathematics is very important to focus on". The students expressed opinions on what they think is *the most important part of mathematics*. Student 1 and 2 said that the most important thing in mathematics

is the basic operations. Student 3 said that the student only used the basic mathematics outside of the classroom and does not focus on different aspects of mathematics. Student 6 said that “I think you only use the basic operations...”

Every year the students have courses in mathematics. Student 3 told the interviewer about a previous course. “... but we didn’t take the whole algebra course because we didn’t have enough time to do it”. Student 2 expressed the same feelings when talking about a previous course with a previous teacher. Student 2 said “In there they used to explain step by step so for that reason we didn’t cover the whole curriculum. For example last year when I was in tenth grade I was learning ninth grade topics... So for that reason it is a little bit difficult to face some topics in eleventh grade that I am supposed to have covered the last year”. A similar issue was brought up when the interviewer and student 4 talk about the teacher. “I think that I have a really good mathematics teacher right now but the teacher sometimes assumes that we have in our mind the previous knowledge, but because of the time we sometimes forget basic things”. According to these statements it seemed as students could fail to benefit from parts of the courses in mathematics and it appears as students will not have any other opportunity to catch up to the rest of the class.

The reason of learning mathematics

The grading system is a little different at every school. One thing they have in common is that at the end of each period a grade is determined for that period and then the students gets a final grade at the end of the year. Since many of the students were finishing comprehensive school this year a couple of the students emphasized the important of the grade. Student 2 said “the grade is important because we finish the year soon and if you have bad grades it’s hard to overcome those bad grades...” Student 1 said “people who are failing have to do another workshop to save the grade”. Student 3 had a possible explanation to why so many students had to “save their grade” in the end of the year, the student said “The year is divided into four periods. If you fail the first period, you can do extra work to overcome those problems but there are students who are lazy and who fail many tasks and waits till the end of the year to overcome all of those problems”.

Since this study was conducted at the end of the school year the observer got to observe a couple of examinations and heard a lot of discussion about examinations and evaluations. In school 1 the observer saw two lessons where the main focus was to set the grade. The examinations where conducted in different ways. One way was that the students who were failing got to do some assignments on the board.

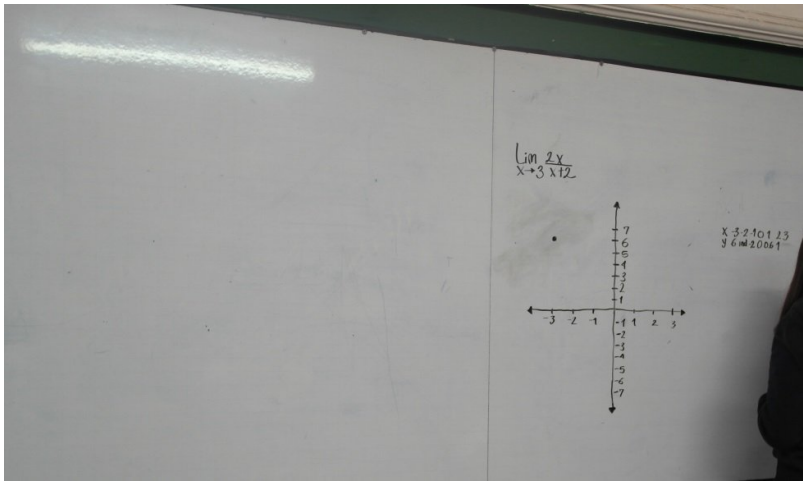


Figure 2 Observation 2 at school 1, examinations on board.

During this lesson the board was divided into three different sections, so that three students could solve assignments at the same time. The whole class was in the class room at the time. The once who were not failing but still wanted to improve their grade pushed their tables closer to the teachers desk and did assignments there so that the teacher could have a good view of the students. Each student got different assignments.

In the interview with this teacher the teacher said that “I prefer it this way because when they are at the board they assume the role that they have to know ...” The teacher does this kind of examinations often but written test were also common. Even when the students takes the written tests the assignments are different for everybody. Teacher 1 said “It is not a long test; it is something that focus on what I really need”. The teacher was talking about what she needs to pass the student in that course. If they can solve chosen exercises they pass.

During the observations the students handed in assignments if they passed the assignments and were satisfied with their grade they did not have to do any more assignments and if no, new assignments were handed out. The teacher summarizes all work during the course in “the list” – as student 1 referred to it. After students handed in their work the teacher would sum up their score of the year to see which grade the student got.

Teacher 3 expressed a positive view about different kind of examinations. When they had something they called workshop the teacher could observe how they work which enabled the teacher to see what the students were learning and how they were learning. This way of working enabled students to reflect concerning their own knowledge, what they know and what they need to work more on. Teacher 3 said “all evaluations are important because they all add something to the learning process”. The interviewer interpreted this as a reason to have a lot of exams and work/workshops, which were added to the total sum before grading.

9	NNN2.7	3.4	2.3	4.0	3.5	3.0	3.0
2	NNN2.0	NNN2.7	4.3	4.0	3.5	3.0	3.0
4	3.0	NNN2.5	2.8	3.0	3.5	3.0	3.0
5	3.0	NNN2.3	3.7	3.0	3.5	3.0	3.0
2	NNN2.5	3.1	3.4	4.0	3.5	3.0	3.0
2	NNN2.4	NNN2.7	2.8	3.0	3.5	3.0	3.0
3	NNN1.6	NNN2.5	4.9	4.0	3.5	3.0	3.0
4	3.0	NNN2.5	3.0	4.0	3.5	3.0	3.0
6	NNN2.8	3.0	3.2	4.0	3.5	3.0	3.0
6	NNN2.0	3.0	4.0	4.0	3.5	3.0	3.0
10	NNN1.8	NNN2.3	4.8	4.0	3.5	3.0	3.0
2	3.3	3.1	2.8	4.0	3.5	3.0	3.0
NNN2.2	NNN2.5	4.3	3.5	4.8	3.5	3.0	3.0
3.0	NNN2.7	3.3	3.0	3.0	3.5	3.0	3.0
3.0	3.2	2.8	3.0	3.0	3.5	3.0	3.0
3.0	NNN2.0	4.0	4.2	3.0	3.5	3.0	3.0

Figure 3 Observation 2 at school 1, The list.

The grading systems at the other schools worked very similar to school 2. Homework, assignments, workshops and oral and written tests were taken into account when the grades were set. At school 1 and 2 the student could get a grade between 1 and 5 where the student passed if he/she got a 3. At school 3 the students could get a maximum of 100 points and to pass they needed 70 points.

During the interview with teacher 2 there was a question about how the teacher prepared for lessons including how many assignments the class was given the teacher said “It depends on the group’s development. So if the group is a fast group we are going to make a lot of activities but if the group is a slow group we are going to reduce the amount of activities”. Teacher 1 said “I like them to solve a lot of exercises”.

Learning/Learners

School is a place for learning, where the students are learners. In this theme, it becomes transparent how teachers and students view the student's place in the education in mathematics. It also shows to a great extent how students perceive their education.

What motivates students

One keyword that was detected was *to like*. Many times it seemed crucial that students liked mathematics in order to even care about the subject. Teacher 1 said “... in general you meet many students that really like mathematics and then there are others that don’t like it and they don’t care about mathematics”. Teacher 2 was asked about what kind of problem the teacher encounters when teaching mathematics. The answer was “Nowadays, not only in mathematics, I have realized that the interests of the students are not the same as the old times... and the studies are considered something you have to do and don’t want to do”.

Some of the students agreed that it is important to like mathematics and have an interest in mathematics in order to be good in mathematics. Student 2 said that in order to be good at mathematics you have to “... like mathematics because if you don’t like mathematics you are not going to take advantage of that”. What the student was saying was that if you do not like mathematics you are not going to take advantage of the education in mathematics, i.e. the things and the opportunities that the school provides

within a student's education. Student 5 said that "I think that anybody can do math, you don't have to be a genius or like a nerd to do math. You just have to like it and to do it, to put pleasure to it, to make it pleasurable and if you can do that then I think that everybody can be good at math". Students 3 said that to be good in mathematics you have to have the right "*attitude*, if you have the motivation to do it, you are going to do it".

Teacher 3 tries to keep the students motivated by allowing them all to answer trivial things at the same time. "So when the questions are very simple or trivial, like what is the value of x , I allow the students to answer at the same time and I think that that keeps them motivated. It gives them the impression that they are participating".

Variables of success

There are a lot of different variables that can make the process of learning easier. This study has found a couple of variables that students and teachers emphasize when they are talking about the education in mathematics.

To pay *attention* is something that every student talked about and also one of the teachers. Teacher 1 talked about some of the problems that were encountered during mathematics lessons. The teacher said that there are problems with "lack of concentration, lack of attention..." On this subject student 1 said "when you are young it is hard to concentrate because sometimes when you are looking at the board and then you are looking out the window and get distracted very easily..."

Student 2 was asked about what was necessary to be able to be good at mathematics; the student responded "First of all pay attention". Student 3 said that one of the student's responsibilities is to pay attention. Student 4 talked about how you develop attention in mathematics class. Student 5 said that the student understand when the student is paying attention. Student 5 also said "and it is not really difficult, what I do have to do is pay attention to what the professors are telling me and do what I am asked to, but it's not complicated". Student 6 said "... but the specifics qualities that a person should have to be a good student in mathematics depends on the amount of attention that you pay".

As quoted above some students talk about *distractions* in relation to attention. Other students mention *concentration* like teacher 1 mentions above. They also talk about *focus*, student 2 continues his quote above about attention with "... Be organized, if you are not organized you lose the way, you are not focused". Student 1 talked more about attention later in the interview when the student said learning mathematics is like learning how to drive a car. "Attention and practice, practice and practice, like in mathematics". Student 1 also said that besides being able to pay attention you should also learn how to listen to succeed in mathematics.

The observations generated a lot of information about the environment in the school and in the classrooms. The class started from the point when the teacher entered the classroom, which was often some time after the actual lesson had started. When the students settled down and the content of the lesson were processed there were a lot of noise. Occasionally from the students in the classroom but mostly the noises came from outside of the classrooms. It would be other students trying to get attention, it could be other students playing in hallway, it could be some random noise that could not be specified by the observer or there could be airplanes passing by. Hence, there were a lot of different things and actions that could make your focus and attention shift.

According to the observer this was the situation across the entire schools. Which made it hard to find some peace and quiet in any of the schools. The situation inside the school was a lot better at school 3, but there were however more distractions from the outside since it was closer to the airport.

Teacher 3 talked about different kind of participation. As mentioned under the category about motivation the teacher allows everyone to answer at the same time when they are answering easy questions. “But when I have to have an answer that has an explanation that needs to have a concept, than I ask them to raise their hand. So that they can actually *participate*”. Participation was something that teacher 2 talked about as well and teacher 2 said that the students’ grade partially was based on their participation.

Importance of having a good teacher

To be able to learn, one often needs different resources. The teacher appeared to be the main source for almost every student. Two students said that before they asked the teacher the students asked their classmates. Even though the two students did not have the teacher as their primary resource when they had question, both of them said that the best way to learn new things was when the teachers explained to them. In fact four out of six students said that the teacher’s explanations or teaching methods was the best way to learn. Therefore it seems like the teacher is viewed as the primary resource in general. Student 6 verified that this was also the case with the student’s parents. “... my dad says that if you have a good mathematics teacher, take advantage of that”.

“If you don’t understand the subject you have to ask the teacher... because he/she is the teacher...” This quote is from student 6 and in general the teacher seemed to have gotten a lot of questions, not because the student understood it better but because the student were supposed to ask the teacher and not his/hers classmates. Almost like the teacher had knowledge that classmates could not possess and therefore the teachers explanations were always superior to others’ explanations. Student 1 said that it were the students’ responsibility to ask the teacher if they had any questions or doubts.

Students often preferred *step by step explanation*, which they referred to a lot during the interviews. This seemed to occur mostly when they were learning methods to solve exercises. One student mentioned that the best way to learn is to do exercises because “... when I am doing the exercises I realize that there is a gap of knowledge”. The student talked about the gaps of knowledge as things that the student did not know how to do and through the exercises the student could detect that and build knowledge from that point.

It seemed important to *do what the teacher told you to do*. To be responsible for the work your teachers assign to you and to answer to your studies. During the observations the teachers handed out assignment/exercises to the students and that was what they should do during that class. The students had a very limited assortment (if any) to choose from. The exercises that was provided from the teacher were the only exercises that were processed during the observed classes.

The teacher were the primary resource, secondly classmates, thirdly the internet and more specifically youtube. The students said that they mostly used the internet/youtube when they were doing homework. Because many of the students did not have anyone who could help them with their homework at home. No other resources were mentioned. Not even in the private school where they were supposed to have books and

were given a lot of study guides. Students in the public schools were not required to buy books, since public schools are supposed to be free, except for the uniforms which they have to buy.

Didactics

Education can have different shapes. This study detected tendencies in the way teachers taught at the different schools that participated in this study.

The structure of the lessons

Student 5 describes an ordinary lesson as following; “With this professor the way that we have been doing it is that we have one class for theory, one class for activities and exercises and one class for the evaluation”. Student 4 describes it as “The teacher arrives, explains the topic and gives us easy exercises. The teacher upgrades the level and makes the exercises more difficult. Then the teacher takes two or three lessons to do the same thing and then we have an examination”. All off the student’s descriptions were very similar to the two above, for all three schools, both public and private. With the exception of repetitions of the topic, reviews of homework and in some cases, time for students and professor to have the possibility to form a social relationship. Student 6 expressed thoughts on this way of conducting education. “Because when it is to structured it is kind of boring and you start to think of other thing instead of mathematics...It is just a *routine* like, see that topic, make an exercise and evaluate it and it’s kind of boring”.

The coordinator of school 3, where student 6 goes to school, said that, one of the problems that they had was with the teachers. The teachers did not want to be innovative within their work. When asked why this was a problem the coordinator responded “because it becomes a routine in the classes and the teacher have the same activity for each class. There are routine activities, there is not any dynamic inside the classes”. The coordinator said that he thought that this effected the students’ motivation, creativity and their results.

Although the students did not express differences in their current mathematics classes, they did express a lot of differences between different teachers and schools. The students especially mentioned how schools and teachers varied in their *methodology*. Antonym words that were used to describe differences between schools or teachers was, slow and fast, easier and harder, dynamic and routine. Student 2 had to change schools recently and said the following. “...this is my first year here and the methodology here is very different in contrast with my previous school... In the other school they used to teach step by step but here they teach with photocopies, worksheets”. The student also said that it was hard to *adapt*. It seems like it is the students responsibility to adapt to the new school. For example student 1 had attended many schools and said the following. “I think I had to adapt to the schools because some teacher think that the students are equal and as one of my teachers said; “I am here just for the payment”, was the teachers exact words”. Luckily, none of the current teachers were described in any similar way. Rather the opposite, they were often described as good teachers.

The teacher at each school was interviewed about how they planned and conducted their lessons. Teacher 1 at school 1 said that is does not take a lot of time to prepare for the classes because “...the classes are already prepared since a long time. Because of my experience I know what to do”. The teacher had already planned a similar lesson for another class earlier, which is why it does not take a lot of time. At school 2 the teacher

had a similar response. Teacher 2 said that “it depends on the topic, it depends on the lesson, it depends on the group but taking into account my experience it takes more or less 30 minutes. I spend this time because of my experience”. Teacher 3 said “I plan my classes weekly not daily. I organize the topics that belong to that week. With that specific group I have three lessons a week so in order to prepare for their week it takes me about 45 minutes”.

As a couple of students pointed out, *students are different* and have different ways to learn and to understand. The students said that the teachers often tried to explain in different ways, so that the students would understand. Students does however still need to adapt to the teachers methods even if the process of adapting is not going very well. Since teachers does not have much time to plan lessons it would be impossible to try to adapt/accommodate for each student.

Tools for teaching

As quoted by the coordinator at school 3 there was a problem with the teachers who were not innovative enough. Their lessons and classes had too much of a routine and consisted of routine activities. The interviewer asked the teachers about what kind of activities they performed with their students. Teacher 1 mentioned “*cooperative learning...working with graphic... tables to enforce factorization... group exercises or individual exercises or something on the board or we can watch a video*”.

Teacher 2 was asked; what do you think is the best way for your students to learn new things. The teacher responded “Particularly I would like to use different tools such as *technology* because I would like to create some contrast to the exercises we have made before. But when we want to use the technology classroom, we can’t because it is occupied”. The teacher also said that “For example next week when we are going work on Pythagoras, we can go outside the classroom and check the distance in said formula”. Furthermore, when students had problems in certain topics. The teacher suggested that they should watch videos on *youtube* on that topic. The teacher also talked about being a *guide* for the student to make them realize and discover where mathematics are in real life. Below is a couple off pictures from the observations at school 2 and examples of the exercises they did. The exercises that were presented did not seem to have any connection to any area in real life.

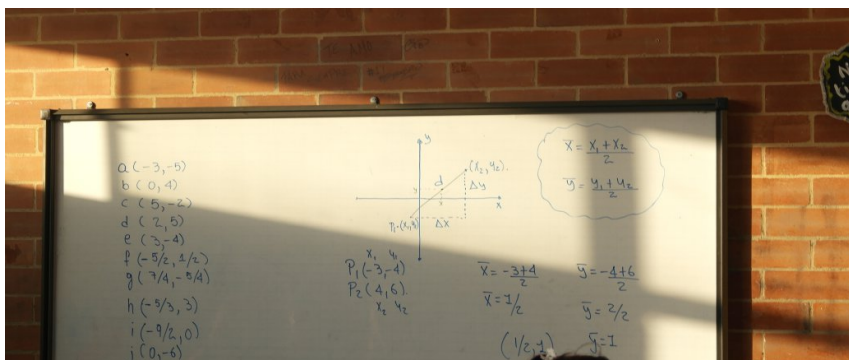


Figure 4 From observation 1 at school 2

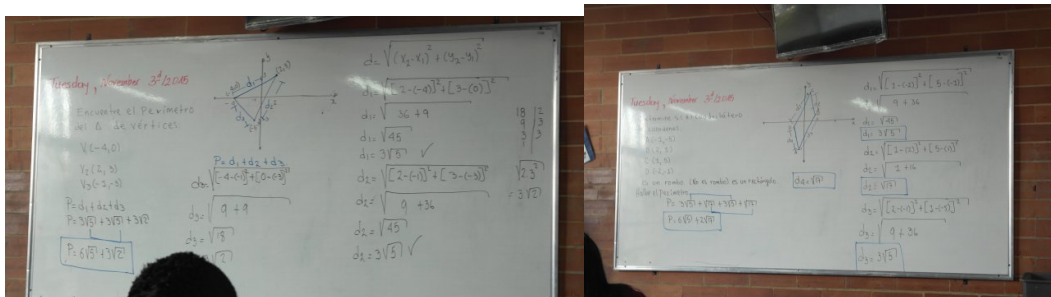


Figure 5 and 6 From observation 2 at school 2

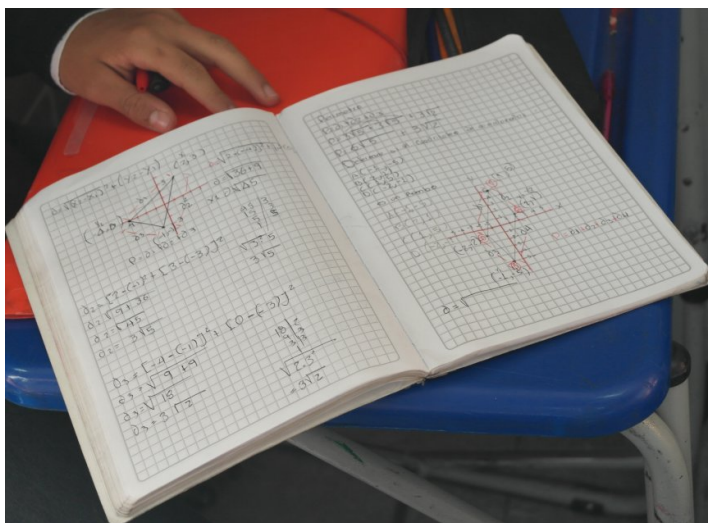


Figure 7 From observation 2 at school 2

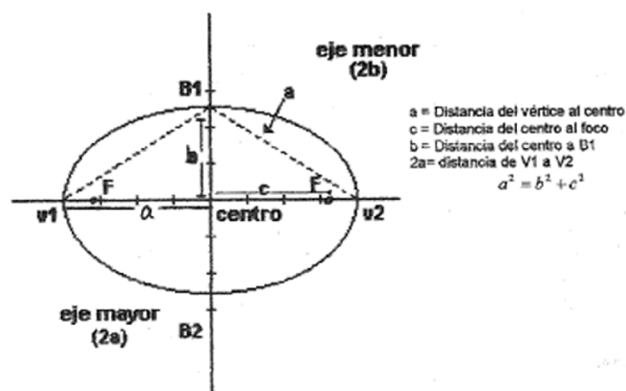
When the observer observed school 3 she saw that teacher 3 handed out some papers at the beginning of the lesson. This was according to the teacher a kind of guide for the students. Teacher 3 tries to make a guide every time the teacher starts a new topic and this was what teacher 3 said about the guide in the interview;

“The idea with that is as I start a new concept or explains new concepts, the students can see it in the guide and also have exercises that are already solved to see how they are solved and to have a clear example of what they are doing... I call it a guiding guide and it will help me guide the class and also help the students through the acquirement of new knowledge”. During the interview teacher 3 also mentioned other tools for teaching but said that the resources were not there and that was why the teacher had to teach on the board.

LA ELIPSE

DEFINICIÓN COMO LUGAR GEOMÉTRICO

Es el conjunto de puntos $p(x, y)$ tales que la suma de sus distancias a 2 puntos fijos llamados focos es siempre igual a una constante llamada $2a$.



Ejemplo 1

$$\begin{aligned}
 F_1(0,2) \quad F_2(0,-2) \quad 2a &= 6 \\
 V_1(0,3) \quad V_2(0,-3) \\
 a &= 3 \quad c = 2 \\
 a^2 &= b^2 + c^2 \\
 a^2 &= b^2 + 4 \quad B_1(\sqrt{5},0) \\
 b^2 &= 9 - 4 \quad b = \sqrt{5} \quad B_2(-\sqrt{5},0)
 \end{aligned}$$

Ejemplo 2.

$$\begin{aligned}
 F_1(1,1) \text{ y } F_2(7,1) \quad 2a &= 8 \text{ eje mayor} \quad C(4,1) \\
 a &= 4 \quad c = 3 \quad a^2 = b^2 + c^2 \\
 V_1 &= (8,1) \quad 4^2 = b^2 + 9 \\
 V_2 &= (0,1) \quad b^2 = 16 - 9 \\
 &\quad b = \sqrt{7} \approx 2.6 \\
 &\quad B_1(4,1 + \sqrt{7}) \\
 &\quad B_2(4,1 - \sqrt{7})
 \end{aligned}$$

Figure 8 From observation 1 at school 3, guiding guide

A couple of students were asked about if different kind of teaching methods occur during mathematics lessons. Student 2 told the interviewer that sometimes they play mental games, like Sudoku, during their lessons. Other times the teacher can not attend the class and they get assignments instead of class, but besides that the classes are similar to each other. Student 5, which was quoted earlier, said that they have one lesson for each stage – theory, exercises and evaluations. The student also said that the plans for the lesson can change if they for example have doubts before tests etcetera but not otherwise. Student 6 said that there is too much structure and routine, and that the teacher needs to be more flexible.

Exercises and assignments

Through the observations the observer got to see a lot of different exercises and assignments and the difficulty level was not easy nor extreme. It was what one could expect for the last and penultimate year of high school. Then again, since the adolescents were 15 and 16 years old this seemed as a pretty hard difficulty level. Below is an example of assignments that was handed out during a class. The students could choose to do the assignments if they wanted to try to raise their grade. Another example of

exercises can be seen from the different photos from the observations and in the guiding guide.

LIMITES LATERALES		
Encontrar cada límite por derecha y por izquierda con la grafica		
1. $\lim_{x \rightarrow 2} x + 3$	2. $\lim_{x \rightarrow 1} \sqrt{x - 1}$	3. $\lim_{x \rightarrow 2} \frac{x-2}{x-2}$
4. $\lim_{x \rightarrow 3} \begin{cases} x+2 & x > 2 \\ 3 & x < 2 \end{cases}$	5. $\lim_{x \rightarrow 0} \frac{1}{x}$	6. $\lim_{x \rightarrow 1} x^2 - 1$

PROPIEDADES DE LOS LIMITES		
Evalúe los siguientes límites indicando la propiedad o propiedades que se aplican en cada paso:		
1. $\lim_{x \rightarrow 3} 77$	2. $\lim_{x \rightarrow 3} (3x - 7)$	3. $\lim_{x \rightarrow 2} (x^2 + 2x - 1)$
4. $\lim_{x \rightarrow 3} \frac{4x - 5}{5x - 1}$	5. $\lim_{x \rightarrow 1} \sqrt{\frac{8x+1}{x+3}}$	6. $\lim_{x \rightarrow -3/2} \frac{4x^2 - 9}{2x + 3}$

LIMITES INDETERMINADOS		
Encuentre los límites factorizando		
7. $\lim_{x \rightarrow 4} \frac{3x^2 - 8x - 16}{2x^2 - 9x + 4}$	8. $\lim_{x \rightarrow 2} \frac{x^3 + 8}{x + 2}$	9. $\lim_{x \rightarrow 0} \frac{\sqrt{x+2} - \sqrt{2}}{x}$
10. $\lim_{x \rightarrow 0} \frac{\sqrt[3]{x+1} - 1}{x}$	11. $\lim_{x \rightarrow 3} \frac{2x^3 - 5x^2 - 2x - 3}{4x^3 - 13x^2 + 4x - 3}$	12. $\lim_{x \rightarrow 2} \frac{x^3 + 8}{x^4 - 16}$

Figure 9 From observation 1 at school 1

Teachers told the interviewer that they tried to adapt the exercises and assignments according to the class. If there was a very skilled class the teacher would give them a greater amount of work and at a higher level, and if they were not they would receive easier assignments. Usually a class started with assignments on a *medium level* and later at a higher level. During the lessons the class only had a *limited amount of exercises*, which have been mentioned earlier.

During the observations there were a lack of usable documents, but the researcher found a couple of relevant documents elsewhere. One of them described the national standards in mathematics and language (Ministerio de education nacional, 2003). The mathematical standards in Colombia have set goals for third, fifth, seventh, ninth and eleventh grade. The different standards are designed in the same manner. First, they are divided into five different abilities, which roughly translates into : *i*. Numerical thinking and number systems, *ii*. Spatial thinking and geometric systems, *iii*. Metrical thinking and measurement systems, *iv*. Probability thinking and data analysis and *v*. Variation thinking, algebraic and analytic systems. This study examined the standards set for eleventh grade. These were presented in a table (Appendix F), which have been adapted from the original document (Ministerio de education nacional, 2003, p. 20 – 21).

In the table the abilities had a different amount of goals. These goals seemed to increase in difficulty. The ability with the highest amount of goals was *iv*. Probability thinking and data analysis, which had nine goals. After that, *ii*. Spatial thinking and geometric systems with six goals. The ability with the lowest amount of goals was *iii*. Metrical thinking and measurement systems, which only had three goals. This amount of goals

where pretty consistent through every grade, i.e. there were for example a low amount of goals for *iii*. Metrical thinking and measurement systems in every grade.

This table shows abilities/procedures that the students should be able to master at each level within the different abilities. The words that were used to describe these abilities were: compare, analyzing, identify, design, to reason, recognize, solving, formulate, interpret, describe, modeling, using arguments, establish and propose. Ergo the table, or standards, contain a variety of abilities and aspects of how students can interact with mathematics. This was the author's interpretation of the original table.

The observer could identify some parts of this table in the observed classes. In school 1 they were working with limits which could be found at level 1 in Variation thinking, algebraic and analytic systems (v.1). School 2 seemed to be processing parts of *ii*. Spatial thinking and geometric systems and the work they did could be identified at level 2 (*ii.2*) and level 3 (*ii.3*), which this study has translated as "2. Identify characteristics of different geometric objects through representation of Cartesian systems (polar spherical, ...)" and "3. Problem solving for geometrical figures' characteristics and conical algebraically". Finally the class at school 3 was processing something that also seemed to be connected to *ii*. Spatial thinking and geometric systems at level 2 and maybe in some extent to level 6 (*ii.6*) which this study has translated as "6. Recognize and describe geometrical curves and location".

The teachers all mentioned some parts of the national standards in the interviews, but no one referred to all of the different levels or abilities nor made a direct connection to areas they were working with and these standards. Every school has its own curriculum and the teachers said that they used the national and the international standards when they were developing the curriculums. Teacher 3 talked about standards and what the teacher said does not agree with this study's translation/interpretation of the national standards. Teacher 3 said:

"In Colombia the mathematical curriculum has three main things, which are the development of logical thinking, problem solving and the use of mathematical language with coding. In order to develop those three things there are so many other little processes that need to occur in order to accomplish them. For example how to do a series, how to do mathematical models...".

Development in mathematics and in other aspects of life

The aim with the education of mathematics is not only limited to mathematical literacy, it also includes the development of different skills the students can use, in and outside mathematics. Below is one quote from each teacher where they are talking about what they are trying to develop in their students.

Teacher 1 said that the aim was that students developed "logic abilities, analyzing abilities, different skills related with their thoughts, problem solving, identity, to overcome fears and taboos".

Teacher 2 said "I want to develop the way that they are thinking" "The idea is to focus on different exercises that develop the students way of thinking and the way they develop different ways of solving problems in their lives". The exercises focus on "analytic thinking and planning for different hypotheses and for different solutions of the problem that I gave them".

Teacher 3 said "...it is not limited to those competencies, you also have to look at the fact that for example the part of the brain that develops the logical thinking or those mathematical abilities is also the part of the brain that deals with affections and emotions... Therefore it's not only about those abilities but also with other things that have to do with the development of logical thinking or those that develop hand in hand with logical thinking".

The students did not mention all of the abilities that the teachers mentioned and sometimes none of the abilities mentioned above. For example, student 1 only talked about operations in mathematics, attention and the ability of listening. The student was however convinced that mathematics was going to be very helpful in the student's life. Student 2 said that you are developing "mental abilities and understand mathematics which you do through paying attention" with mental abilities the student meant the ability to become faster in order to solve exercises.

Student 3 said that the student did not know what abilities they were developing and also said that the teacher did not talk about which abilities they were developing. Student 4 talked about developing "attention. As a student I consider that it is very important to develop autonomous work... some of the students develop that skills, but it depends on the student".

Student 5 had some similar ideas to the teachers about what students are developing. The student said "I have learned to solve problems, to think logically and I am not only talking about math problems, I am talking about our life problems. Because in life you have to think logically to solve those problems and that is what mathematics fosters..." The student also said that you learn to "think on your feet... think fast and think well" in addition to that the student said that you learn to reason well. "To be resourceful, for example if you have a problem and don't know how to solve it you have to find different ways and figure out how to solve it either way. I think that math teaches me to find those alternative ways to solve that". Student 6 said that they are developing "the normal ones. We have learned the basic operations ... and we are working on interactive skills. We don't use the topics we are learning in a specific ability, we learn the topics in our subjects... but we don't know the functionality of it in our lives".

Culture

There are a lot of aspects that have an impact on the education of mathematics and the culture is definitely one of those aspects. This study has found a couple of cultural aspects that could have an impact on education.

Social and cultural problems

Teacher 1 was quoted earlier when the teacher talked about the social and cultural difficulties the students are facing. Here is a story from student 1 which attended school 1, as teacher 1 worked at.

"For example a few days ago I was in a Starbucks and a girl came into the Starbucks wanting a glass of water because she did not have enough of money for a coffee. She was hungry but the people in the starbucks didn't care because they were full. Usually people who have money go to Starbucks, people from the high society. Then the girl that was asking for water asked a different girl that was buying a cup of coffee to buy her a coffee as well. Since the girl came into the Starbucks people felt uncomfortable and people started looking at her very badly. The girl was drinking the coffee that the

other girl bought for her and a waitress of the Starbucks told her that: girl, can you leave this place? Literally, she pushed her outside the Starbucks and she fell and her coffee went on the floor. So you focus on that, you focus on the difference between people who have a bachelor and the people who don't and if you have a bachelor you are able to kick people who doesn't, but the fact is that you don't know why the other people doesn't have a bachelor".

During an interview with the principal at school 1 the principal said that the students face a lot of problem. "Drugs, alcohol, groups of thieves, insecurity around the school... there is not an authority image at home. Some of the students are aggressive, because of the conditions at home". At school 2 the coordinator and an English speaking teacher were interviewed. The coordinator said that the biggest problems they had was with drugs. Another problem is with the parents of the students. The coordinator said that the parents do not really care. The teacher who was interviewed said that another problem was that a lot of students from the lower socioeconomic classes *does not have goals* or dreams about continuing their education or get a good career.

Colombian people often refer to them self and their population as *lazy*. This is something they perceive as a cultural thing and when visiting Colombia this was something that the researcher heard in a lot of different places. An example from a student was that almost everybody waited till the last minute to do their homework because they were lazy. During the observations some student had moments where they did not do much work and other who went to the bathroom and did not come back to the classroom, but during exercises and presentations most of the students seemed focused.

Politics

Both students and teacher agreed that the politics around the education and the education in mathematics needs to be updated. Teacher 1 said that "the school system is very basic... we need better politics in the education". "There is not any development process to advance or overcome... The curriculum doesn't change in mathematics education". The teacher also talked about how Colombia needs to work on breaking down barriers and to be a country which help its population to overcome the social difficulties it is facing. Teacher 2 expressed a wish to increase the amount of *hours* that students have in mathematics and the teacher think that this would increase their knowledge.

Resources were never literally mentioned as a main problem but it was brought up time to time in different contexts. For example the teacher 2 wanted to use computers as a part of the methodology and teach through that media. This was never possible to do since there was a lack of resources. When teachers talked about their planning they often talked about how their plans depended on resources, among other things. The coordinator at school 3 mentioned resources as a problem as well did the principal at school 1 and another teacher at school 2.

But it is not just the classrooms and the teaching that are suffering from a lack of resources. The principal at school 1 told the interviewer that not long ago teachers were protesting because of their salaries. All the teachers that were interviewed had two jobs; one of them even owned a cab as an income. This might have affected how much time the teachers could afford to spend at the schools they worked at.

The school

Through the unstructured observations and through unstructured interviews with principals and coordinators this study could assess the *quality* of the school. School 1 scored eight point five out of twelve points on the quality questions (Appendix C), school 2 scored six points and school 3 scored ten points. School 3 was located in a pretty good neighborhood in the northeast of Bogota, school 2 was located in the northwest of Bogota and school 1 in the southeast of Bogota. Neither school 1 nor 2 were located in dangerous neighborhood but it was not safe there during the evening. According to a couple of people including the observer, school 1 was located in the worst neighborhood in comparison to the other schools. School 2 seemed to lack sufficient space for its students and no one of the schools provided *offices* for the teacher. When teachers were working at the school, the only place that were available were their classrooms.

A group perspective

During the interviews there were many discussions about different kinds of *groups*. All of the teachers said that they adapted their lessons to the group. Teacher 3 said "... we are not talking about personalized education. We are talking about a group education. So it is really hard and difficult to keep everybody at the *same level* since they have so many different rhythms..." Student 6 talked about advancing all together as a group, and about helping the ones who do not understand. Student 3 described how the students and the student's friends formed a small group during lessons and helped each other to understand and to solve exercises. Student 1 said the student always asked one of the classmates for help and in turn the student helped other classmates.

Interaction

During classes students and teachers interact with each other through different manners. This study mainly focused on communication and the established relationships between students and teachers. During the interviews the respondents told the interviewer about the relationships and how they communicated with each other. Teacher 1 said that the teacher likes to have a close relationship with the students and that the teacher talks with the students as much as possible, sometimes even about students problems. Student 1 and 2 who had teacher 1 as their teacher said that they did not have a close relationship, it was simply a *teacher-student-relationship*. They both also said that they trust the teacher but only as a teacher in mathematics and would not talk with the teacher about their own personal problems.

Teacher 2 said "I am not their friend. My relationship with them is teacher-student, no more... It is not necessary, I don't usually talk with my students..." The teacher did not talk with the students a lot except when there seemed to be a problem, for instance if students starts to perform at a lower level. Student 3 said the it was important to have a good relationship because otherwise the student would not ask for help. "It depends on the relationship between the teacher and the student, if the student has a really good relationship or confidence with the teacher the student is going to ask for help, but if the student doesn't have a good relationship or confidence with the teacher the student is going to be quiet". The student said that half of the class had a good relationship with the teacher, and the other half did not. This student though their relationship was good but preferred to talk to another teacher if there were any problems besides mathematics. Student 4 which also is a student of teacher 2 said that the student trust the teacher with mathematics but otherwise not. The student said that they had a normal relationship.

Teacher 3 emphasize the importance of having an *emotional connection* with the students, in order to create an environment where the students want the teacher to attend the class. The teachers describe this as “a positive welcoming for knowledge to happen” and said that without that connection it is so much harder to make knowledge happen. Teacher 3 said that the teacher talks with the students every day about different aspects of their lives, including their problems. Student 5 says that they always have some time to interact with the teacher before they start the lesson, which was important. According to student 5 they did not have a normal teacher-student-relationship, they were more like friends. This created a good environment for learning. This student said that the student could talk with the teacher about anything and that the student trusted the teacher. Furthermore student 5 also said “If I have a good relationship with them, then I have a good relationship with the topic or with the class itself”. Student 6 also had a good friendly relationship with teacher 3 and could talk with the teacher about a lot of things. But the student would not talk to the teacher about truly personal problem, only the ones which were school related.

The communication that occurred during classes could be observed during the observations. Since there was only one observation at school 3 there is a limited amount of data from that school. Communication between teacher and student was mainly one-way communication. The teacher had a lecture with assignments for the student to solve. Many of the teachers’ questions followed the initiation-reply-evaluation model (IRE). In school 3 the answers were occasionally a little longer and contained more of an explanation. The teacher asked if the students had any question one time during observation 5, but this was actually the only time the observer heard a teacher ask his/hers students if they had any question. The observer did not see this, neither longer explanations nor asking students, at any other school. However the communication still followed the IRE model.

The observations at school 1 took place during examinations. The observer did not notice any kind of communication between teacher and student that were more than a couple of sentences. Sometimes the teachers answer was only a nod or a yes or no. This could be because there were examinations. The students however did interact, talk and explain to each other a lot during the classes. Many times in attempt to try to help other students to understand and learn. In this class the female students seemed to talk about mathematics more than the male students.

The observations at school 2 showed that their communication much like the other schools also mainly followed the IRE model. The teacher asked question to the class and did explain the answer many times after the students had given a simple and short answer. There were barely any questions from the class and the teacher did not ask if they had any questions. The teacher usually said ready or okay. To mark the ending of an explanation.

Time

As mentioned earlier teacher 2 would like to have more hours for teaching. Time was also a category that was found when analyzing the observations. A lot of the times during lessons teacher said thing like, *hurry up* and are you *ready*. It seemed like the teachers wanted the exercises to be solved quickly. Many times when teachers wanted students work the students were not even close to finished.

Even though some teachers might have wanted more time, the observer felt like they did not use all of the time that they had at their disposal. When the observer was going to observe a class she had to wait for the teachers to arrive to the classrooms at multiple occasions. Sometimes she waited fifteen minutes after the class was supposed to start. Other times when the observer was accompanying the teacher to the classroom from the teachers' lounge the teacher waited to go to the classroom several minutes after it started, even though the teacher was ready to go long before that.

Summary

In order to summarize the result this study will try to answer the research questions.

How do the teachers plan and conduct mathematics education?

The teachers spent about 15-30 minutes to plan each class. Since they had big classes they did not have any time to individualize the education. Even though teacher said that they worked with different methods the students generally seems to think that the lessons were very similar. Lessons consist of lectures (theory) and solving exercises (practice) or examinations. The examinations does vary from written test, to assignments that they hand in, to solving problems on the board, but the assignments seemed to be pretty similar either way and did not appear to be created/constructed in a way that made the national standards reachable.

What kind of didactic methods dominates the education in mathematics?

It seems like every school had a very traditional way of teaching. Homework had a natural place in all of the classes, but in different ways. The teacher was the main source of knowledge especially for school 1 and 2 which had students that were from lower socioeconomic conditions. The teachers taught to the groups and not the individuals and with different views of how important a relationships were for learning. If the teachers could, they would change a lot of the current curriculum and standards and the results show that teachers wanted to receive a high quantity of work from the students.

How do the students describe their perception of their education in mathematics?

The results indicates that the students think that mathematics is difficult but both useful and necessary. However, they do not seem to know why it is useful or necessary and it seem to appear as an abstract and disjointed subject. It is the students responsibility to adapt to the education and not the other way around. This seemed difficult for some students as they mentioned how different methodologies were hard to get used to. If one likes mathematics and pays attention they can be successful at it, but otherwise no.

Discussion

This chapter will aim to discuss the results that was presented above and suggest further studies, but before that this study will discuss the methodology of this study.

Methodological discussion

The sample of this study was small for this kind of design. In total there were a lot of participants, however in each part of the study the amount of participants were limited. Therefore it is difficult to generalize the results, but as mentioned in the subchapter “quality requirement” this study aims to reach some transferability, which should be possible even with a smaller sample size.

The interviews

All of the interviews were conducted somewhere in the school, usually in a empty classroom or an office. There were no calm/quiet group activity room at the school, which is why the interviews were conducted in classrooms/offices. The interviews took place during school hours, this was good because it gave the participants bigger opportunities to participate. An effect of this was that there was a lot of noise during the interviews since all other students, teachers, and personnel were active in the school. Sometimes noises were very loud and actually interrupted the interview. Usually this was not an issue but sometimes the interviewer, the interpreter or the respondent got distracted because of all this noise. In the majority of the interviews the interview was interrupted by someone. For example, the teachers were curious when the students were interviewed. When the teacher were interviewed he or she sometimes needed to receive a call or message from somebody. A couple of times someone wanted to use the classroom that was used for the interviews, but fortunately the interviews never had to move. At one occasion the interview was actually interrupted and had to stop. The interpreter did not have any more time and the interview was incomplete. Luckily the interview could continue at another time and was completed.

The problem with the interrupted interviews was sometimes a consequence of the fact that the interviewer was very unexperienced and was therefore not able to estimate the actual time for the interviews. The interview had been tested before the actual interview began, but there were a lot of factors that were not taken into account. For example; the fact that the everything had to be said twice since an interpreter had to be used. Another factor was that a lot of times the respondent did not understand the question and it had to be repeated again, sometimes after the respondent had already answered. The interviewer never interrupted the respondent, that is why she chose to ask some questions again after she had already got an answer. An example is when there was a discussion about evaluations. Respondent often talked about points but the question was about abilities. The interviewer then asked again in a different way to see if the respondent would answer differently.

One thing that has been important for the interviews is the interpreter. Without the interpreter this study could not have preformed the data collection through the chosen method. Therefore the interpreter had a significant part in the data collection. This study had two different interpreters, simply because the regular interpreter’s schedule conflicted with the interviews.

The regular interpreter did not speak English as a first language. Occasionally this complicated things because the interpreter’s vocabulary was sometimes limited. This

was usually solved by a short discussion on the question in hand, which also took a lot of time during the interviews. Another problem was that the interpreter's accent was sometimes hard to understand. Since the interviews were recorded, the effect of this was that it took a lot of time and effort to understand what the interpreter was saying but it was not a crucial factor for the quality of the study. However, when listening to the interviews the researcher found that sometimes the interpreter actually asks the respondent the wrong question. Ergo not the question that the interviewer ask to the respondent. Consequently this has an impact on the study in form of validity and reliability. It is the researcher's beliefs that this did only happen a very limited amount of times, however it should not have happened at all.

The second interpreter was fluent in English and Spanish and had a great vocabulary. The only problem that occurred was that sometimes the interpreter explained what the respondent said instead of translating what the respondent said.

The interview guides (Appendix A & B) were created in advance and even though they were supposed to contain few but open questions they did not. This is also a consequence of the fact that the interviewer was unexperienced. On the other hand the questions that this study used enabled a lot of information from the respondents.

The observations

The fact that the observer was unexperienced could be both a disadvantage and a benefit. It is a benefit because the unexperienced is usually more sensitive to impressions. The observer was also an outsider to the field, which is good since an outsider can analyze the field better than a person who is used to the field. Fangen (2005) says that the observer would like to assume a position without prior knowledge. For this reason it is sometimes easier to be an unexperienced observer, because he or she does not know anything. At the same time the researcher should be able to be selective with the observations. Since observations provide a big amount of information the researcher have to select which parts that are more important to concentrate on. This is easier for a more experienced observer (Fangen, 2005). That is why the observer tried to observe the school before initiating the structured observations. Thus the observer could get acquainted with the field, which made it easier to be selective.

Every class were supposed to be observed twice in the classroom during a lesson in mathematics at each school. In school 3 that did not happen because of a change of plans. The teacher did not feel comfortable having the observer there. It was supposed to be a ordinary lesson in mathematics but as mentioned earlier the students participated in a ceremony that occurred during the lesson. The teacher said that the activity they were going to do during the short lesson would be too chaotic to observe, which was why the teacher did not feel comfortable. There were no more opportunities to observe this class, which resulted in only one observation at school 3. This affects the studys reliability in a negative way.

All the classes were in Spanish and since it would have interrupted the class to have an interpreter there to interpret, there was no interpreter. The observer spoke a little Spanish but not fluently. When the observer did not understand a scenario she asked and most of the time got a explanation she understood. The thing that could affect the validity of this study is what she did not notice because she did not speak the language. Therefore there might be factors that have not been analyzed that could have been of importance to this study. If this was the case that would have a negative effect on the

validity. This kind of method design can compensate for this sort of problem. Through the interviews the researcher can compensate for the things that might have been overlooked during the observations.

Some of the students, the teacher and the people who worked on the school spoke English. During the observations some additional interviews/conversations could be conducted. In school 2 the interview with the coordinator was conducted in the teachers' lounge. There were some teachers and students in the room at the same time. Therefore there was a risk that the coordinator did not feel that he could speak freely. Same goes for the conversation that the observer had with an English teacher shortly after the interview with the coordinator.

Examining documents

When the design of this study was chosen the researcher did not think about the large amount of time that the selection of document would consume. The researcher was handed a big amount of documents without much guidance within the documents. They all were in Spanish and since there was no recourse for translating all of the documents there was not as much examining of documents as planned. Some documents were requested by the researcher from the school and the school/teacher was going to send them but they never arrived. The researcher did however obtain some documents that could be used as a complement to the rest of the collected data.

Time and place of the study

The study was conducted in three different schools, two public school and one private school. The two public schools were on opposite sides of the city. This fact contributes to the reliability of the study in a positive way. Ideally there would have also been two private schools but there was not enough time, because of two reasons: First, this study is rather small and the second one is that the study took place at the end of the semester and when data collection was completed at the third school almost every school was closed until January. The period that the study was conducted during also affects the study. During this time a lot of the students were finishing comprehensive school. A consequence of this was for example that many students said that the grade was important because they were finishing the year soon. The time might influence the students and teachers to express things that they do not usually think about that much. However grading is a big part of the education therefore it is this researchers perspective that it was a great opportunity because the process of grading was able to be examined thoroughly.

The ages of the participants in the interviews were not always in the guidelines of the study, because sometime it was not possible. Sometimes students were sick, or occupied with tests etcetera, in some classes there were only a few in the right age-group and since students were chosen by the researcher in collaboration with the teacher sometimes the researcher had to compromise on the age.

Result discussion

The view of mathematics

During the interviews all of the teachers said that they were trying to make connection between mathematics and the students' daily life and future life. They seemed to prefer doing this through application in mathematics and through technology. Unfortunately there were no resources for teaching through technology and the observations did not

reveal any use of applications. The majority of the connections to mathematics seemed to occur through conversations when the students asked. The students saw mathematics as a necessity and help for their daily and future lives, but they did not appear to know how it could help them in either way. They usually exemplified with the use of money and/or the use of basic operations. They seemed to be limited to those examples. Solely student 5 talked about mathematics as something more in relation to the student's daily life. For example, the use of logic and how it would help you solve problems in life, both present problems and future.

The results of this study indicate that many of the students do not really know why it is important to study mathematics nor how it can help you in your life. Mathematic literacy was defined in this study as "The capacity to identify, to understand and to engage in mathematics and make well-founded judgements about the role that mathematics plays, as needed for an individual's current and future private life, occupational life, social life with peers and relatives, and life as a constructive, concerned, and reflective citizen" (OECD, 1999, p. 50). According to this definition only one of the students actually show that the student possess or have at some level reached mathematic literacy. The image of mathematics that the teachers were presenting to the students does not seem to agree with the students' image of mathematics. This indicates that the process of internalization a correct image of mathematics have not been successful. Therefore mathematics becomes isolated, disjoint and abstract to the students. All of the teachers had a very good education and did therefore probably have a good view of mathematics. In addition to that they all had a lot of years of experience and seemed to possess a profound knowledge about mathematics. A profound knowledge is necessary for this procedure (Yackel & Cobb, 1996). Hence, something else in this procedure must have been missing.

According to Yackel and Cobb (1996) it is the teacher's responsibility to try to enable their students to internalize an accurate understanding of mathematics. This happens through communication and this communication needs to occur on a daily basis (Skott et al., 2010). The level of communication varied between the teachers. Teacher 2 even said that communication with the students was not a necessity. On the other side of the spectrum teacher 3 said they had a little time each class just for interacting and for communicating. However, if this communication have a misdirected aim it might not be useful to the extent that students develop an accurate view of mathematics. This time it does not seem as the teacher's attempts have had a positive impact. If the teacher does not understand the students' mathematical thinking he might not be able to help his students in a desirable way (Skott et al., 2010).

Another thing that seems to be contributing to this abstract view of mathematics is that the different fields in mathematics are not connected. The apprehension of mathematics appears to be that mathematics consists of topics and different areas. During lessons they never seemed to process mathematics as a whole. This is a consequence of their curriculum/standards that divides mathematics into courses processing isolated parts of mathematics during a limited period (Ministerio de education nacional, 2003). A possible consequence of the separation of the different areas/topics within mathematics could be that it enhances the perception of mathematics as isolated, disjoint and abstract. This could make it more difficult to connect mathematics to everyday life, too. Since a lot of real uses of mathematics are not isolated to only one area or topic. According to Skott et al. (2010) mathematical content that do not apply in real life can not be translated into real life situations. Therefore students who have not experienced

mathematics as a field, merely as different topics, could not be expected to connect it themselves and therefore might not be able to see the use in their daily life.

The view of mathematics might affect the students' reason for studying mathematics. To pass the course seemed to be the most important reason for about four out of six students. The way students are graded and evaluated also contributes to the importance of the grade.

The teachers all grade in a similar way. They have assignments and tests during the courses and set a grade depending on which percentage the students have reached at the end of the school year. The result indicates that the quantity is more important than the quality. Even though the teacher grade each assignment/test with a score, it is still important that the students complete a high amount of work to be able to have a good grade. When teacher 1 presented the grade from "the list" the teacher summarized the student's work during the year, which consisted of points and added it up to a different point which indicated which grade the student got. This only shows a grade and not which qualities the student possesses. Furthermore, teacher 1 said this about the test that are used in the class "It is not a long test; it is something that focus on what I really need". This kind of examination form does not make the evaluation into a learning process but simply a way for the students to pass. Teacher 3 said that the teacher had a lot of different examinations because they all added something to the learning process but they all seemed to be processed in the same way, where they were summarized as points. This kind of examinations are categorized as summative assessments as opposed to formative assessments (Andrade & Cizek, 2010). As Hattie (2009) showed in his study, formative assessments could be a powerful tool in education. When summative assessments are the only kind of assessments the teachers lose an opportunity to have a positive impact on their students' academic achievements. If students and teacher both see information from assessments as valuable this could be beneficial to the students results (Andrade & Cizek, 2010). Therefore teacher in Colombia should use the information that their many examinations provide and use it to create formative assessments which in turn could help students reach the predetermined educational goals.

Learners/Learning

In mathematics it appeared to be important that you liked the subject in order to be good at it. If not, you were not going to take advantage of the education that was provided in mathematics. This implied that if you did not have an interest in mathematics you could not be good at it. This was important for both students and teacher. It was said that if you like mathematics you would be motivated in the subject.

The motivation seemed to be the actual problem because that was what liking mathematics resulted in. From this point of view the motivation has to come from within the student, which implies that the school and the teachers do not have any responsibilities to motivate student. Motivation is an important concept of education (Vallerand et al., 1992). Therefore teacher and school should aim at providing education which enhances students' motivation. One teacher tried to motivate the students but this teacher did it in a very monotonous way, which according to the teacher's student was boring. It seemed like the teacher did not succeed with creating motivation in the class, the result was possibly the other way around. The communication that was used to increase the students' motivation can be categorized according to the IRE-model. This model is believed to diminish learning potential (Skott et al., 2010). This agrees with

research that shows connections between motivation and learning and achievements (Vallerand et al., 1992) In other words this kind of communication appears to diminish the students' motivation and in turn their learning potential, which would have a negative effect on the students' results. The schools did not appear to enhance students' motivation either. None of the schools got a maximum score on the quality questions, but some schools scored higher than others. According to Post (2011), the quality of the school can affect the students' motivation. If the schools affected the student's motivation negatively, it would make it more difficult for students to try to be motivated. Furthermore, motivation can also be a good predictor of concentration (Katz, Eliot & Nevo, 2014).

There was also a lot of focus on just that, focus; mainly the ability to pay attention and not get distracted during the lessons. This task is once again placed upon the students. Actually many of the students said it was their responsibility to pay attention and be concentrated. All of the assignments that were observed were mainly monotonous and did not in the eyes of the observer inspire to pay attention. Outside of the classroom there was a lot of noise that contained a lot of variations. Audio with a lot of variations could easier interrupt the students' concentration (Groome, 2010). Since a lot of students perceived mathematics as difficult it is even more important that they are not interrupted, because assignments with a higher difficulty level require more concentration (Groome, 2010). This is a problem that originates from the school's quality. With more sound-proofing, many of these distractions could be avoided.

One student said that when you are not paying attention you will easily get lost and then you might fail the exercise. From the previous part of the discussion it was concluded that a lot of completed exercises was a very good thing. The students get an enormous amount of responsibility but not the conditions to manage these responsibilities. These situations/responsibilities can be interpreted as stressors and when stressors are presented students have to have appropriate resources to deal with them. Some students might find their own way to cope with the stressors but for those who do not it could result in stress (Holt, 2012). If students experience stress this could diminish their academic performance and eventually lead to exhaustion (Holt, 2012). In addition to this Deutsh et al. (2013) showed that "Importance of learning efforts in the eyes of the student" could only account for 8.5 percent of variance with individual efficiency as an outcome variable. This could be understood as: students did not think it mattered whether they put a lot of effort into their learning. This study has concluded that the students have many responsibilities and are expected to contribute a lot to their own education. This becomes an issue if the students do not think that their efforts matter. Then they will probably not apply themselves. Therefore, if the responsibility lies within the students there will most likely not occur any kind of development, if the student believe that his/hers efforts does not have an impact.

The result indicated that the teacher was viewed as the primary resource in the classroom. This study believes that was why it was so important to have a good teacher. Furthermore this study has some explanations to why the teacher was the primary resource. One explanation is that the teacher was in general very knowledgeable and was very good at explaining the exercises to the students. Another explanation is that the teacher was, except for the classmates, the only source of exercises, theory and new knowledge during the lessons; since the schools did not have wifi nor books the students could not attain knowledge through anything else than the teacher. Furthermore there were only a few exercises that were handed out, which the student had to do.

Therefore students could not evaluate exercises and choose exercises but rather just do what they were told and preferably fast and correct, with not much time to reflect over their own work. In addition to that, students were often told to ask the teacher rather than their classmates.

Yackel and Cobb (1996) argue that each student can achieve intellectual autonomy and it is the teachers' responsibility to guide their students through the process of reaching intellectual autonomy. When the teacher becomes a knowledge base of what is right and what is wrong and there is not any time to reflect, students might have difficulties with the development of this ability. When there is not any intellectual autonomy it is hard to evaluate their own work, but also exercises and assignments that students are not used to. That could be an explanation for the low PISA results.

Even though there usually are more important predictors than class size, this might still be important because of the fact that these classes have very limited options of resources. The teacher was always the primary resource but there were at some schools up to 35 to almost 40 students per class. A class with up to 35 to 40 students needs more than one resource. In the cases the study examined there were generally more students in each class at the public schools. This could also be an explanation to why private school education is perceived as better.

Didactics

As already mentioned many times the lessons sometime appear boring and routine. The description of the lessons that the interviewer got from the students showed that every teacher had more or less the same routine. Both students and coordinators perceived this as a problem, it was said to affect the students' motivation, creativity and results. Earlier this study argued that the students must have a lot of responsibility regarding their attention and motivation. If the teachers are performing acts that seems to lower student motivation this is very contra productive and it creates conditions where students would struggle to meet their expectations.

The results indicated that it was the students' responsibility to adapt to the teachers' methodology. Student who are not used to certain kind of sociomathematical norms/methodology can encounter problems (Yackel & Cobb, 1996). This becomes visible in this study since those student who were new had difficulties adjusting to the sociomathematical norms/methodology. According to Loewenberg Ball et al. (2008) the teacher need to have knowledge of content and students to understand the students as learners. With that knowledge teacher can be able to use flexibility to create terms in which students can learn and develop as learners. In the observed classes they were not talking about personalized education and therefore they there were no flexibility in creating different conditions for different students. This indicates that the teacher are not using the knowledge that they need in order to meet the demands of the MKT-model and therefore could not be seen as truly fulfilled teacher (Loewenberg Ball, et al., 2008).

The teacher used very little time to plan the lessons. This could explain why their lessons become routine and repetitive. The teachers' planning was reduced because of the fact that all of them had experience as teachers. They had 10, 20 and 25 years of experience. Since they had worked as teachers for a long period of time they had taught these classes before, this made it easier to plan classes and lessons.

It is not clear how much planning time the teachers' schedule allows but it seemed like the teacher chose of their own accord to spend a short amount of time on this task. Ergo they could spend more time if they wanted to. Then they would have more time to be innovative or create assignments that are new to the students and thereby break away from the routine classes. Or work on creating a more dynamic education where the student could interact with the subject in another way. There would also be more time to make the education more individual based. A teacher said that the teacher often creates different assignments for each student. Then the assignments are handed out randomly. With the extra time these assignments could be individualized to help the students develop different skills instead of consisting of monotonous exercises. In that process the teacher could meet additional demands of the MKT-model, assuming they have the knowledge of content and students (Loewenberg Ball, et al., 2008). However, there seemed to be some obstacles when teacher were working at the school. For one thing, they did not have offices, which might have contributed to the fact that they did not spend that much time planning the lessons. Second, they did not have the resources to do what they wanted to. Thirdly they all had a second job, possibly because of the low salaries. The principal at school 1 said that teachers had protest about the salaries earlier during the year. This might have had an effect on the teachers' motivation. Since it has been argued that the quality of the school can affect students' motivation (Post, 2011), it might also affect teachers' motivation.

When teachers were interviewed they said that they implemented different kinds of exercises but the students thought that the lessons were pretty much the same. This could have different explanations. One is that in interviews people sometime say what they would want to do and not what they actually do. Another explanation could be that the teachers have a more profound knowledge of mathematics and this enables them to see differences in another way than the students. Or the students may not be able to pay close attention to the small differences because of all the distractions in their environment.

Students and teachers also have different opinions on which abilities the students are developing. Some students do not even think that they are developing anything besides the ability to do basic operations and achieve a level of comprehension about different measurements. The abilities that are stated by the teacher are few and sometimes vague.

In standards (NCTM, 2000) they mention five abilities that students should have the opportunity to develop; problem solving, reasoning and proof, communication, connection and representation. Out of these five only two were mentioned by name, problem solving and communication. Applications were mentioned a lot which could stand for representation. Analytic thinking was mentioned too. In standards analytic thinking is a part of reasoning and proof. Finally they talked about developing the way the students thought but not in a specific way. Connection is however never mentioned, in fact when a teacher is asked about mixing different parts of mathematics the teacher said that it never happens. Even though teachers often referred to standards, including international once, as inspiration/sources when they talk about developing curriculums the results of this study indicates that they might not be updated in this area.

In the national standards (Ministerio de education nacional, 2003), their goals appear clear and precise, but they do not translate directly into the international standards. The five abilities actually translate directly to some way of thinking, for example numerical thinking, spatial thinking, etcetera. That could be why the teachers talk a lot about

getting to students to develop different way of thinking, the teacher could have been referring to Colombia's national standards. However the curriculum consists of a total of 27 goals within these five abilities and those were barely mentioned at all. Similarly to the international standards the teacher were somewhat close when they were talking about them, but none of the teacher mentioned it directly, which indicates that they did not have a lot of knowledge of the national standards. The observations did show that even though the teacher did not mention the national standards directly they did work with them in some sense.

To not be able to put words on what they were trying to develop in their students could be why the students experienced that the teacher never talked about abilities. To be able to simplify something the teacher needs to have clarity in their communication. According to Hattie (2009) teacher clarity is one of the most important things that teachers have to do. Since the teachers did not seem to do that, it could have a negative effect on students' achievements.

According to the results the teachers included other abilities as well; it is not just about mathematics. The students need help developing their identity and overcoming issues related with their status/strata. It is about developing other parts of the brain and its abilities while developing skills in mathematics. The teachers said they wanted to be guides to the students. This is a necessity for students in order to develop intellectual autonomy (Yackel & Cobb, 1996). This study has already shown results that indicate that the majority of students had not developed intellectual autonomy. In that sense the students had not been properly guided. When one is a guide to another person they perform as a leader and to be a leader to another a social bond is required (Svenningsson & Alvesson, 2010). Since communication play a big part in the creation of a social bond (Aspelin, 2010) and one of the teacher did not feel the need to talk with the students, that might explain if the teacher could not functions as a guide. Only Teacher 3 expressed that the teacher wanted a social bond, not just a teacher-student-relationship. One of this teacher's students was the only student who showed signs of a deeper understanding of mathematics. This student could go to the teacher with any problem. This indicates that teacher 3 has a social bond with the student and might therefore be able to serve as a guide to this student.

Culture

In the beginning of this study social conditions in Colombia were described. According to Deutsh et al. (2013) one of the variables with the most influence over students' success in education was human capital of parents. This variable measures the socioeconomic level of the students' home.

Students, teachers and personal all referred to socioeconomics as a problem. According to them students from the lower stratum of socioeconomics were in risk of getting involved with criminal activity, having parents who did not really care about their children's education and none present parents/authorities at home. They also talked about the fact that a lot of children/adolescents did not have goals or dreams about future educations or good careers. Socioeconomic status could have a big effect on the results in mathematics, studies have shown that it could be up to 0.70 (Hattie, 2009). The result of the study indicate that there were a lot of students from lower socioeconomic statuses, which according to Hattie (2009) could have a negative effect on students' achievements.

None of the students from the public schools had anyone at home that could help them with their homework. The students from the private school said that they both had someone who could help them at home. The parents of the students from the private schools also had more well paid jobs. Murillo and Martinez-Garrido (2014) have shown that students from lower socioeconomic statuses does not benefit from doing homework that consumes a lot of time. Maybe that is because they do not have anyone at home that can help them with their homework. The students that were from lower socioeconomics often talked more about their mothers. Deutsh et al. (2013) mentions a poverty trap which describe the effect of mothers with low education who transmit their knowledge and beliefs to their children. Since schools usually did not offer any help with homework this poverty trap could affect the students from the lower socioeconomic statuses and therefore affect their achievements in mathematics.

Gender inequalities were mentioned in the theoretical background and gender was also mentioned as a significant variable in mathematics education (Landguiden, 2014; Deutsh et al., 2013). This study did not detect any major differences between men and women. Even though there is not any support for gender as a predictor in this study, it should not be dismissed. Since this could depend on the methodology of this study. It might be explained by the fact that this study was conducted in an urban environment and in urban environment there are usually less inequalities between men and women (Landguiden, 2014). Therefore, gender should not be dismissed as a predictor of achievements in mathematics or as a variable which can affects students' course of education in mathematics.

Colombian people often referred to themselves as lazy. As an outsider this is a little tricky to completely comprehend since it is a part of their culture and cultures takes a lot of time to fully comprehend. The way they perceive themselves might also be a reason for why they act as they do. This is called a self-fulfilling prophecy (Holt, 2012). Therefore one reason for performing poorly in mathematics could be that they perceive them self as lazy. This is a part of their social norms and also a part of the students' sociomathematical norms. When it is rooted in the cultural norms it is hard to change but this study believe that it is necessary to try to change the sociomathematical norms in this area so that the student can not use that as a reason for not trying. It lies within the teacher's responsibility to try to change that kind of sociomathematical norm (Yackel & Cobb, 1996). Therefore teachers in Colombia might have to work more on changing that kind of behavior than in other places in the world, since this norm was perceived as a national norm.

The politic around the education is according to teachers not as updated as it should be, this includes the curriculum. For this reason it might be difficult for teachers to incorporate things they think that the students would benefit from. Especially since they all worked two jobs and did not spend a lot of time planning and preparing for the courses. After reviewing the national standards of Colombia the researcher found it to be quite updated and that it was in sync with the international standards except for the abilities concerning connection. In the Colombian standards no demands on connections were found. Another differens is that the Colombian curriculum centers more around the content of the course, whereas the international standards focus purely on the abilities regardless of the context. In the international standards the abilities are perceived as essential, whereas in the national standards the ability to do something with a certain area becomes more essential, i.e. both content and abilities are equally important. Which this study thinks results in a bigger focus on knowing/memorizing

certain content within mathematics. This can affect the students' perception of mathematics (Skott, et al., 2010). Nevertheless it still contains a variety of abilities that are essential for intellectual autonomy and mathematical literacy (Yackel & Cobb, 1996; OECD, 1999), but it is this study's opinion that it would need some flexibility and a more holistic approach to fulfill its purpose.

It was important to students and teacher to advance as a group, and to keep everybody at the same level. This seems to be another sociomathematical norm. According to MKT (Loewenberg Ball et al., 2008) teachers should have knowledge about content and students. This involves knowledge of the students as a group but also knowledge about students as individuals. Teacher 3 pointed out that they are not performing personalized education. This indicates that they do not have knowledge about the students as individuals only as a group, in that case individualized education becomes impossible. According to Löwing and Kilborn (2002) individualized education is needed to help students with their individual problems in mathematics. As a couple of students mentioned during their interviews: students are different, and some teacher will not adapt to that. They rather see the students as the same. The teacher might use different techniques from time to time but there is no individualization at all. Therefore it could be difficult for students to develop within mathematics.

As mentioned earlier communication is important. Communication occurred differently in the different classes, both between teacher and students but also in other kinds of interaction. The main findings were detected in the interaction between teacher and students. Communication is considered to be the most fundamental founding-stone in a social bond (Aspelin, 2010). Social bonds are not just essential to be a good leader or a guide; they also have an intrinsic value. Student 3 exemplified this in a good way when the student explained who asks the teacher questions and who does not. Those who had a good relationship with the teacher felt comfortable with going to the teacher for help and the others asked their classmates and hoped to get a good explanation/answer. Hence, there are many consequences if the teacher do not talk and communicate with their students in a good manner (Aspelin, 2010).

Hurry up and ready were two keywords that were found by analyzing the observations. The results indicated that quantity was many times more important than quality. This sociomathematical norm will affect the students' perception of mathematics (Skott, et al., 2010) and in fact one of the abilities that students mentioned was the ability to solve problems fast. The results indicate that their focus will be on how to do it fast. This might result in students not focusing on how to do it well when they are learning mathematics, and eventuate in not learning how to do it well and the miss out on the accompanying skills.

Post (2011) argues that the quality of the school affect mathematical achievements, when the quality decreases so does mathematical achievements. The study adapted Post's (2011) twelve questions to measure the quality through the observations. None of the schools got a maximum score, they scored six, eight point five and ten out of twelve. The private school got the highest score but the school could still increase in quality. The observations also revealed that the staff, like the principal, coordinator and teacher, thought that their school was underfunded. This was the case for all schools. According to the data from the observations and from prior research from Post's (2011) study, the schools quality might be partly responsible for students' low mathematical achievements.

Summary

Below is a summary of the three research questions, which were: How do the teachers plan and conduct mathematics education? What kind of didactic methods dominates the education in mathematics? How do the students describe their perception of their education in mathematics?

The teachers spent a short amount of time planning the lessons. They usually used planning from previous years and focused their teaching on the whole group rather than the individual. The teachers used national and international standards as inspiration but did not seem to have complete knowledge about neither the national nor the international standards. The teachers all had a small period of time in the beginning of the year where they further developed their curriculum, which was when the planning started. In Colombia all the courses in mathematics are divided into different areas within mathematics. This affects the teachers' planning and the results indicate that this eventuates in that mathematics becoming disjoint. This also limits the exercises and assignments that teachers use during their lessons. When the national standards were compared to the international standards it showed that the international standards have a stronger focus on the abilities/competencies that the students should develop, whilst the national standards focused on both competencies and on mathematical content. This probably affected the teachers when they planned their classes. Thus they might have put more emphasis on the content then they would have otherwise. It can be possible that the teacher tried to plan according to the standards, but since they seemed to lack knowledge about the standards it might be difficult to create new exercises and lessons. This might be why they leaned on their previous planning, since they had used it before and probably felt comfortable with it. This seemed to result in that the students perceived the lessons as routine. The teachers planned for their students to do a lot of exercises. Therefore it seemed as many of the teachers focused on quantity, which in turn could affect the students' perception of mathematics. The teachers all conducted themselves in a similar manner, including the communication they used. Their communication was categorized by this study according to the IRE-model. This kind of communication combined with the routine lessons would most likely affect the students' motivation negatively, which in turn would in all probability affect the students' result negatively. It is however also likely that the teachers' planning and manner were affected by their motivation. The schools' quality have been shown to affect students' motivation, this study have concluded that it is also possible that the schools' quality could affect the teachers motivation, which could also be a reason to why the teacher chose to use their previous planning.

To answer the question about what kind of didactic methods that is most common, this acquired a more holistic view. Initially by examining the culture within which the teachers work. The culture does not seem to believe that an individualized education is possible. This seemed to have multiple effects on the education. One effect was that in the classroom at hand where the teacher targeted their teaching to the whole group. Secondly the assignments were never designed according to the students as individuals. Thirdly, the feedback were merely on an individual basis when the student asked the teacher a direct question and even then it was not certain that the student could comprehend the answer. Otherwise students did not seem to receive feedback on an individual level. The lack of individualized education created terms which indicated that the students had to adapt accordingly to the education and to the teachers and not the other way around. In addition to examine the culture contextual circumstances also have to be examined, for example the quality of the school. Since there were no

individualized education and the quality of the schools was low, these factors combined could be the reason why the school did not offer additional help for students how wanted or needed it. Additional help was only offered to those who were failing. Another factor that effected the education was the national standards. As the planning and conducting of the classes were affected by the standards, so were the didactic choices when the teachers decided which methods to use to create good conditions for the students. The didactic choice of quality or quantity is one which also goes hand in hand with planning and conducting the lessons.

The students gave many descriptions about how they experienced and perceived mathematics. Which mainly centered on that it was useful, but they did not know why it was useful. This might be explained by the communication that occurred between teacher and students. Since the teachers had difficulties with explaining to the interviewer what they were trying to develop within the students, they may have even greater difficulties explaining it to the students. Since the abilities which students develop in mathematics are a major reason to why it can be useful in real life this could affect the students' perception of mathematics. Furthermore, the students seemed to think that mathematics was disjoint. This study believes that this perception of mathematics was an effect of the national standards which works as guidelines when teacher create their curriculums. This might also contribute to why students do not know why mathematics is useful, since a lot of real life mathematics consists of combination between different areas of mathematics. One of the other major perceptions was that if you liked mathematics, you would be motivated and therefore be more successful in the subject. The origin of the motivation was from within. It was therefore the students' responsibility to try to like mathematics or create the motivation on their own. The students also had other responsibilities that were out of their direct control, for example adapting to the school and teachers with different methodologies. This study believes that these responsibilities may put students under a great deal of pressure. This could lead to stress, which in turn could affect the students' results negatively.

This study was based on the PISA-results. Above are a lot of factors which could have an effect on the students' achievements in mathematics and in turn on the PISA-results. However, one factor in particular may affect the result more direct, namely that neither the standards nor the curriculums have any requirement of that the different areas in mathematics should be connected. Since the PISA-test mainly focus on real life situation this could have a great impact on the results of the PISA-tests. Furthermore the PISA-tests also acquires students to use different kind of abilities and since the teacher seemed to lack knowledge of standards they may face difficulties with creating assignments that would foster all of the different aspects that are mentioned in the standards in terms of competencies. The assignments that were examined did only concern a small part of the national standards and the international standards. Because of this fact the students can not be expected to develop the competencies that are excluded from the assignments that were processed during lessons, especially since all of the classes in this study perceive the teacher as the primary resource and in extent the only true source of knowledge.

Further research

This study is limited to three classes/schools in a urban area in tenth and eleventh grade from the middle and lower part of socioeconomic statuses. Therefore this study can only present a view of that specific category. To get a more general view of the mathematics education in Colombia further research is needed. Since this study only observed and

interviewed students in grade tenth and eleventh further research are needed in lower grades in Colombia. As this study took place in a urban environment further research should include rural areas to acquire a more holistic view of mathematics education in Colombia.

The sample of this study is small and to reach a higher level of reliability a greater sample is required. The validity also needs to increase in order to strengthen the result in further studies. Since no real pilot studies were performed the validity could increase a lot if further studies have the opportunity to conduct pilot studies before the actual study proceed.

One of the findings in this study was that in the examined classes there was no individualization of the education. Prior research has shown that individualized education in mathematics is preferable and students can benefit a lot from it. The teachers' focus were on the group and it had never crossed their mind that they could create terms in which they could use individualized education. The fact that the teachers had not though about introducing this into their classes is interesting and therefore further research is needed on why teachers do not consider this when they are creating their lessons.

Formative assessments can be viewed as a part of individualized education because it is adapted to each student. Among others Hattie (2009) has shown that formative assessments can be a very powerful tool within education. Therefore it would be interesting to see why this is not a part of Colombian education in mathematics.

Another interesting phenomenon is why the Colombian standards do not include connection within mathematics as content or ability. Amongst others Loewenberg Ball, et al. (2008) mentions horizon content knowledge – which is knowledge about how the different areas in mathematics is connected – as a main knowledge for teachers. It is also clearly stated in the international standards as a competence. Still this does not seem to occur in Colombian education in mathematics, which this study believes is a field for further research.

References

- Andrade, H.L. & Cizek, G.J. (red.) (2010). *Handbook of formative assessment*. New York: Routledge.
- Aspelin, J. (2010). *Sociala relationer och pedagogiskt ansvar*. Malmö: Gleerups.
- Denscombe M. (2009). *Forskningshandboken – för småskaliga forskningsprojekt inom samhällsvetenskaperna*. Lund: Studentlitteratur.
- Denscombe, M. (2004). *Forskningens grundregler: samhällsforskarens handbok i tio punkter*. Lund: Studentlitteratur.
- Deutsh, J., Dumas, A. & Silber, J. (2013). Estimating an educational production function for five countries of Latin America on the basis of the PISA data. *Economics of education review* Vol. 36, p. 245 – 262.
- Fangen, K. (2005). *Deltagande observation*. Malmö: Liber ekonomi.
- Gapminder (2008). Retrieved from: <http://www.gapminder.org/> 2015-09-28.
- Globalis (2015). Retrieved from: <http://www.globalis.se/Organisationer/Foerentationerna-FN> 2015-09-17.
- Groome, D. (2010). *Kognitiv psykologi: processer och störningar*. (1. uppl.) Lund: Studentlitteratur.
- Hattie, J. (2009). *Visible learning: a synthesis of over 800 meta-analyses relating to achievement*. London: Routledge.
- Holt, N. (2012). *Psychology: the science of mind and behaviour*. (2., [rev., European] ed.) Maidenhead: McGraw-Hill Higher Education.
- Katz, I., Eilot, K. & Nevo, N. (2014). "I'll do it later": Type of motivation, self-efficacy and homework procrastination. *Motivation and Emotion* vol 38 (1), p. 111 – 199.
- Landguiden (2012). Retrieved from: <http://www.landguiden.se.proxy.lnu.se/Lander/Sydamerika/Colombia> 2015-09-27.
- Landguiden (2014). Retrieved from: <http://www.landguiden.se.proxy.lnu.se/Lander/Sydamerika/Colombia/Sociala-Forhallanden> 2015-09-27.
- Landguiden (2014). Retrieved from: <http://www.landguiden.se.proxy.lnu.se/Lander/Sydamerika/Colombia/Utbildning> 2015-09-27.
- Loewenberg Ball, D., Thames, M. H. & Phelps, G. (2008). Content Knowledge for Teaching: What Makes It Special? *Journal of Teacher Education* Vol. 59 (5), p. 389 – 407.

Löwing, M. & Kilborn, W. (2002). *Baskunskaper i matematik: för skola, hem och samhälle*. Lund: Studentlitteratur.

Millenniemalet (2015). Retrieved from: <http://www.millenniemalet.nu/alla-barn-ska-ga-i-skolan/> 2015-09-01.

Ministerio de education nacional (2003). La revolución educativa – estándares básicos de matematicas y lenguaje educación básica y media. [pdf] Retrieved from: http://www.colombiaaprende.edu.co/html/mediateca/1607/articles-70799_archivo.pdf 2015-10-01.

Murillo, J. F. & Martinez-Garrido, C. (2014). Homework and primary-school students' academic achievement in Latin America. *International Review of Education*. Vol. 60 (5), p. 661 – 681.

Nationalencyklopedin (1990). Retrieved from: <http://www.ne.se/uppslagsverk/encyklopedi/lång/norm> 2015-10-08.

National Council of Teachers of Mathematics (NCTM) (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (NCTM) (2015). Retrieved from: <http://www.nctm.org/About/> 2015-05-09.

Organization for Economic Co-operation and Development (OECD) (2014). *PISA 2012 Technical Report* [PDF] Retrieved from: <http://www.oecd.org/pisa/pisaproducts/PISA-2012-technical-report-final.pdf> 2015-09-02.

Organization for Economic Co-operation and Development (OECD) (1999). *Measuring Student Knowledge and Skills: A new Framework for Assessment*. France, Paris: OECD.

Peaceworks (2015). Retrieved from: <http://peaceworks.se/nyheter/fred-och-forsoning-inom-rackhall/> 2015-01-06.

Post, D. (2011). Primary school student employment and academic achievement in Chile, Colombia, Ecuador and Peru. *International Labour Review*, Vol. 150, p. 255 – 278.

Skott, J., Jess, K., Hansen, H. C. & Lundin, S. (2010). *Matematik för lärare*. Polen: Gleerups.

Svenningsson, S. & Alvesson, M. (2010). *Ledarskap*. Liber: Malmö.

Trost, J. (2005). *Kvalitativa intervjuer*. Lund: Studentlitteratur.

UNICEF (2016). Retrieved from: <https://unicef.se/fakta/utbildning> 2015-04-15.

UNESCO-UNEVOC (2014). Retrieved from:
http://www.unevoc.unesco.org/go.php?q=World+TVET+Database&ct=COL#par0_2
2015-10-08.

United Nation (UN) (2000). Retrieved from:
<http://www.un.org/millenniumgoals/education.shtml> 2015-09-17.

United Nation (UN) (2015). Retrieved from:
<http://www.un.org/sustainabledevelopment/education/> 2016-01-15.

Vallerand, R. J., Pelletier, L. G., Blais, M. R., Brière, N. M., Senécal, C & Valliérer, E. F. (1992). The academic motivation scale: a measure of intrinsic, extrinsic and amotivation in education. *Educational and Psychological Measurement*, vol. 52, p. 1003 – 1016.

Vetenskapsrådet (2011). *God forskningssed*. Stockholm: Hermerén, G. [pdf] Retrieved from: www.cm.se/webbshop_vr/pdf/2011_01.pdf 2015-03-03.

Yackel, E. & Cobb, P. (1996). Sociomathematical Norms, Argumentation, and Autonomy in Mathematics. *Jurnal for Research in Mathematics Education*, Vol 27(4), p. 458 – 477.

Appendix

Appendix A Interview students.

Opening questions

- Demographic questions

Perception of mathematics – Motivation of mathematics

- Describe what mathematics is to you?
- Which aspect of mathematics is the most interesting/fun?
- What is the best way for you to learn new things in mathematics?

Sociomathematical norms

- Describe an ordinary lesson in mathematics.
- Do you know how your teacher evaluates your work?
- What abilities/qualities are you developing in mathematics class?
- If you do not understand something in mathematics, what happens then?
- What are your responsibilities as a student?

Social relation with the teacher

- Describe your social relationship with your teacher.
- What should a teacher be like?

Life outside school

- Does your teacher set homework?
- Where are you when you do your homework?
- Do you think that homework is useful?

The use of mathematics

- How can you use mathematics outside of the classroom?
- Why do you think you have to learn mathematics?

Finishing questions

- Is there anything you would like to add?
- Why do you think that Colombian students had low results at the Pisa-tests?

Appendix B Interview teachers

Opening questions

- Demographic questions
- How come you chose to be a teacher?

An ordinary lesson in mathematics

- Can you describe an ordinary lesson with this class?
- How much time does it take to plan/prepare for a lesson?
- Do you encounter problems when you teach? What kind of problems?

Question about the observed lessons

Different questions depending on the previous observations.

Sociomathematical norms

- Describe what mathematics is to you?
- How do you think your students are using mathematics in their everyday life?
- What do you think is the main reason for students to learn mathematics?
- On what basis do you evaluate/ grade your students work?
- What kind of abilities are you trying to develop in your students?
- What do you think is the best way for your students to learn new things in mathematics?
- Describe how a perfect lesson should be.
- Do you often set homework in your class?
- Do you plan and teach differently depending on what area you teach?
- Do you involve your students' thought when you are teaching?
- What do you think the students focus is on when they are answering a question/solving a problem?

Teacher's responsibilities

- How would you describe mathematics place/position in relationship with Colombia?
- What are your thoughts on your curriculum?
- What are your responsibilities as a teacher?

Social relation

- What do you think about the teacher-student social relationship in your class?

Communication

- How often do you talk with each student?
- What kind of conversations do you have with your students?

Finishing questions

- Is there anything you would like to add?
- Why do you think that Colombian students had low results at the PISA-test?

Appendix C Unstructured interviews

Quality of school

The quality of the schools was measured by twelve questions adapted from Post (2011, p. 267). These are yes or no question and answers if the school has the following items:

- Electricity,
- Running water,
- Indoor plumbing,
- A telephone,
- Sufficient bathrooms,
- A kitchen,
- A lunchroom,
- A library,
- Nutritional feeding programs,
- Medical services,
- Transportation, and
- Free textbooks

Additional questions about:

- The structured of the school and its education. For example; amount of students, resources, hours students attend classes, etcetera
- After school activities
- Programs to help students advance in their subject beyond their regular classes
- Safety and well-being
- Problems that students encounter in their daily life
- Pedagogical guide lines
- Socioeconomic stratum of the students
- Working conditions for teachers
- Teachers

Appendix D Observation guide

Guidelines

I do not know anything. That is why I have to take notice of everything and little by little as I learn more about the field, I can be more selective. I have to be critical toward my analysis and my own interpretation throughout the entire process. The ideal approach is to always question the things that appear obvious. It is acceptable to find knowledge through the things I see/observe, I do not have to lean on previous knowledge

Framework:

Communication – what kind of language is used? Are there patterns?

Relationships – how does the participants interact with each other?

Social norms – what seems normative in the classroom?

Sociomathematical norms – what is the students' reaction to mathematics and how do they appear to perceive it?

Homework – what is its position?

Teachers' knowledge (knowledge from the book, from the head or by heart)

Teachers' awareness about development of abilities

Body language – could it reflect the atmosphere?

Themes:

- Socialization – isolation
- Atmosphere: positive – negative
- Discussion (about mathematics)
- Focused – unfocused
- In what way do students answer questions during class: short answers – long answers
- In what way are questions asked by the teacher. Leading questions – questions that require critical thinking
- Discipline
- What is mathematics?

Appendix E Accompanying letter

Hello!

My name is Rebecka Rundquist and I am a student at Linnaeus University in Växjö, Sweden, where I study to be a teacher in mathematics and psychology. I am conducting a study in which I am examining the education in mathematics in Colombia. The purpose of this study is to describe how education in mathematics in Colombia can operate and which patterns are common. The study mainly consists of interviews and observations. Therefore I ask if you are interested in participating in this study. The interview is expected to go on for a maximum of 60 minutes.

To participate in this study is completely optional. Hence if you at any time during the study feel like discontinuing, you can do so without providing reason or cause. Your answers will be anonymous and the data from this interview will only be used for this study.

If you have any questions concerning the interview, the study or are interested in the result you can contact me at: xxxxxxx@student.lnu.se or at +57319xxxxxxx

Thank you for your participation.

Estimado Participante,

Mi nombre es Rebecka Rundquist y soy estudiante en la Universidad de Linnaeus en Växjö, Suecia, donde estoy estudiando para ser profesora de matemáticas y psicología. Actualmente estoy realizando una investigación relacionada con la enseñanza de las matemáticas en Colombia. El objetivo de esta investigación es el de describir la metodología utilizada en la enseñanza de las matemáticas en el país e identificar patrones comunes. La investigación se basará principalmente en entrevistas y observaciones de clases. Cada entrevista tendrá una duración de aproximadamente 30 minutos.

Si usted desea participar en la entrevista, debe diligenciar el formulario localizado al final de esta carta. La participación en esta investigación es completamente opcional, por lo tanto, si en algún momento desea dejar de participar en la entrevista, lo puede hacer sin necesidad de ofrecer motivos o razones. Sus respuestas serán totalmente anónimas y la información recolectada solo será utilizada para lograr los objetivos del presente estudio.

De tener alguna duda o pregunta, o de estar interesado en recibir los resultados de la investigación, puede contactarme al correo electrónico xxxxxxx@student.lnu.se o al número de celular: + 57 319xxxxxxx.

Gracias por su participación.

Yo (nombre)_____ participo en la investigación liderada por Rebecka Rundquist de forma voluntaria. Entiendo que puedo cesar mi participación en cualquier momento sin necesidad de ofrecer motivos o razones y que todas mis respuestas serán manejadas de manera confidencial. Se me ha proporcionado una explicación del proyecto e información de contacto de la entrevistadora en caso de tener preguntas en un futuro.

Firma: _____

Fecha: _____

Appendix F National standards

<i>i.</i> Pensamiento numérico y sistemas numéricos	<i>ii.</i> Pensamiento espacial y sistemas geométricos	<i>iii.</i> Pensamiento métrico y sistemas de medidas	<i>iv.</i> Pensamiento aleatorio y sistemas de datos	<i>v.</i> Pensamiento variacional y sistemas algebraicos y analíticos
1. Analizar representaciones decimales de los números reales para diferenciar entre racionales e irracionales.	1. Identificar las propiedades de las curvas en los bordes obtenidos mediante cortes (longitudinal y transversal) en un cono y un cilindro.	1. Diseñar estrategias para abordar situaciones de medición que requieran grados de precisión específicos.	1. Comparar estudios provenientes de medios de comunicación.	1. Utilizar las técnicas de aproximación en procesos infinitos numéricos.
2. Reconocer la densidad e incompletitud de los números racionales a través de métodos numéricos, geométricos y algebraicos.	2. Identificar características de localización de objetos geométricos en sistemas de representación cartesiana y otros (polares, esféricos,...).	2. Resolver y formular problemas que involucran mediciones derivadas para atributos tales como velocidad y densidad.	2. Justificar inferencias provenientes de los medios o de estudios diseñados en el ámbito escolar.	2. Interpretar la noción de derivada como razón de cambio y desarrolla métodos para hallar la derivada de funciones básicas.
3. Comparar y contrastar las propiedades de los números (enteros, racionales, reales) sus relaciones y operaciones (sistemas numéricos).	3. Resolver problemas en los que se usen las propiedades geométricas de figuras cónicas de manera algebraica.	3. Justificar resultados obtenidos mediante procesos de aproximación sucesiva, rangos de variación y límites en situaciones de medición.	3. Diseñar experimentos aleatorios (de las ciencias físicas, naturales o sociales) para estudiar un problema o pregunta.	3. Analizar las relaciones y propiedades entre las expresiones algebraicas y las gráficas de funciones polinómicas y racionales.
4. Utilizar argumentos de la teoría de números para justificar relaciones que involucran números naturales.	4. Usar argumentos geométricos para resolver y formular problemas en contextos matemáticos y en otras		4. Describir tendencias que se observan en conjuntos de variables relacionadas.	4. Modelar situaciones de variación periódica con funciones trigonométricas.

	ciencias.			
5. Establecer relaciones y diferencias entre diferentes notaciones de números reales para decidir sobre su uso en una situación dada.	5. Describir y modelar fenómenos periódicos del mundo real usando relaciones y funciones trigonométricas.		5. Interpretar nociones básicas relacionadas con el manejo de información (como población, muestra, variable, estadígrafo y parámetro).	
	6. Reconocer y describir curvas o lugares geométricos.		6. Usar comprensivamente algunas medidas de centralización, localización, dispersión y correlación (percentiles, cuartiles, centralidad, distancia, rango, varianza, covarianza y normalidad).	
			7. Interpretar conceptos de probabilidad condicional e independencia de eventos.	
			8. Resolver y formular problemas usando conceptos básicos de conteo y probabilidad (combinaciones, permutaciones, espacio muestral, muestreo aleatorio, muestreo con reemplazamiento).	

			o).	
			9. Proponer inferencias a partir del estudio de muestras probabilísticas.	

Adapted from Ministerio de education nacional (2003, p. 20 – 21).



Linnéuniversitetet
Kalmar Växjö

Fakulteten för teknik
391 82 Kalmar | 351 95 Växjö
Tel 0772-28 80 00
teknik@lnu.se
Lnu.se/fakulteten-for-teknik